Colored Simultaneous Geometric Embedding

1 Question

Let $G_1, \ldots, G_h$ be $h > 1$ outerplanar graphs having the same number n of vertices. Suppose that the vertices of each graph are colored using $1 < k < n$ distinct colors that we denote by the integer numbers $1, \ldots, k$, and suppose that, for each color $i \in \{1, \ldots, k\}$, the number of vertices colored i is the same in $G_1, \ldots G_h$. We ask whether there exists a set S of n points such that:

- Each G_i ($i = 1, \ldots, h$) admits a planar straight-line drawing with the vertices mapped to the points of S.
- For each point $p \in S$, the vertices of $G_1, \ldots G_h$ mapped to p have the same color.

2 Observations

For $k = 1$ the problem above described coincides with the classical simultaneous geometric embedding problem of outerplanar graphs without mapping, and it has been positively answered [1]. For $k = n$ the problem coincides with the classical simultaneous geometric embedding problem of outerplanar graphs with mapping, and it has been negatively answered [1]. The special case of three paths and $k = 2$ could be a good starting point.

References

[1] P. Brass, E. Cenek, A. Duncan, A. Efrat, C. Erten, D. Ismailescu, S. Kobourov, A. Lubiw, and J. Mitchell. On simultaneous planar graph embeddings. In *Proc. 8th Workshop on Algorithms and Data Structures (WADS 2003)*, Lecture Notes Comput. Sci., pages 243–255, 2003.