LMST Drawability Problem

1 Question

Let S be a set of points in the plane. The *unit distance graph* of S, denoted as $U(S)$, is the graph obtained by connecting those points that have relative distance less than or equal to one. For a point $u \in S$, denote by $N(u)$ the set of points that are adjacent to u in $U(S)$. Also, denote by $MST(N(u))$ the minimum spanning tree of the subgraph of $U(S)$ induced by the points in $N(u)$.

A *Local minimum spanning tree* of $U(S)$ is a specific proximity graph that can be defined in two different ways:

- $LMST(U(S))^+ = \{(u, v) \in U(S) : (u, v) \in MST(N(u)) \cup MST(N(v))\}$
- $LMST(U(S))^- = \{(u, v) \in U(S) : (u, v) \in MST(N(u)) \cap MST(N(v))\}$

Of course, we have that $LMST(U(S))^- \subseteq LMST(U(S))^+$.

Let G be a planar graph. We ask whether it is possible to map the vertices of G to a set S of points in the plane such that either $LMST^-(U(S))$ is isomorphic to G or $LMST^+(U(S))$ is isomorphic to G.

2 Observations

Local minimum spanning trees have been introduced in the literature of ad-hoc wireless networks [1]. It is proved that, if the unit distance graph is connected, then the local minimum spanning trees are connected and planar. Also, the degree of every vertex is at most 6 and there are no 3-cycles.

References

[1] N. Li, J. C. Hou, and L. Sha. Design and analysis of an mst-based topology control algorithm. In *IEEE INFOCOM 2003*, 2003.