# Point-set Embeddability

# General Problem

**G**

1
2
4
3
5
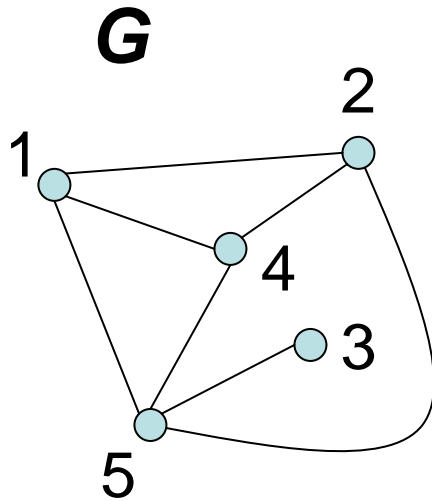
*G* is point-set embeddable on *P*
(*G* admits a planar drawing on *P*)

4
2
1
3
5

**P**
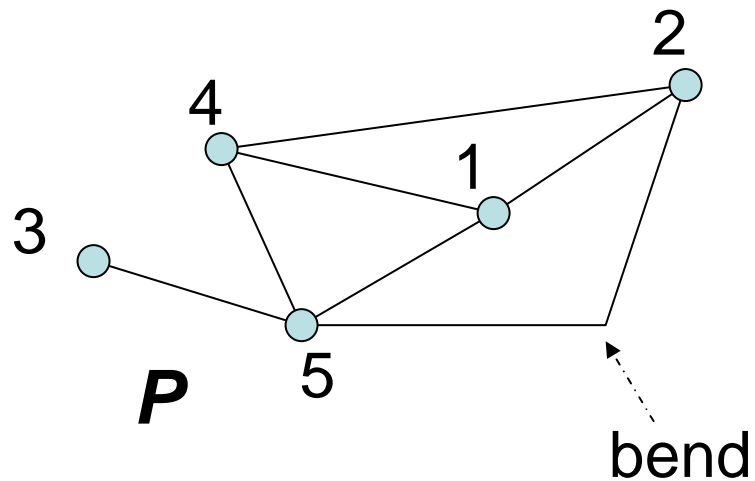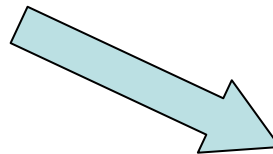
bend

# Preliminaries

• A planar graph with *n* vertices is point-set embeddable with <u>no bend</u> per edge on <u>any</u> set of *n* points in general position iff it is <u>outerplanar</u> (Gritzman, Mohar, Pach, Pollack, AMM'91)

• A planar graph with *n* vertices is point-set embeddable with at most <u>two bends</u> per edge on <u>any</u> set of *n* points (Kaufmann and Wiese, JGAA' 02)

• Two bends are necessary for non-hamiltonian graphs and sets of collinear points
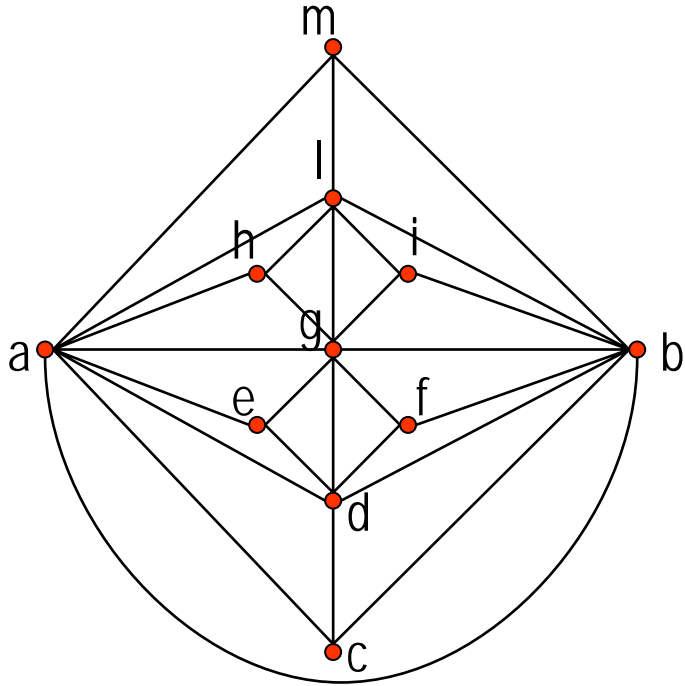
# Open Question

Let *G* be a **planar graph** with *n* vertices and let *P* be any set of *n* non-collinear points. Does G admit a planar drawing on *P* with at most **one bend per edge**?

**Observation**: if *C* is a convex curve and if *G* is a planar graph, *G* admits a planar drawing with at most one bend per edge on *C* (Di Giacomo, D., Liotta, Wismath, CGTA'05)
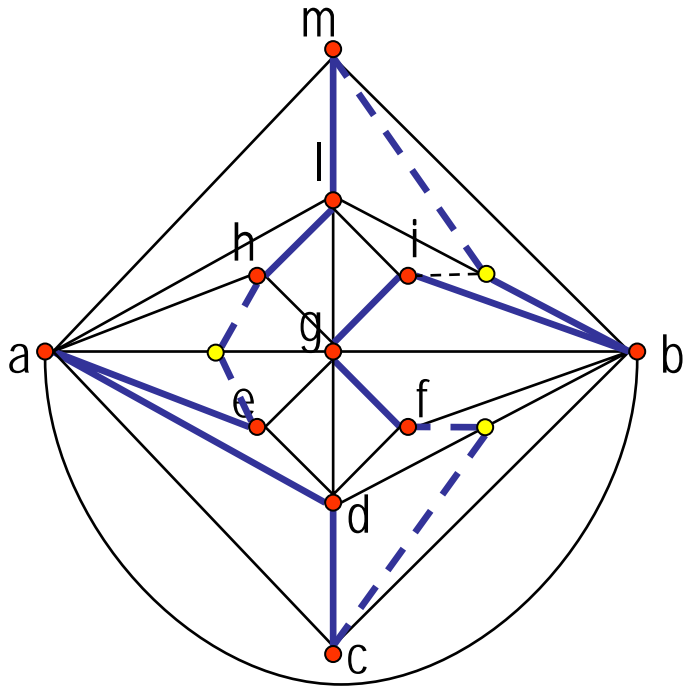
# Two bends always suffice
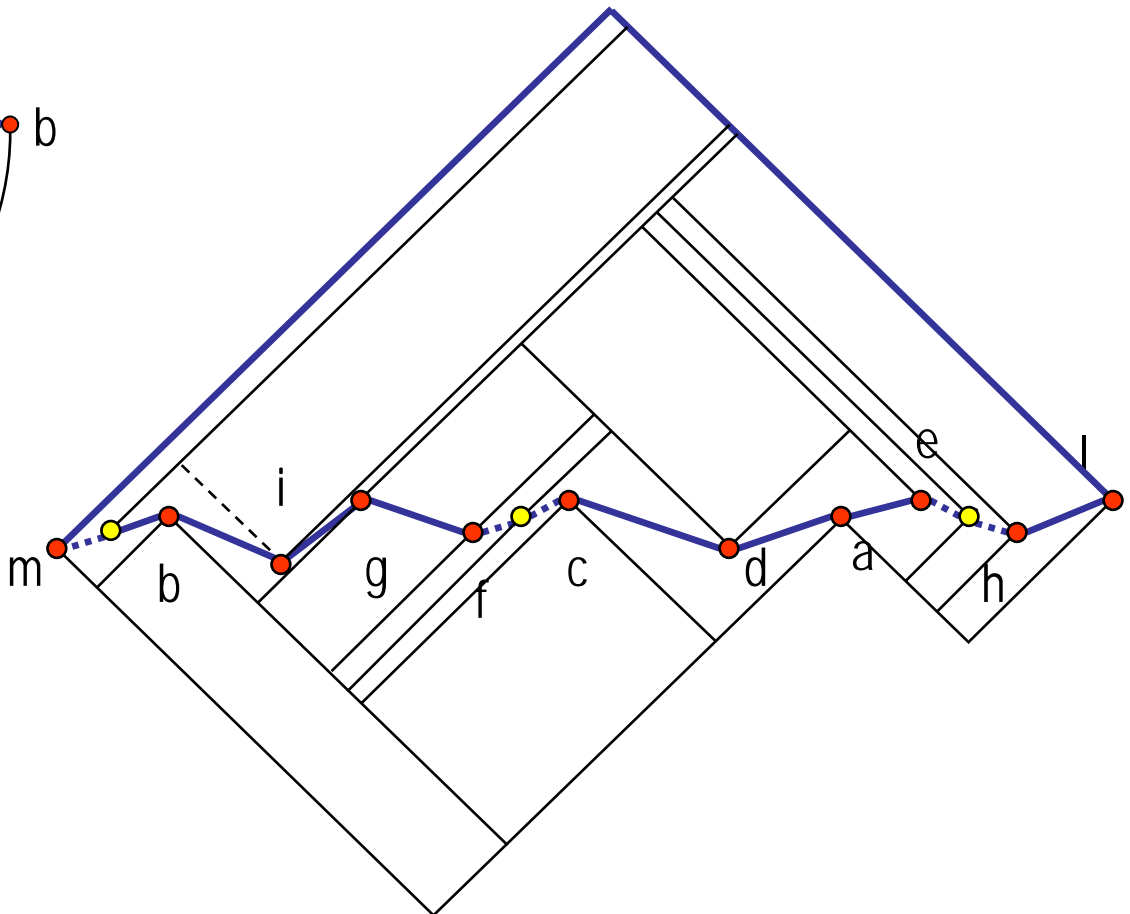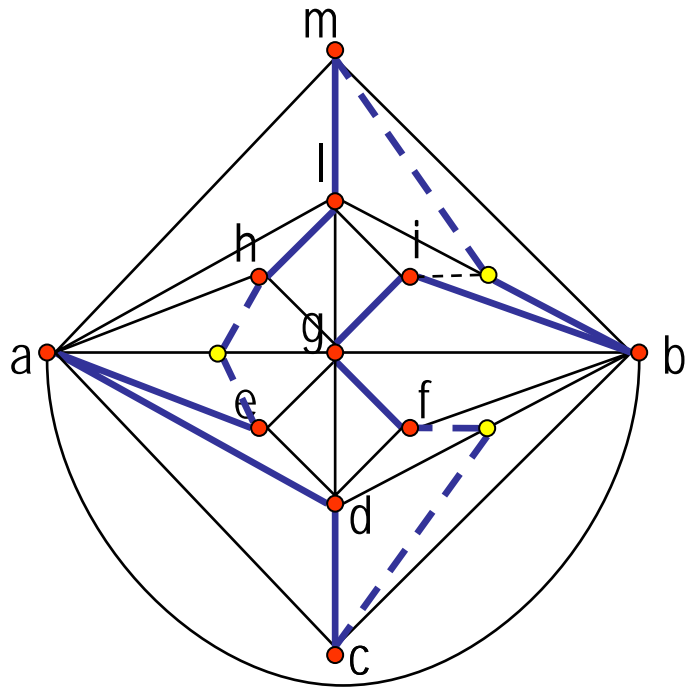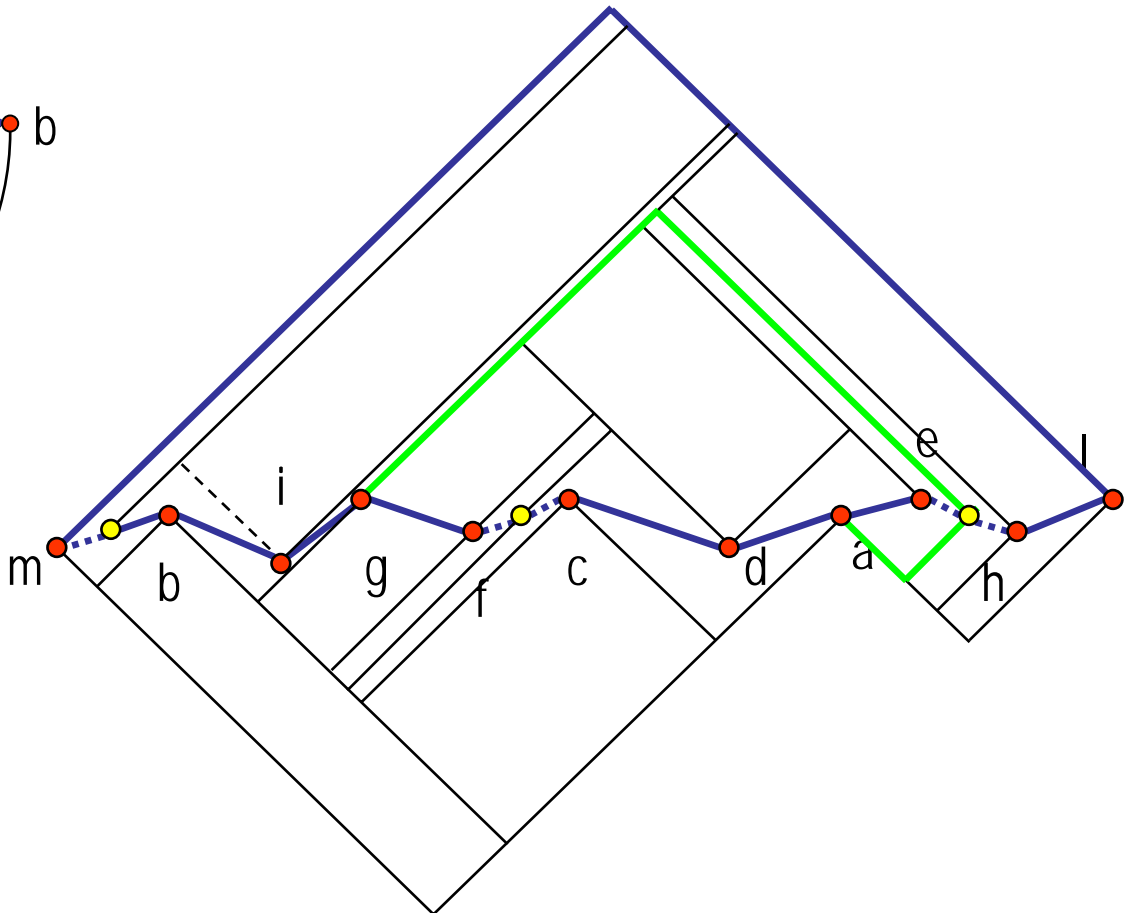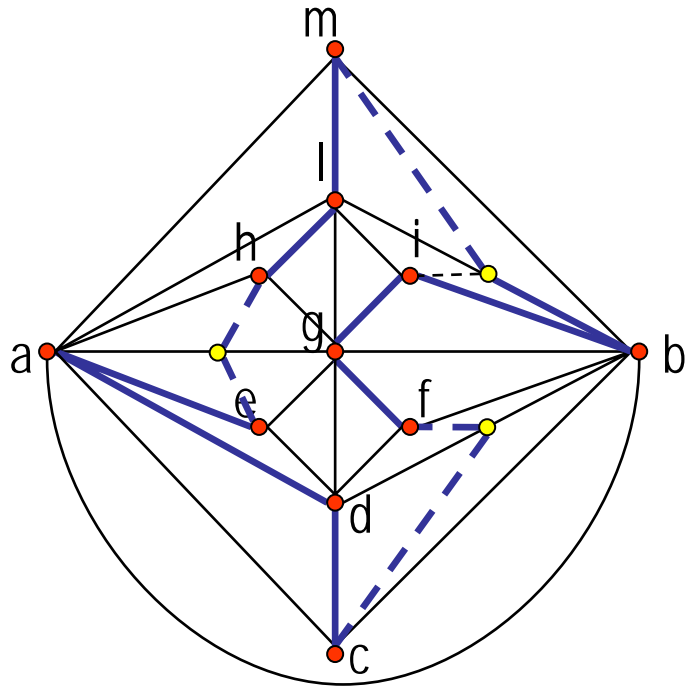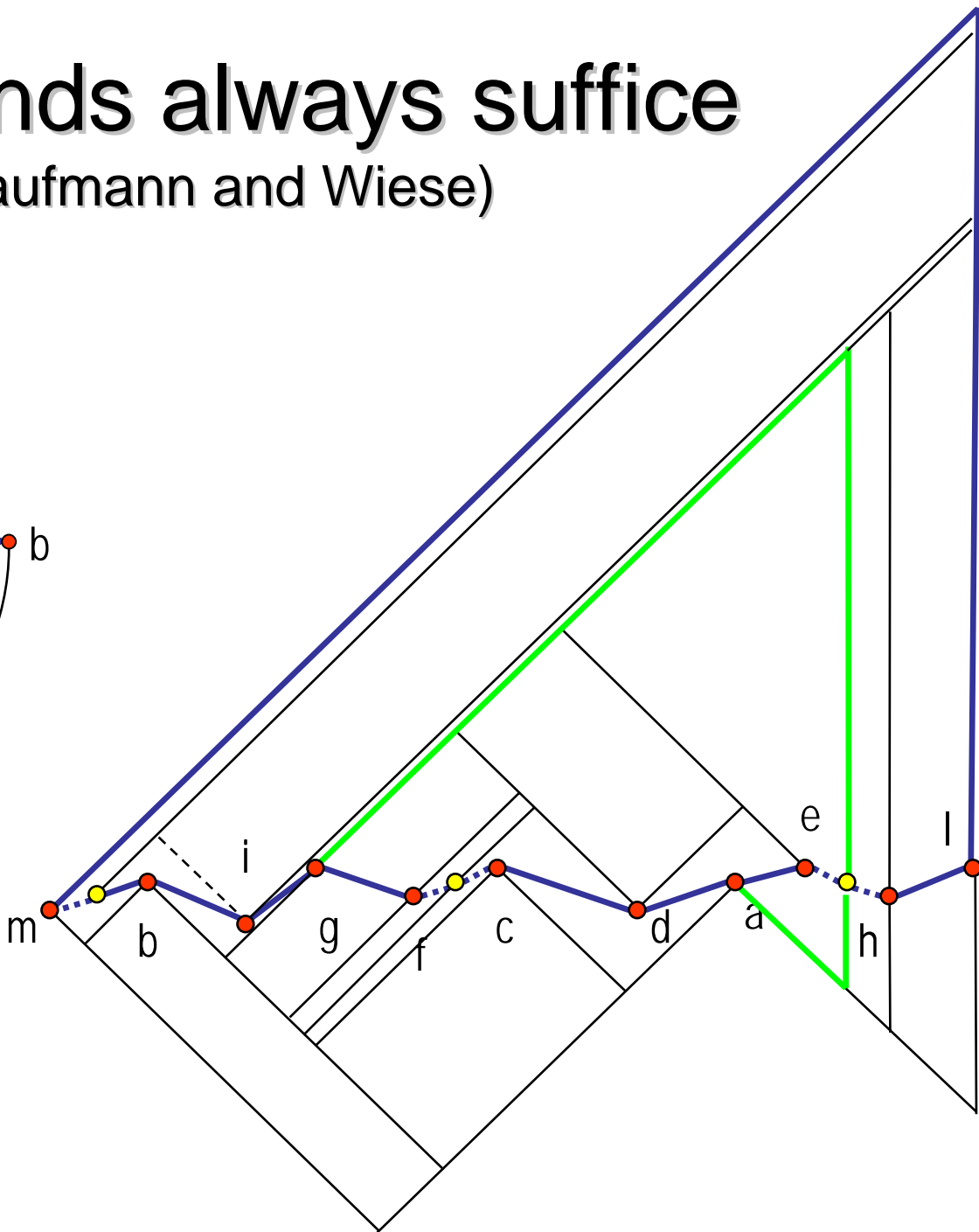## (Kaufmann and Wiese)

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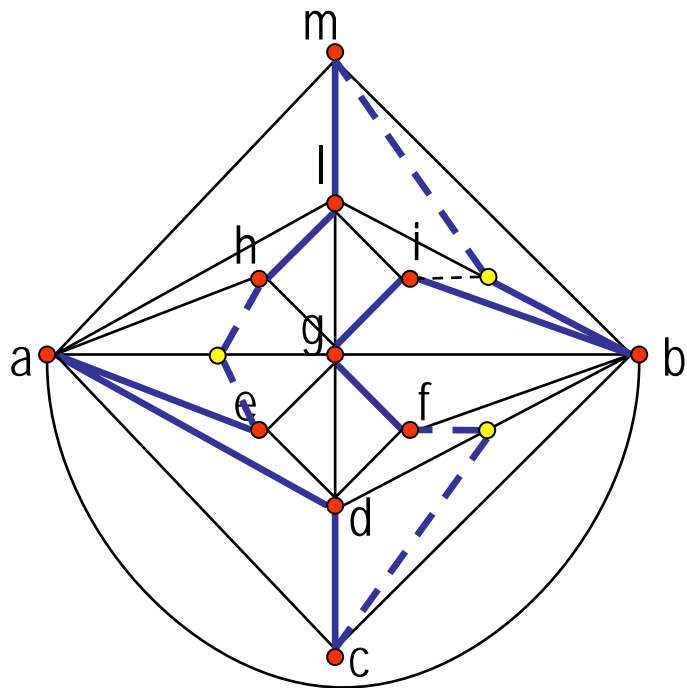# Two bends always suffice
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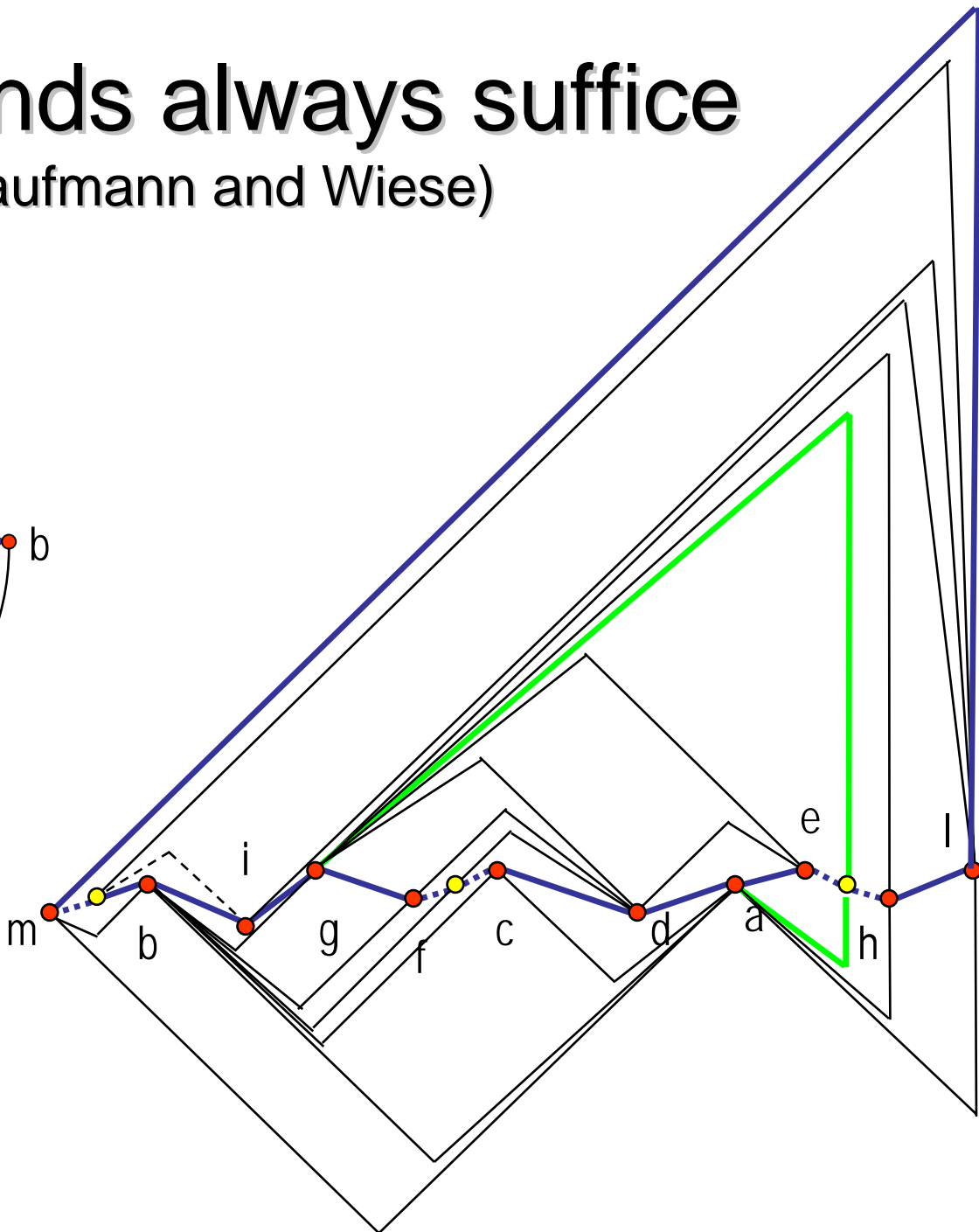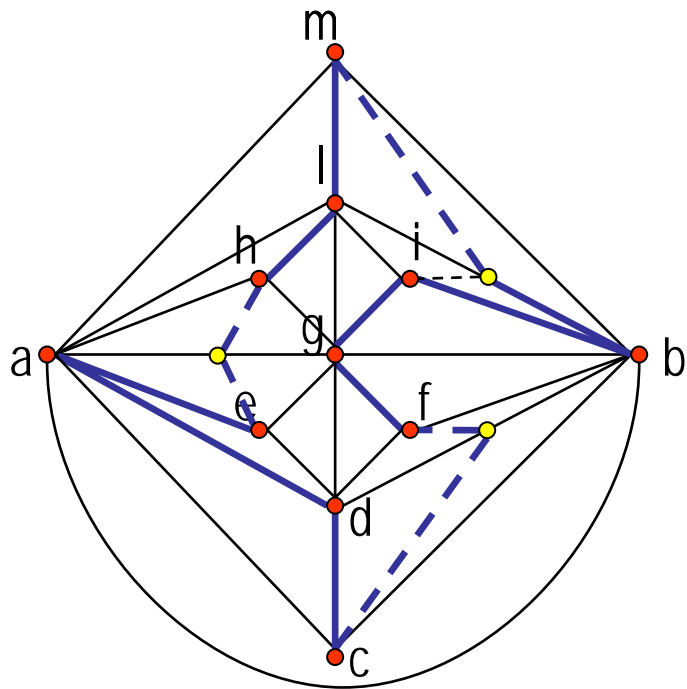## (Kaufmann and Wiese)

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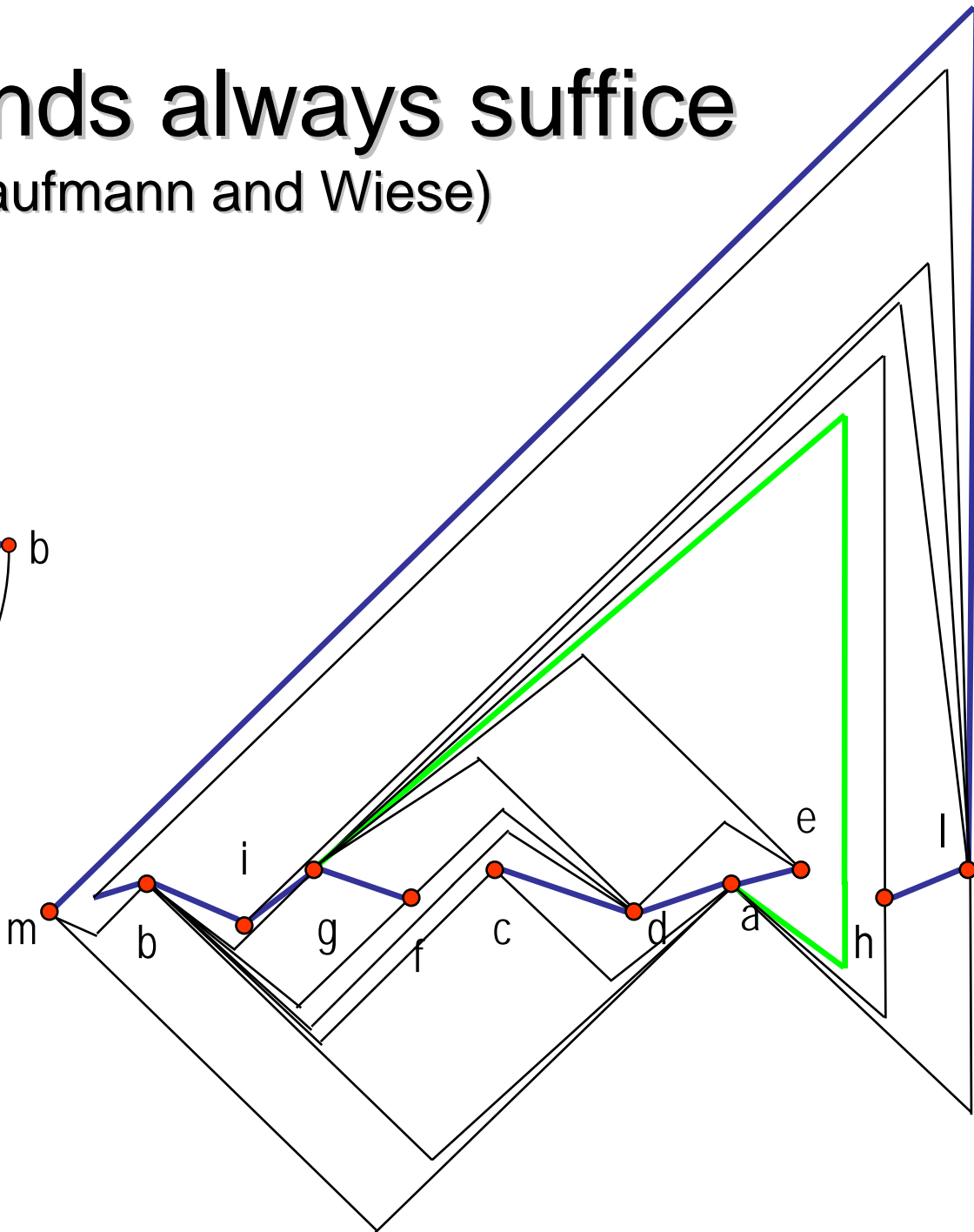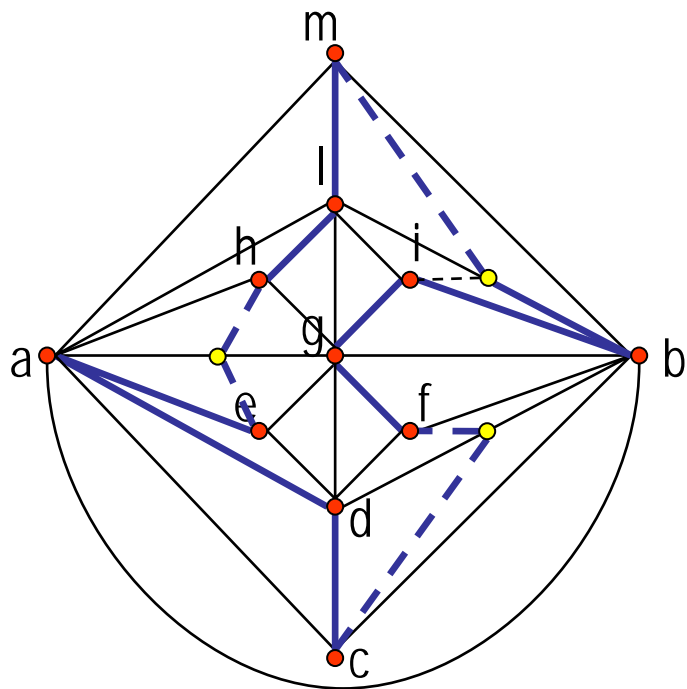
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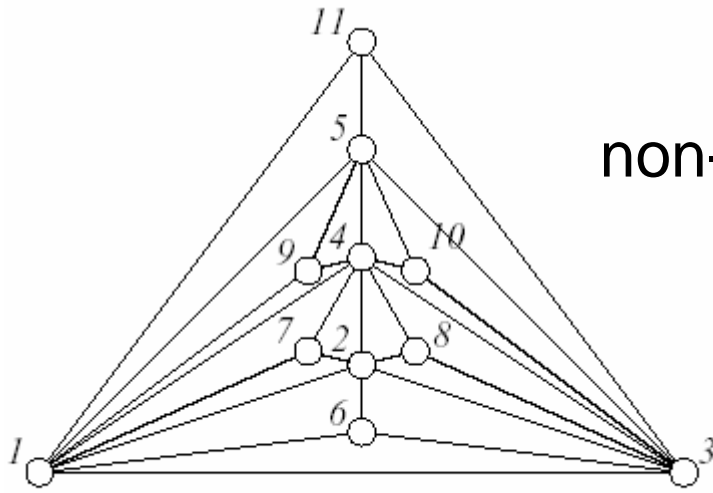# Two bends always suffice
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# Two bends may be necessary
## for a set of collinear points



non-hamiltonian graph

A graph has a two-page book embedding iff it is sub-hamiltonian (Bernhart and Kainen, 1979)