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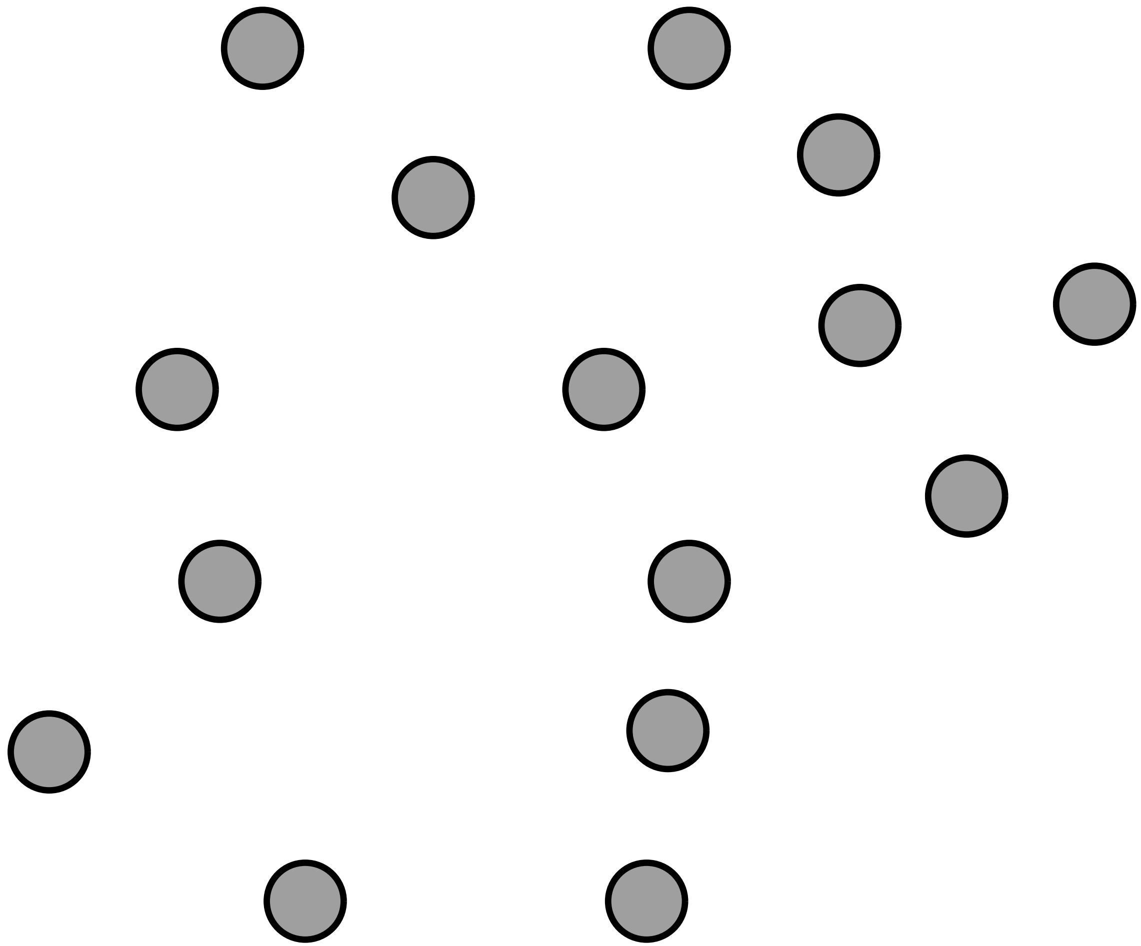
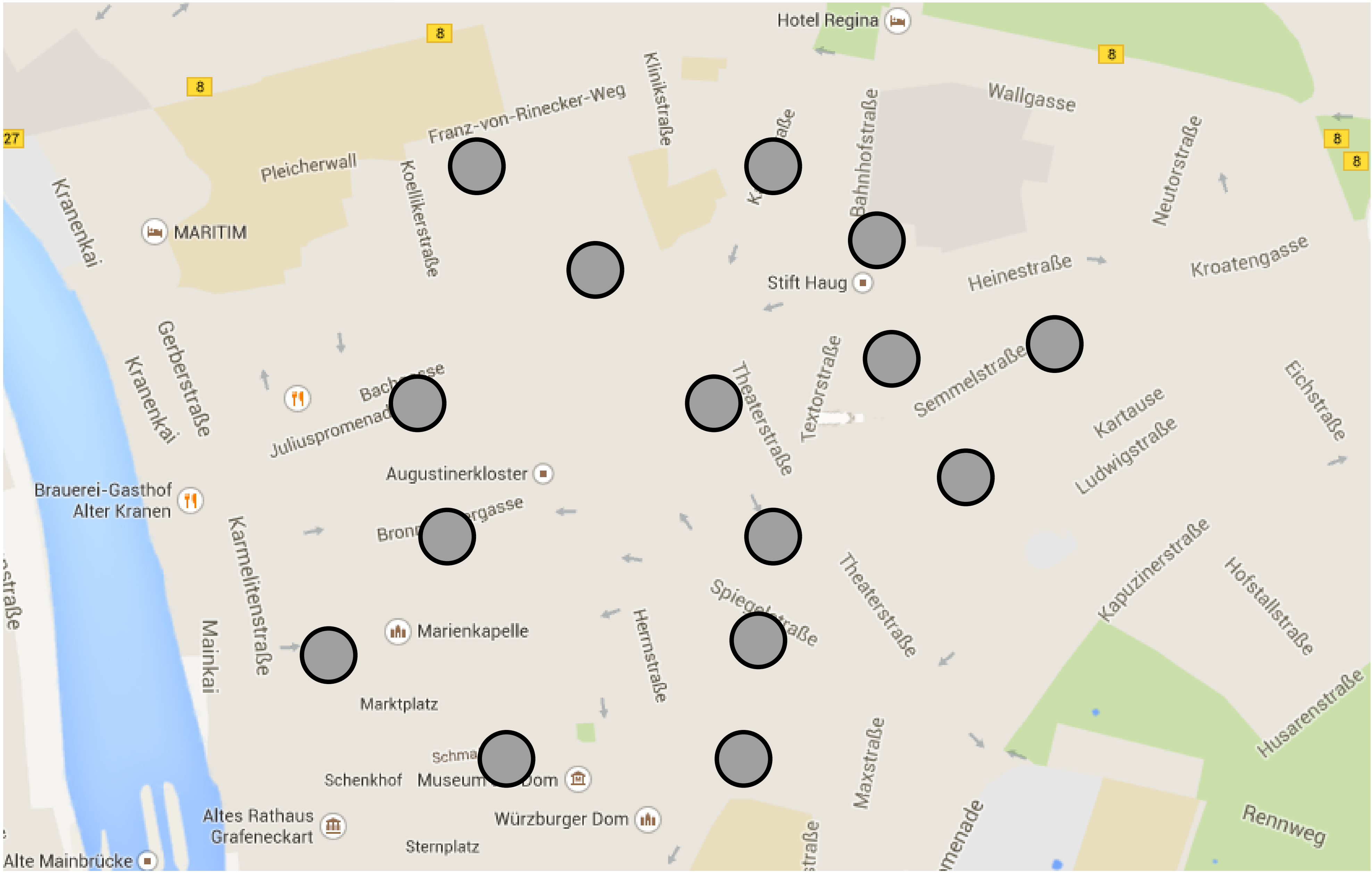
MapSets: Visualizing Embedded and Clustered Graphs

Sergey Pupyrev
University of Arizona

Joint work with Alon Efrat, Yifan Hu and Stephen Kobourov





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Franz-von-Rinecker-Weg
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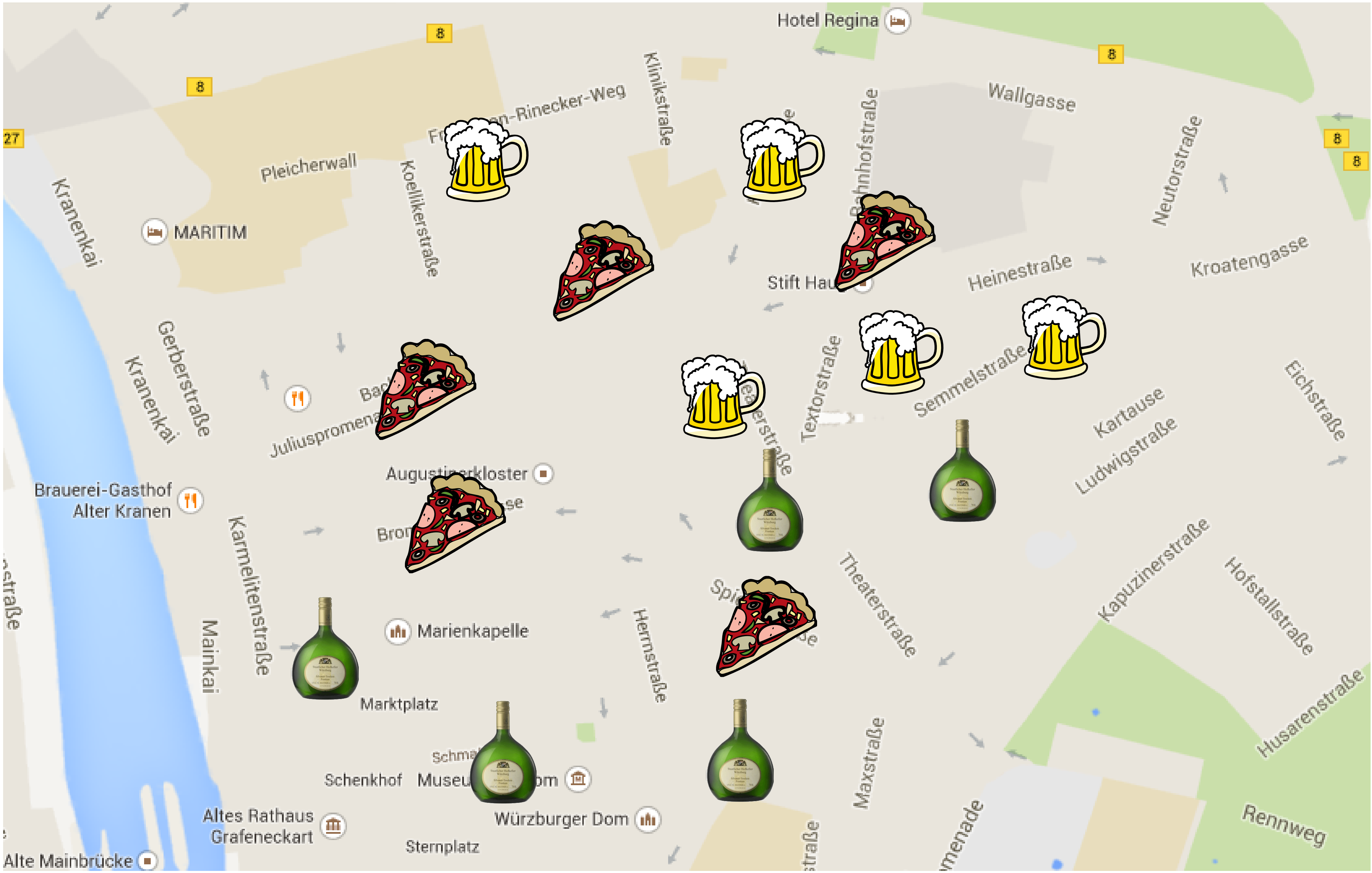
Bahnhofstraße
Pleicherwall
Koellikerstraße
Kranenkai
MARITIM
Neutorstraße
Stift Haug
Heinestraße
Kroatengasse
Gerberstraße
Kranenkai
Textorstraße
Theaterstraße
Semmelstraße
Juliuspromenade
Kartause
Ludwigstraße
Eichstraße
Augustinerkloster
Brauerei-Gasthof
Alter Kranen
Karmelitenstraße
Kapuzinerstraße
Hofstallstraße
Theaterstraße
Herrnstraße
Marienkapelle
Mainkai
Marktplatz
Husarenstraße
Maxstraße
Schenkhof
Würzburger Dom
Altes Rathaus
Grafeneckart
Sternplatz
Rennweg
Alte Mainbrücke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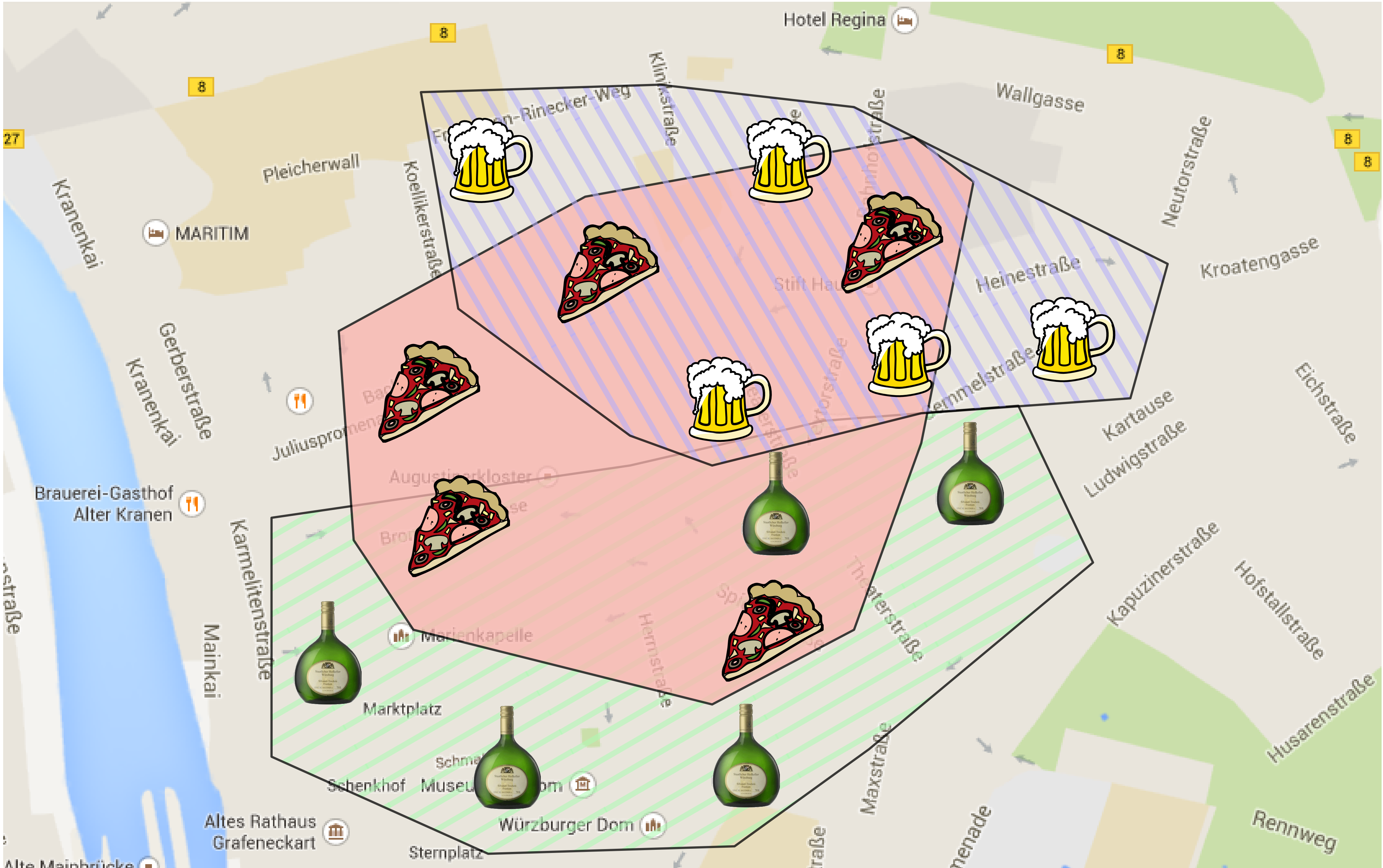
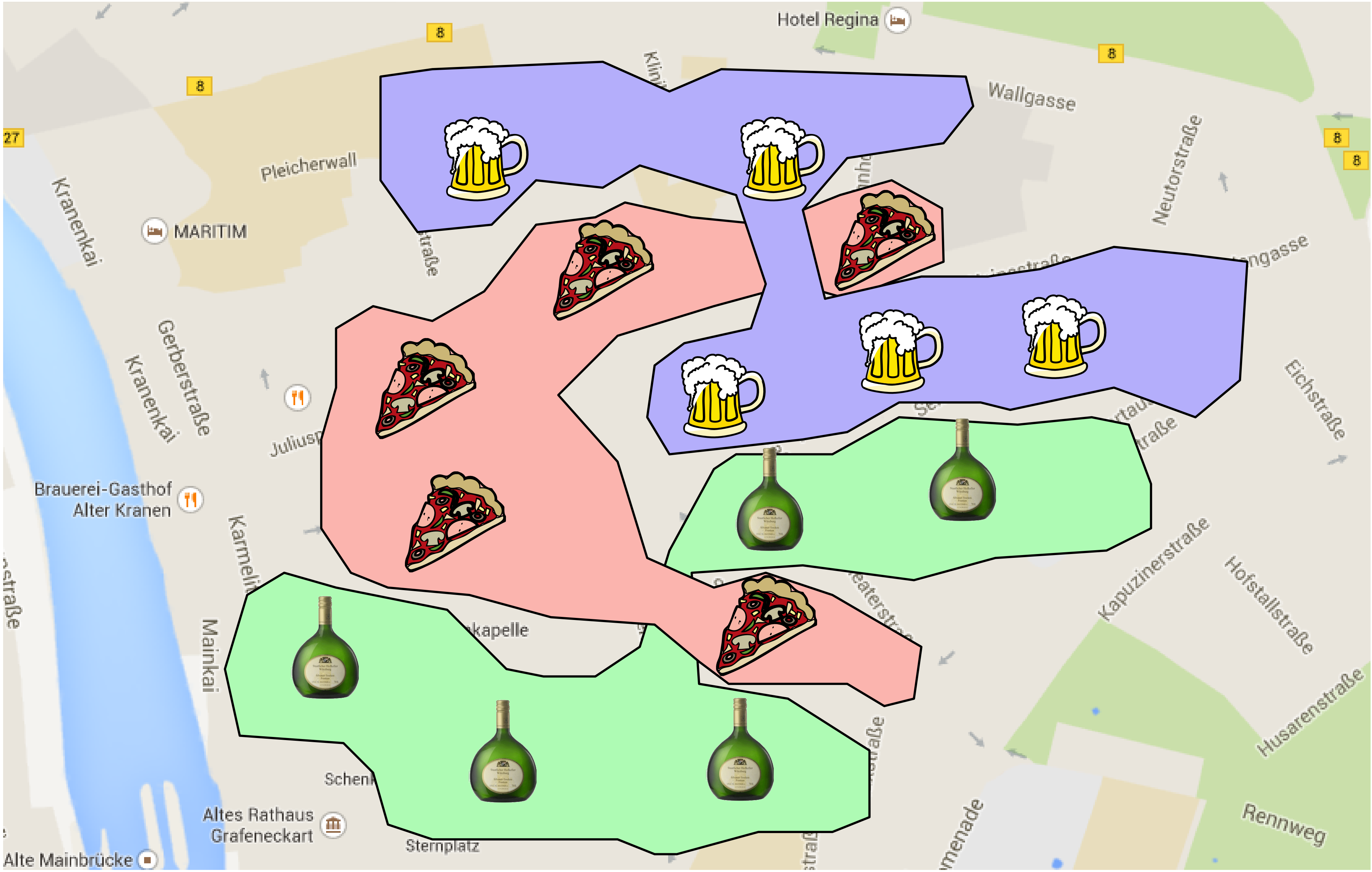
Euler diagrams

[Simonetto Auber Archambault, CGF'09]



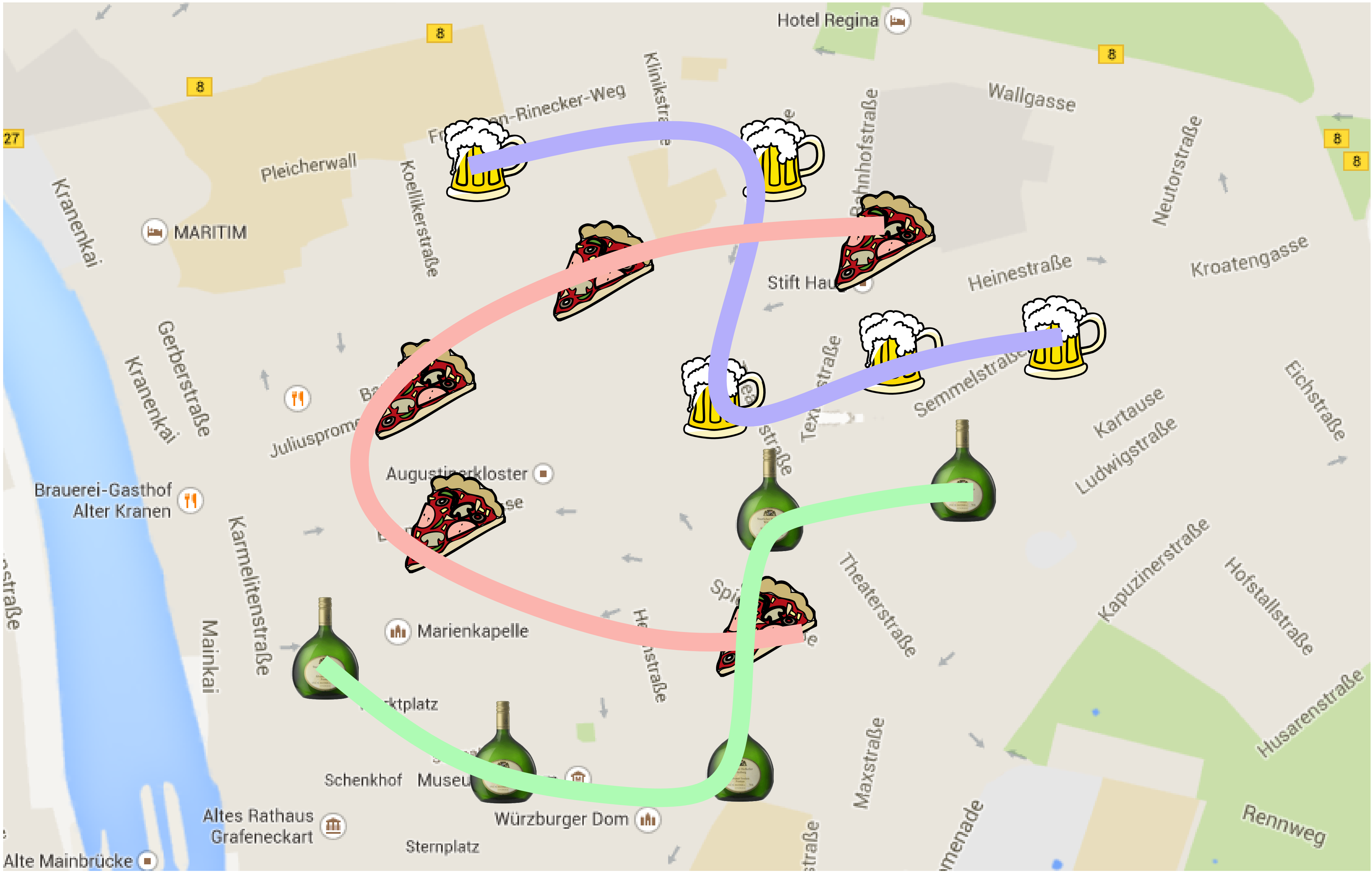
BubbleSets

[Collins Penn Carpendale, TVCG'09]



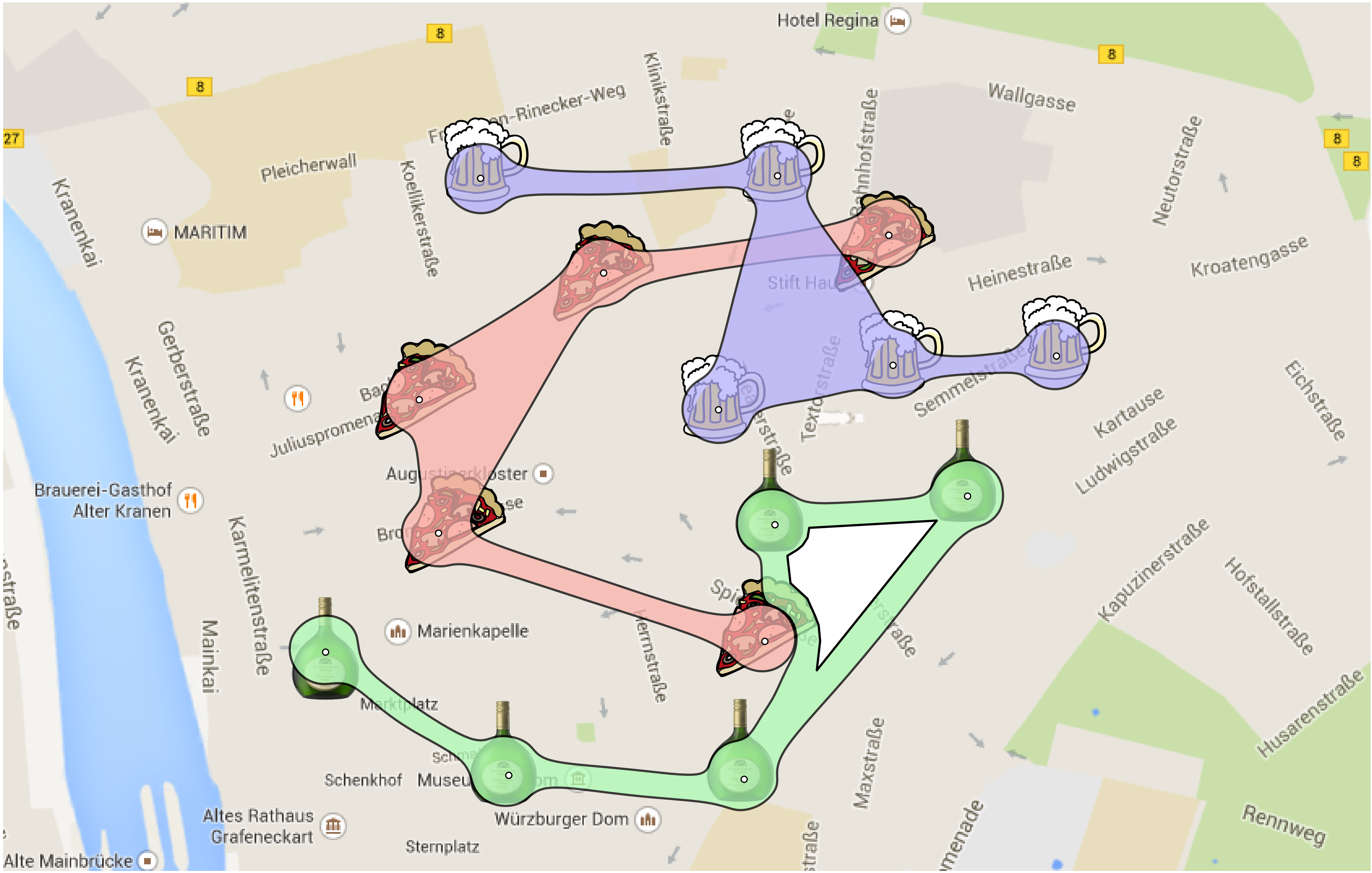
LineSets

[Alper Riche Ramos Czerwinski, TVCG'11]

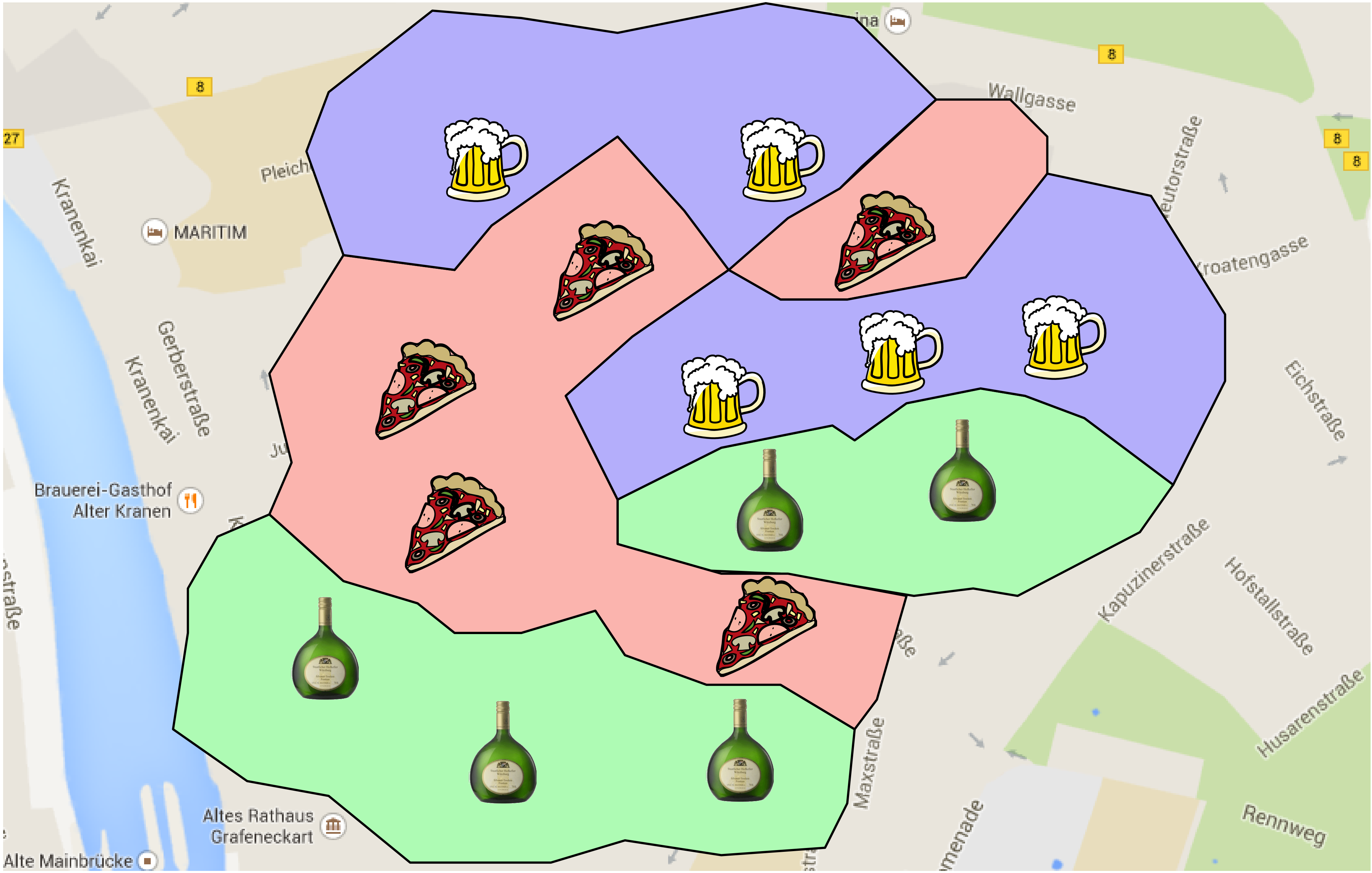


KelpFusion

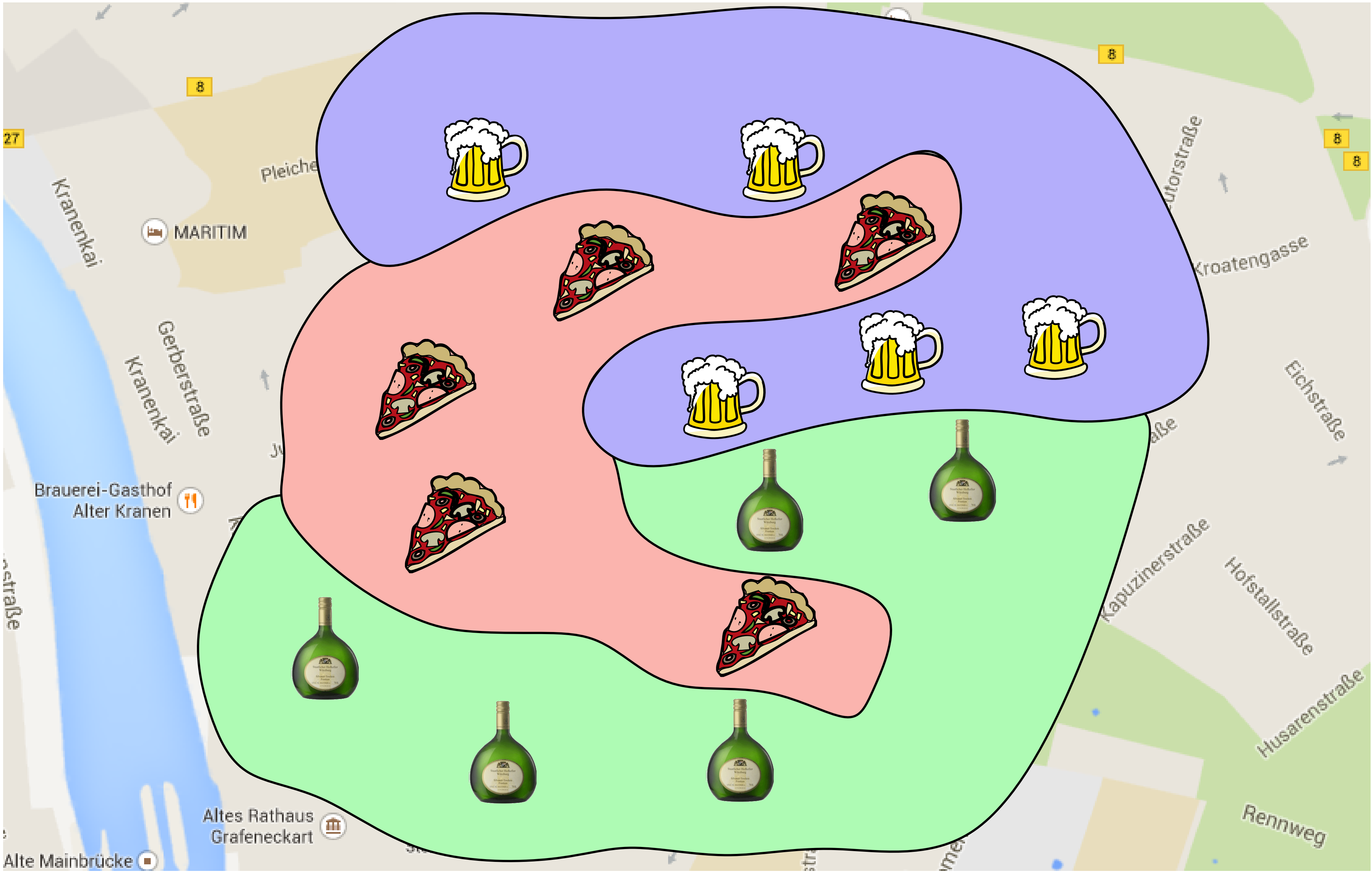
[Meulemans Riche Speckmann Alper Dwyer, TVCG'13]



GMap (Graph-to-Map) [Hu Gansner Kobourov, CGA'10]



a *better* solution



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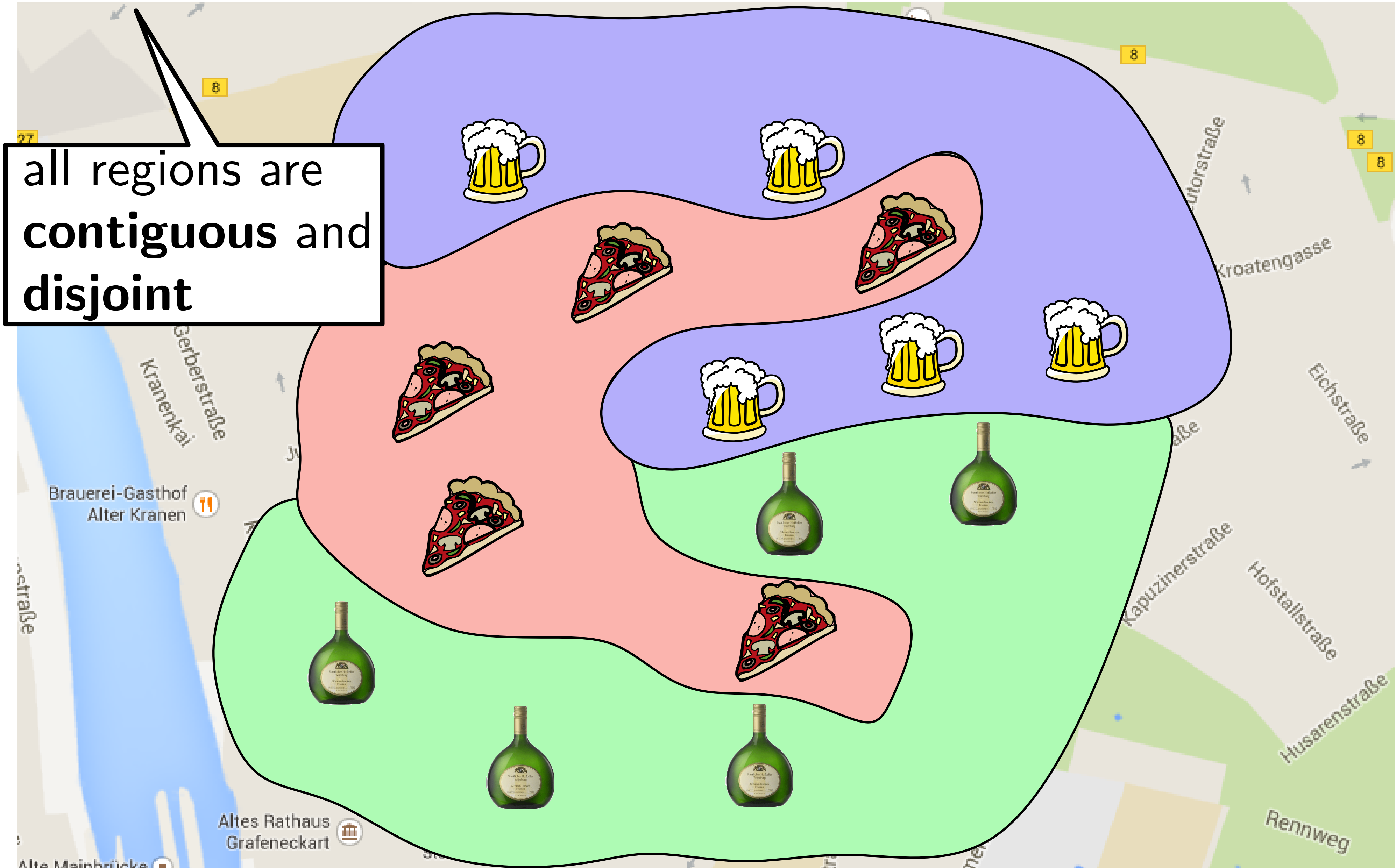
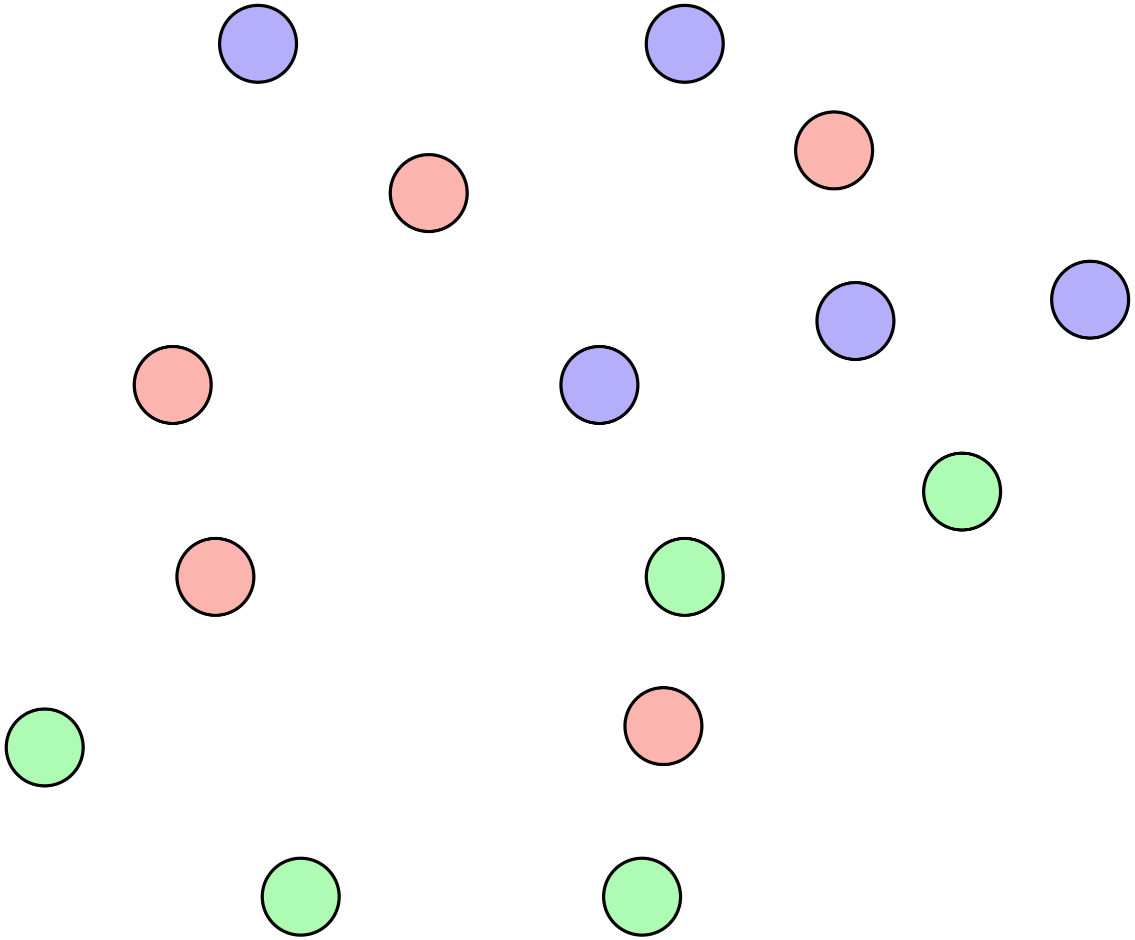


There is always a solution...



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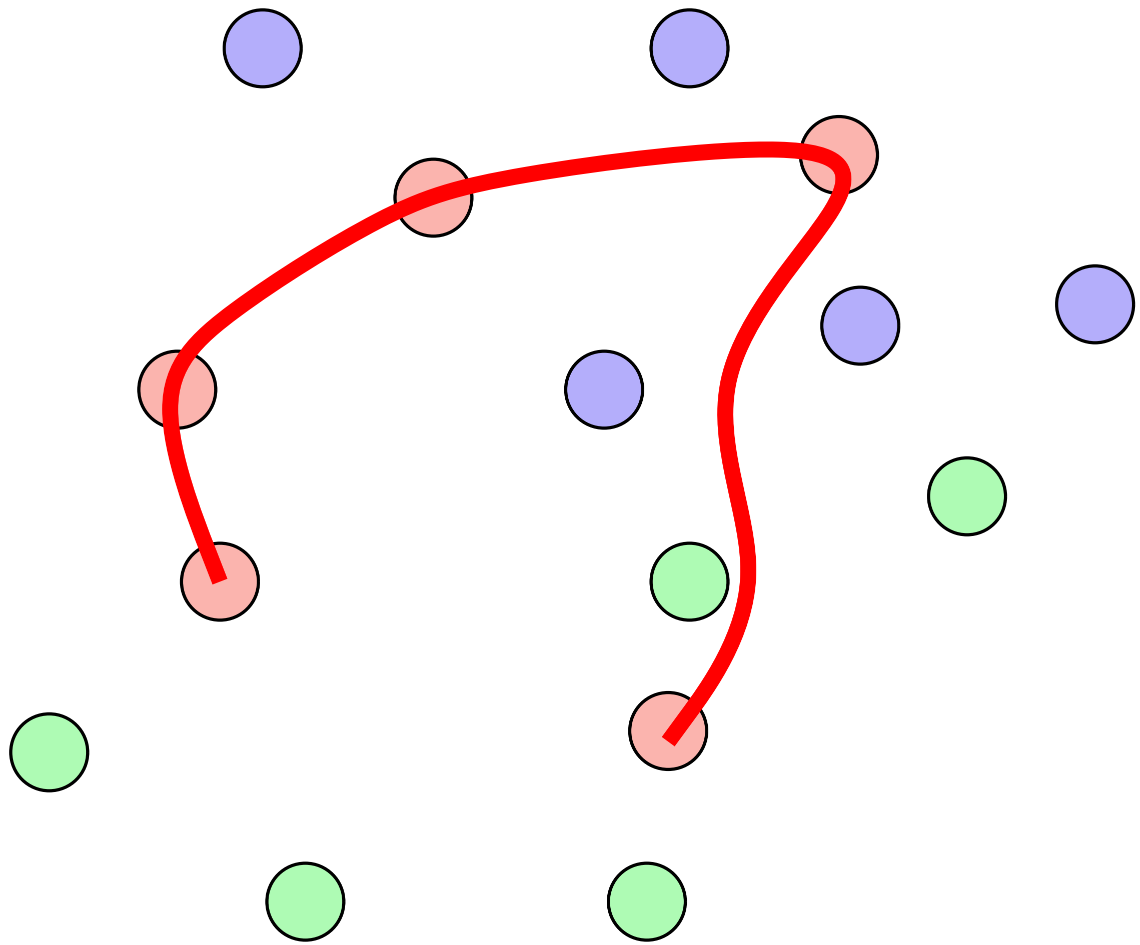

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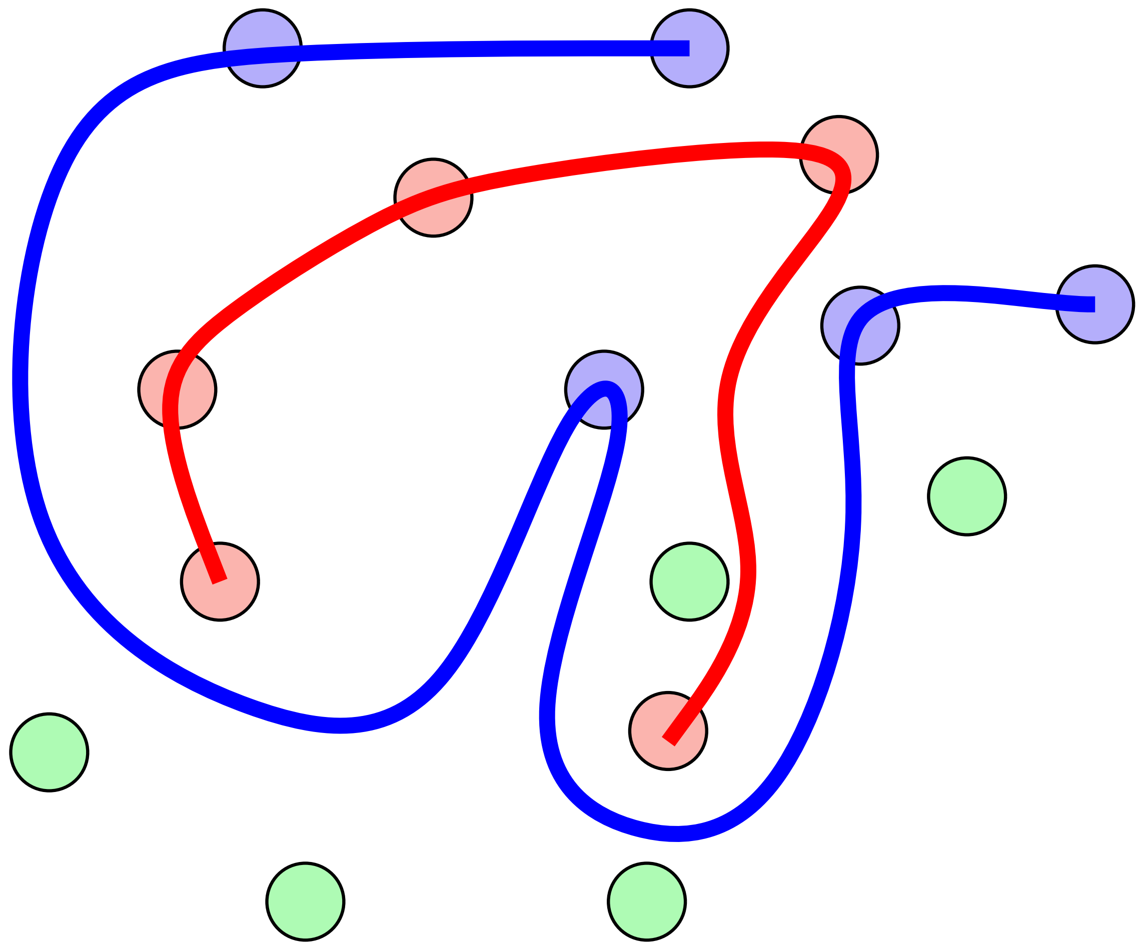

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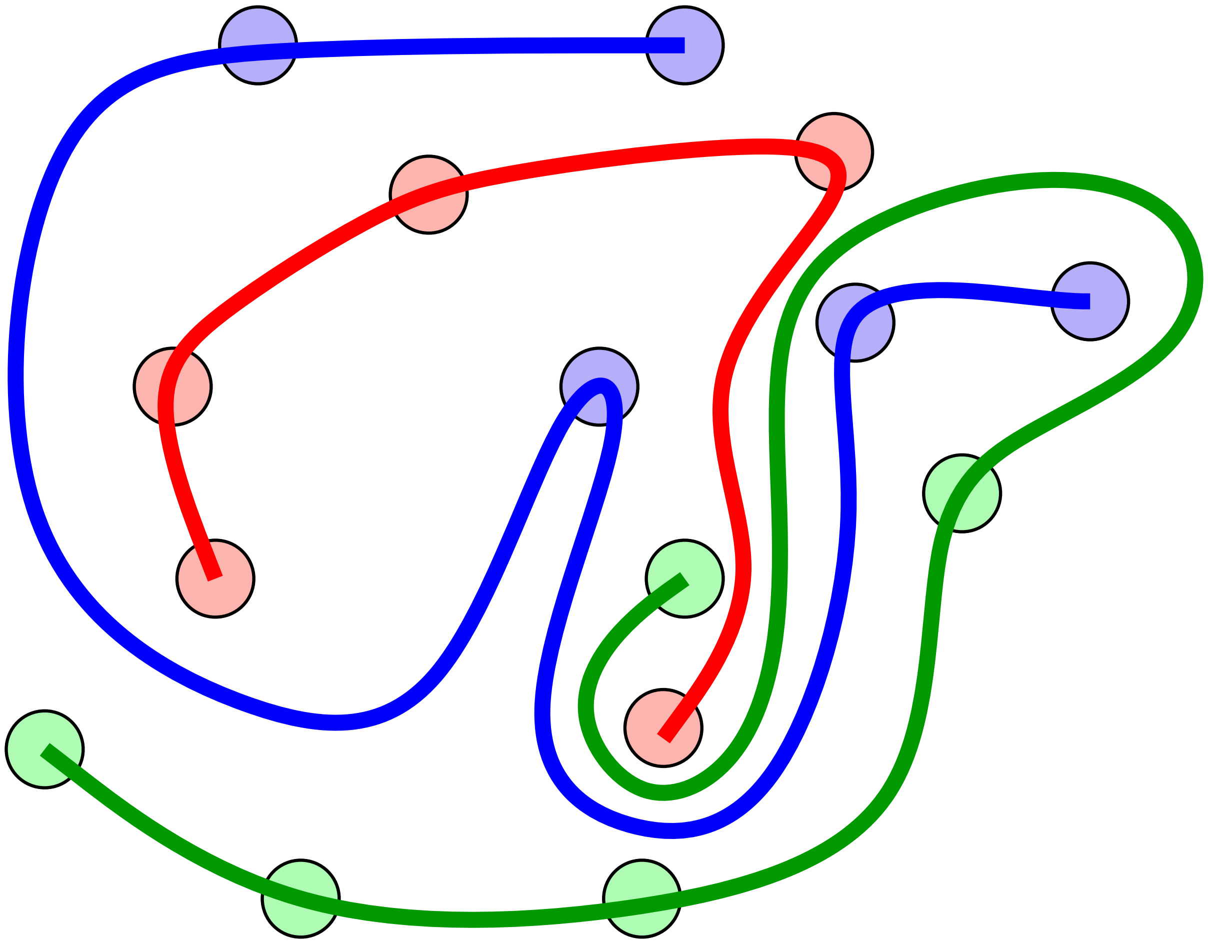
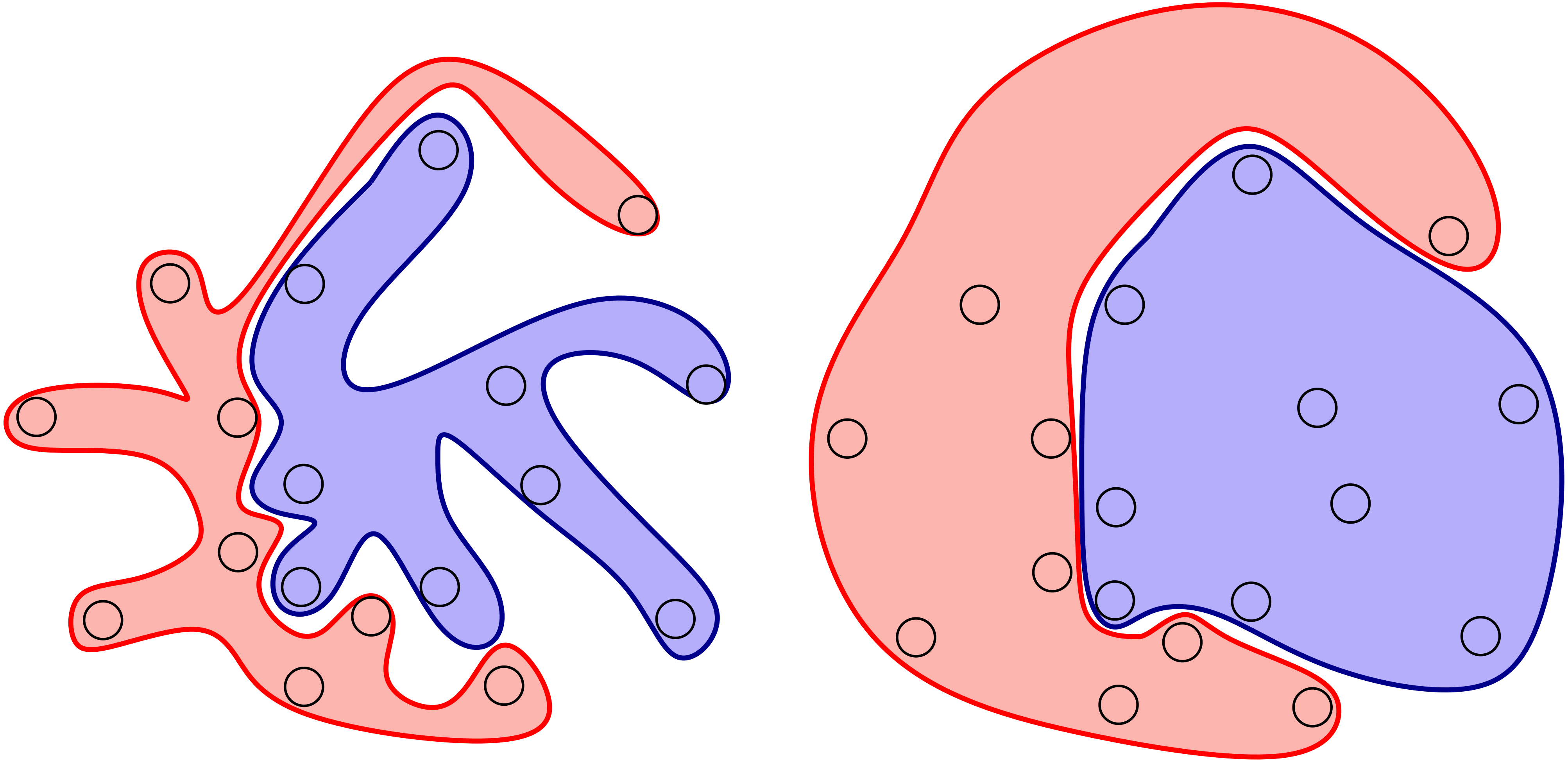


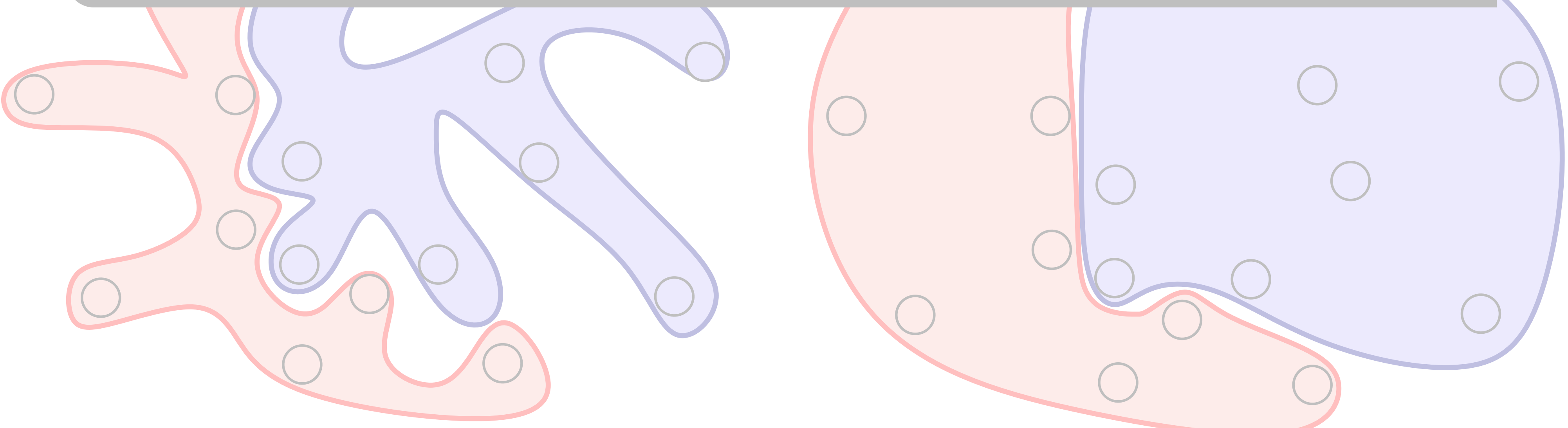
...but not all look the same!



...but not all look the same!

Main Question

How to construct *disjoint contigous* regions, that are as *convex* as possible?



...but not all look the same!

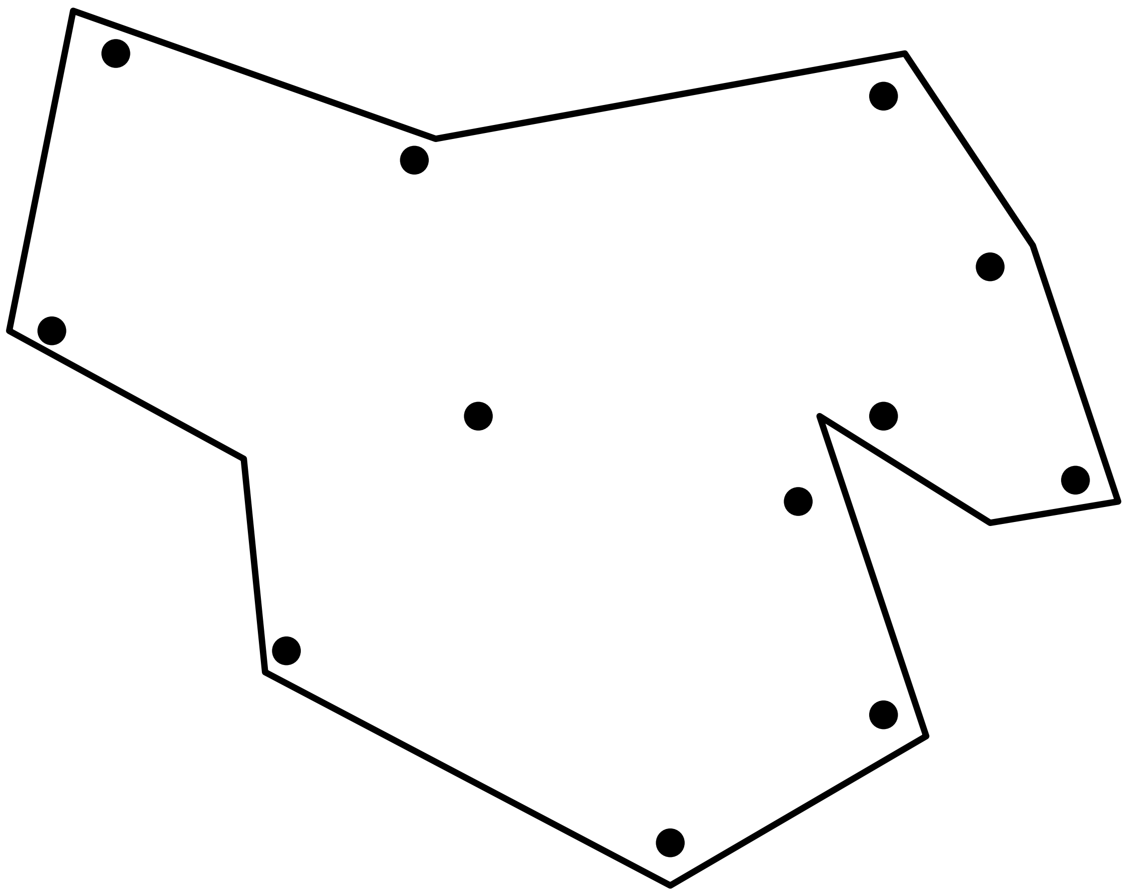
Main Question

How to construct *disjoint contigous* regions, that are as *convex* as possible?

Result

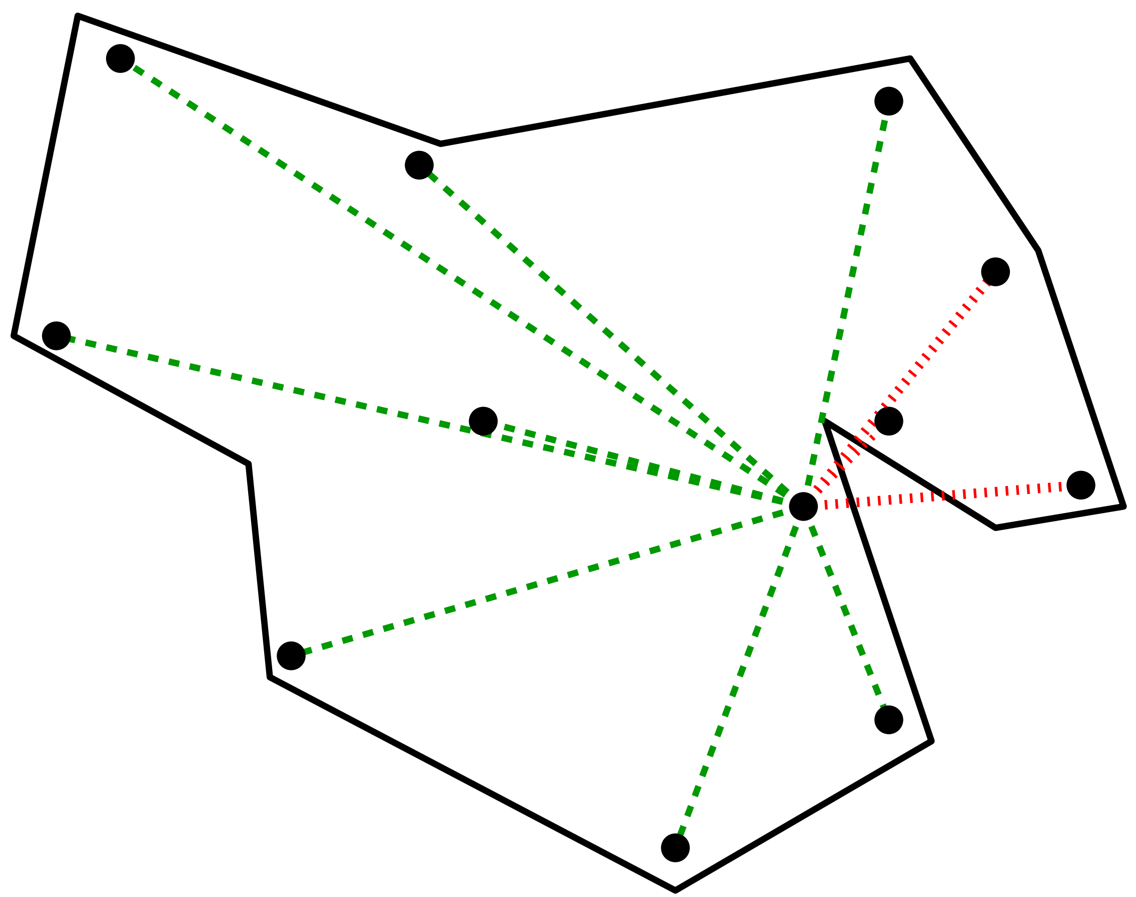
MapSets:
– available at `http://gmap.cs.arizona.edu`
– guarantees non-fragmented non-overlapping regions
– based on a novel geometric problem aiming at optimizing convexity

How to measure *convexity*?



How to measure *convexity*?

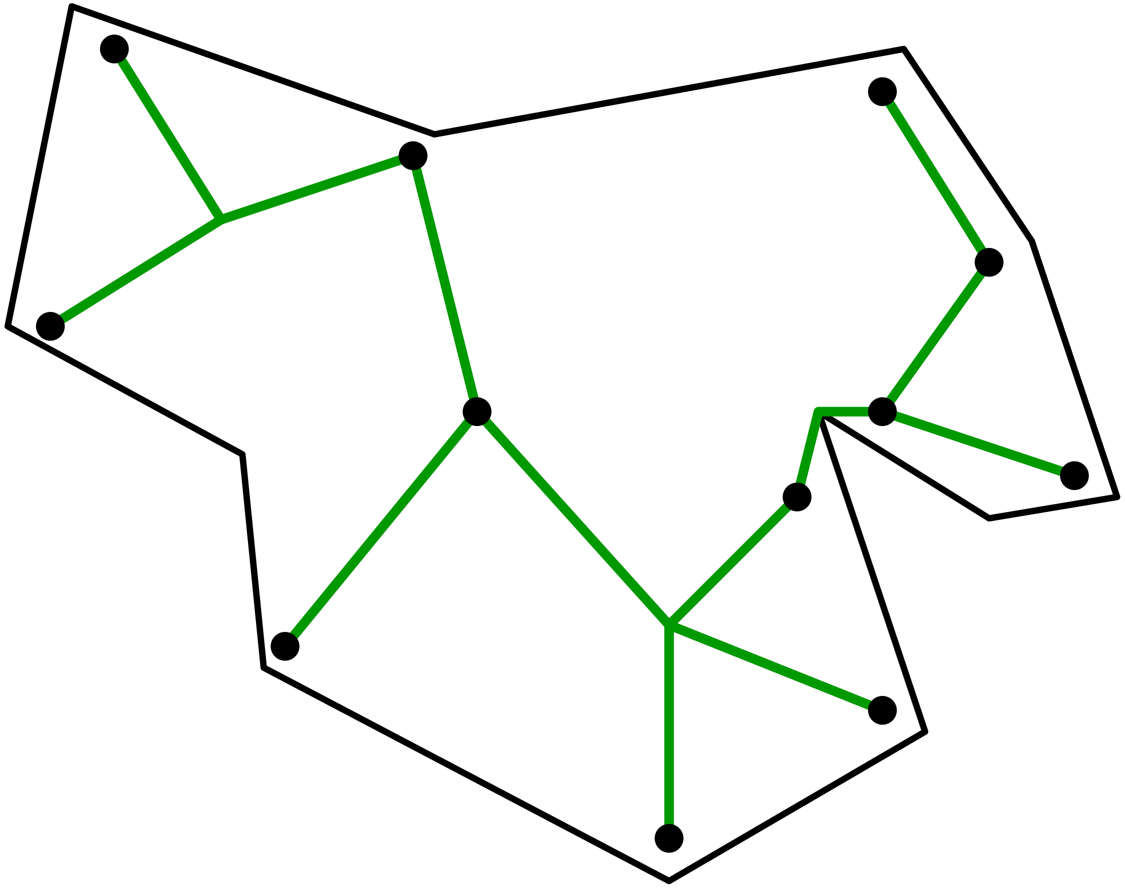
Def.(*visibility-based*): how many points "see" each other



How to measure *convexity*?

Def.(*visibility-based*): how many points “see” each other

Def.(*ink-based*): length of the shortest spanning tree inside the polygon



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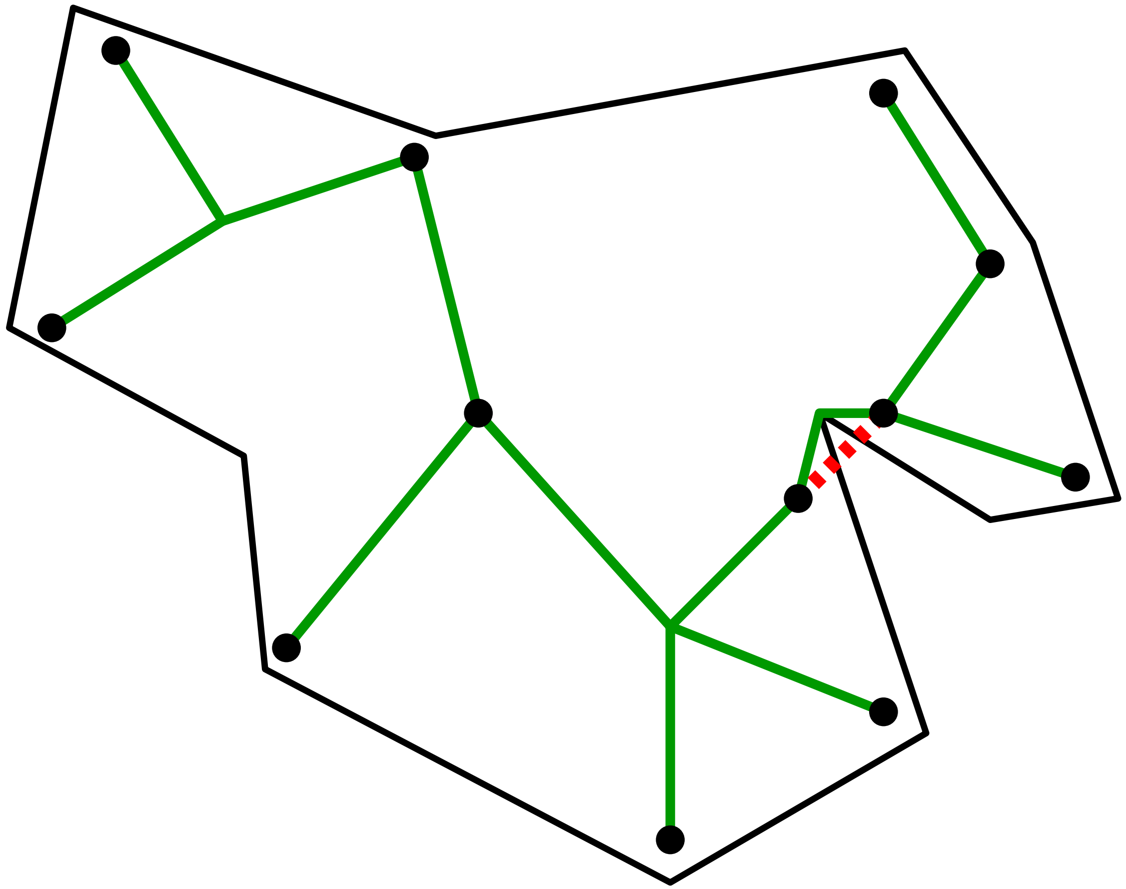

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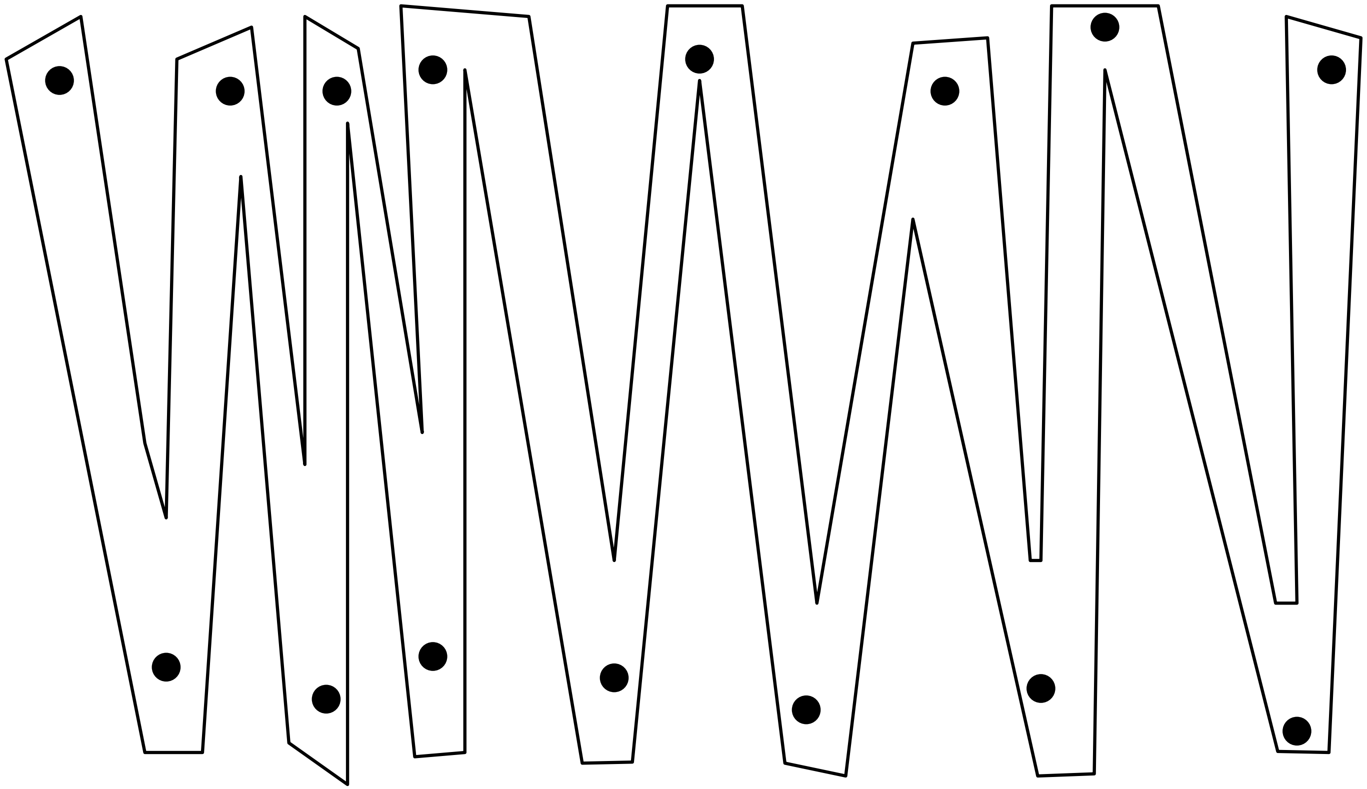

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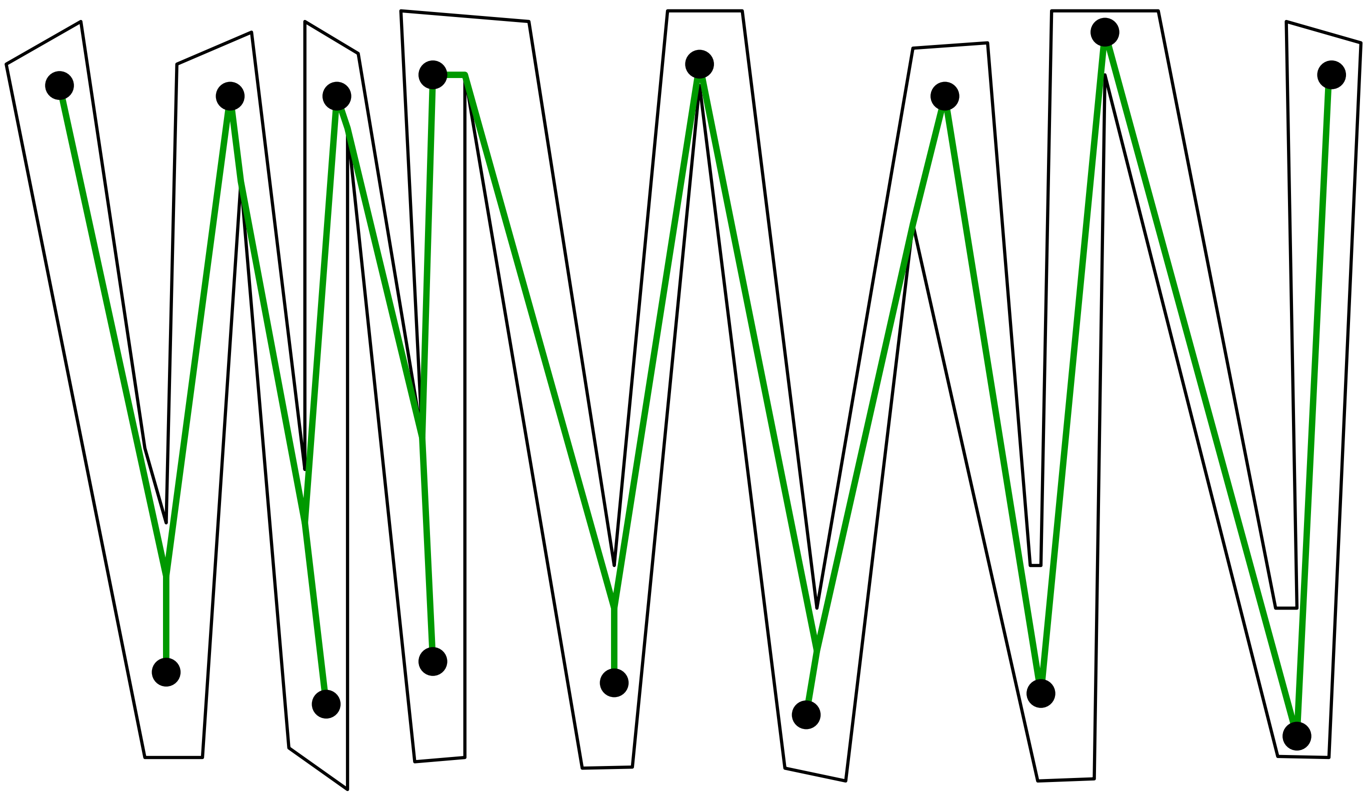

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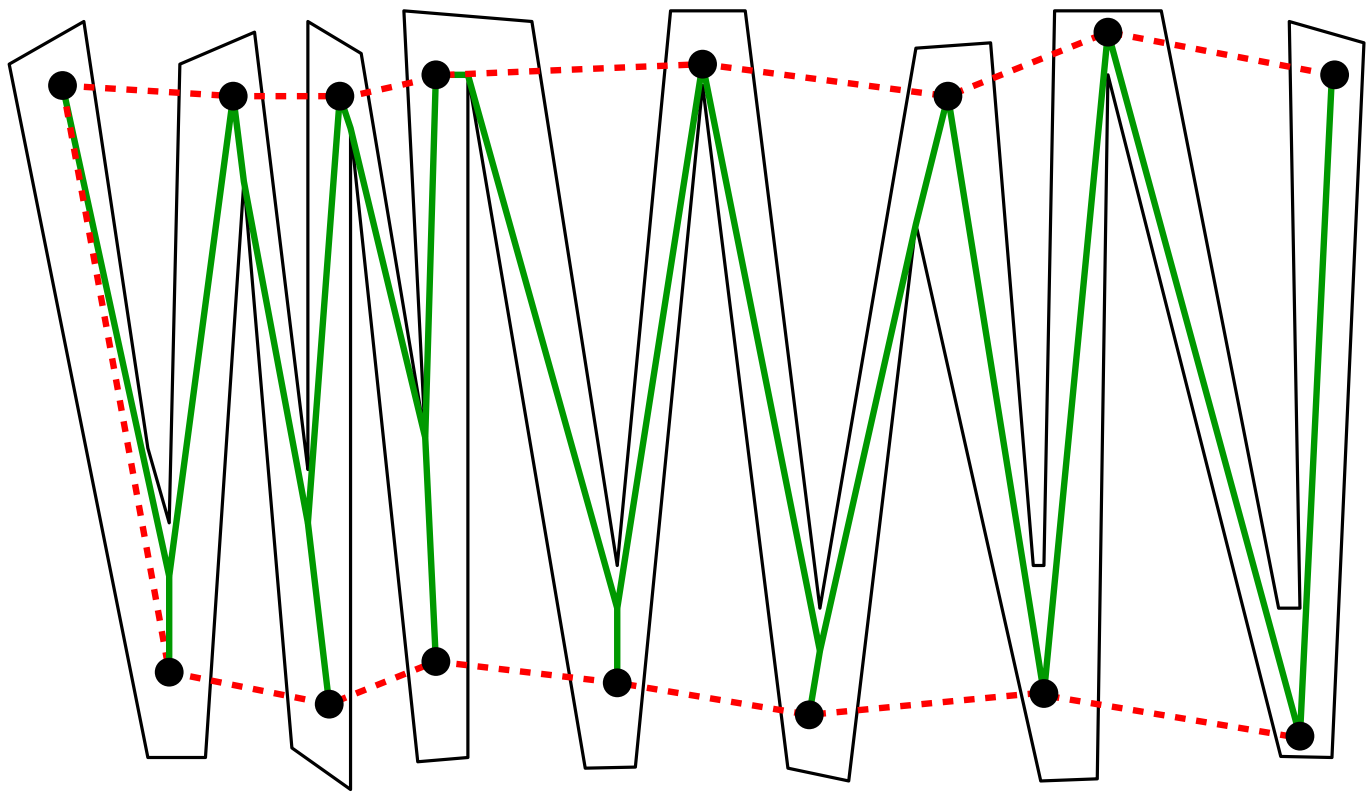
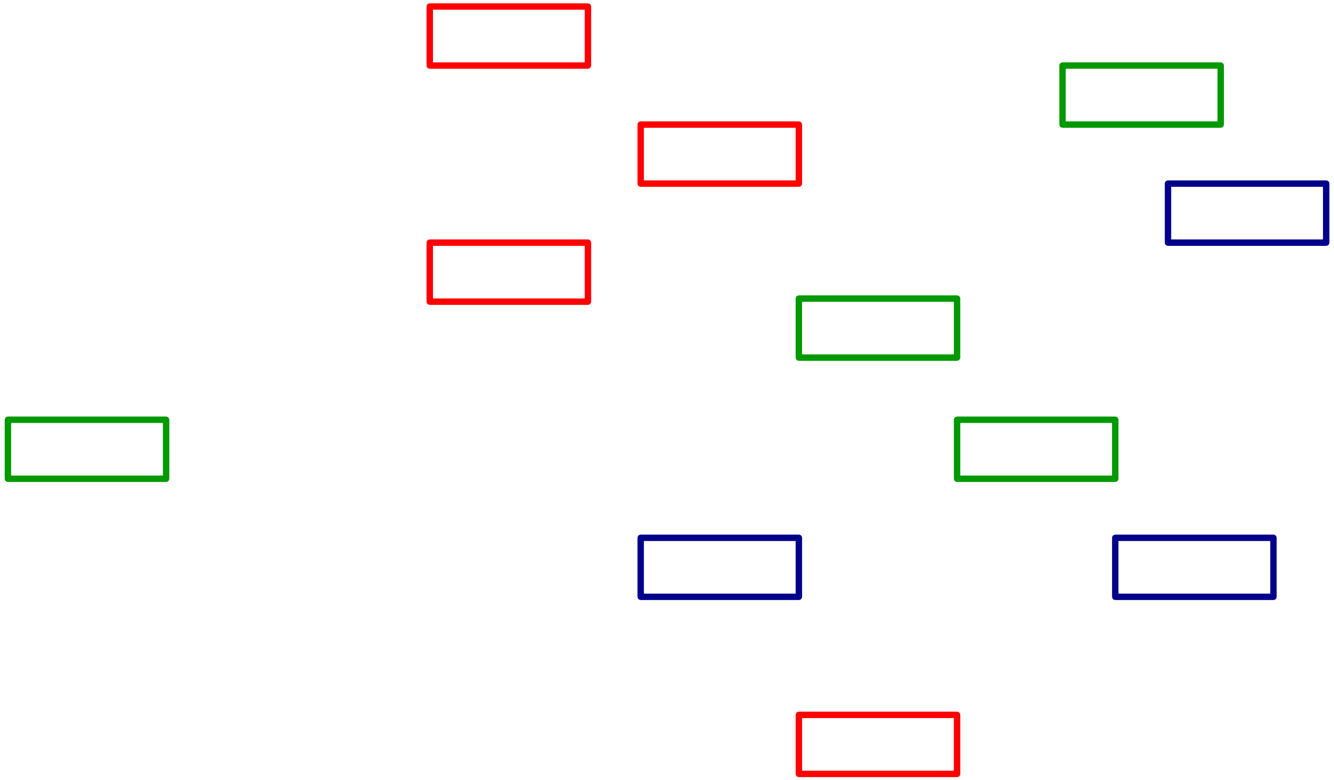
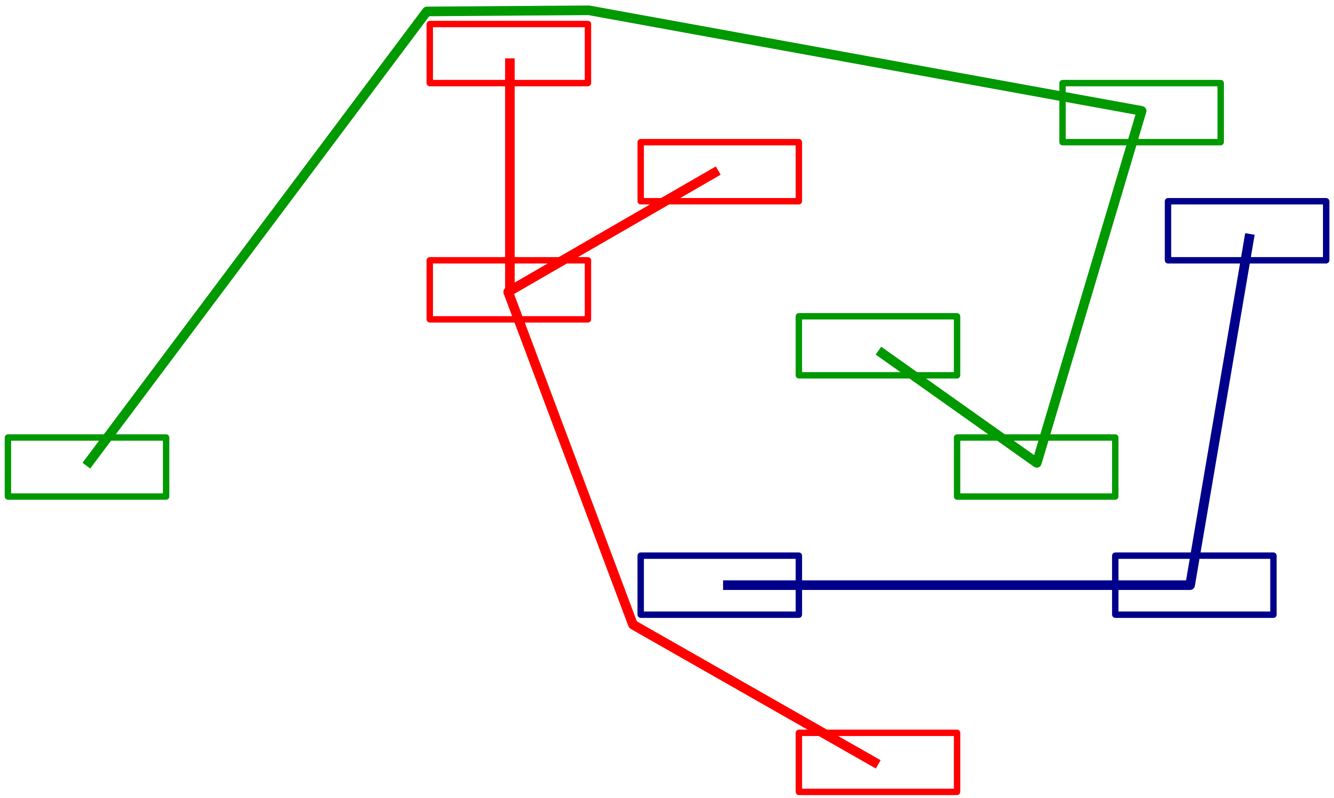


Input

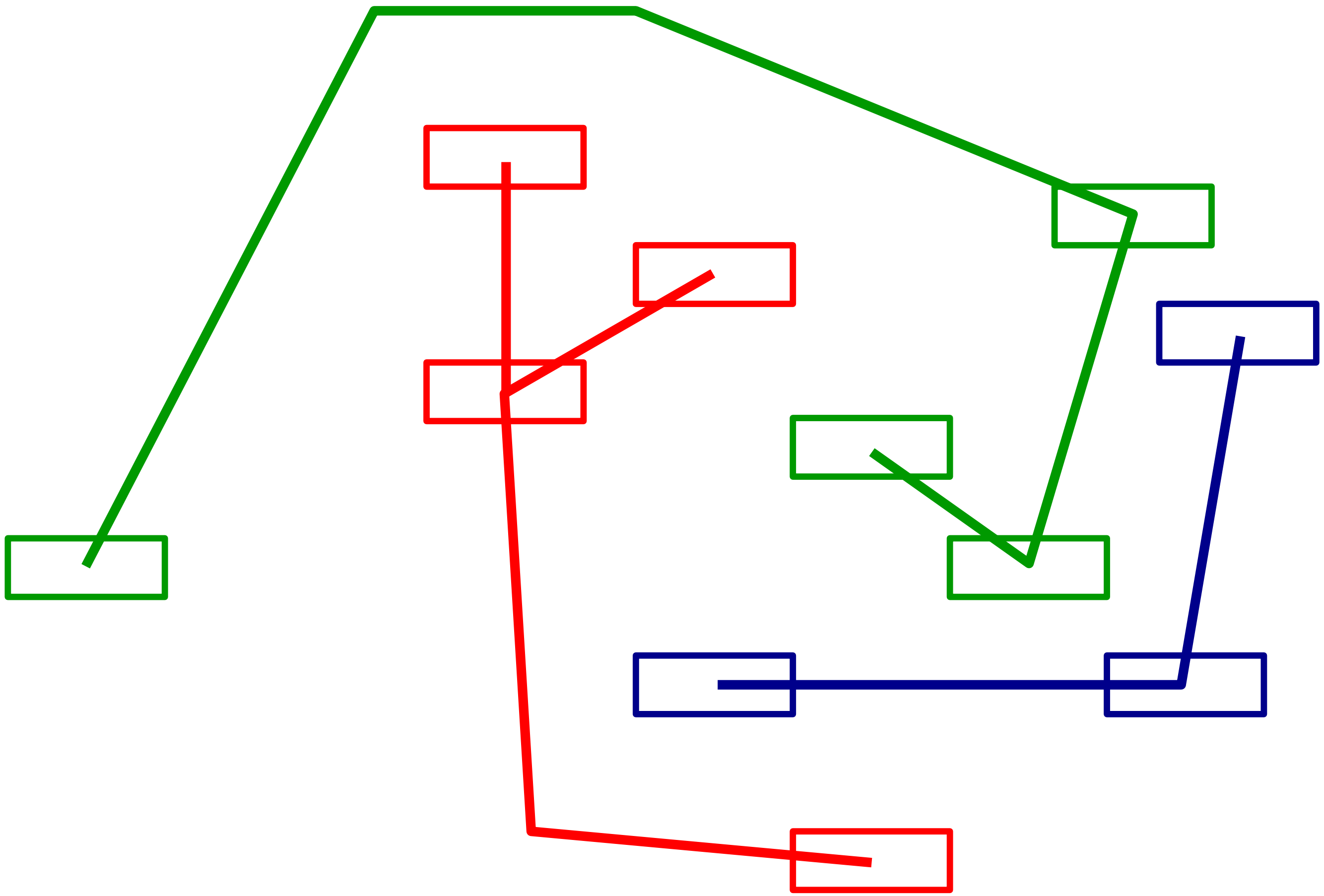


Step 1: Tree Construction

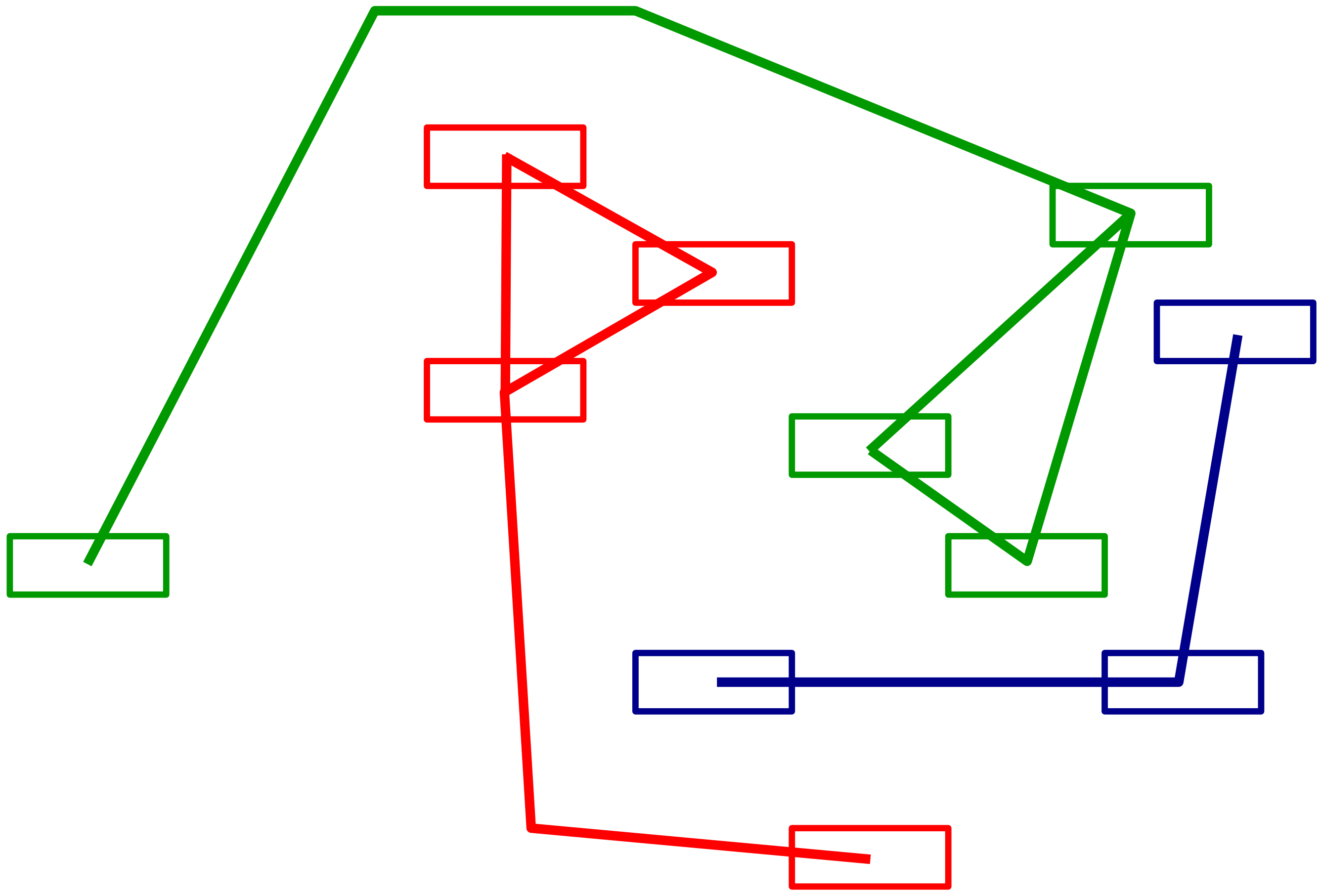
(optimizing ink-based convexity)



Step 2: Force-directed Adjustment



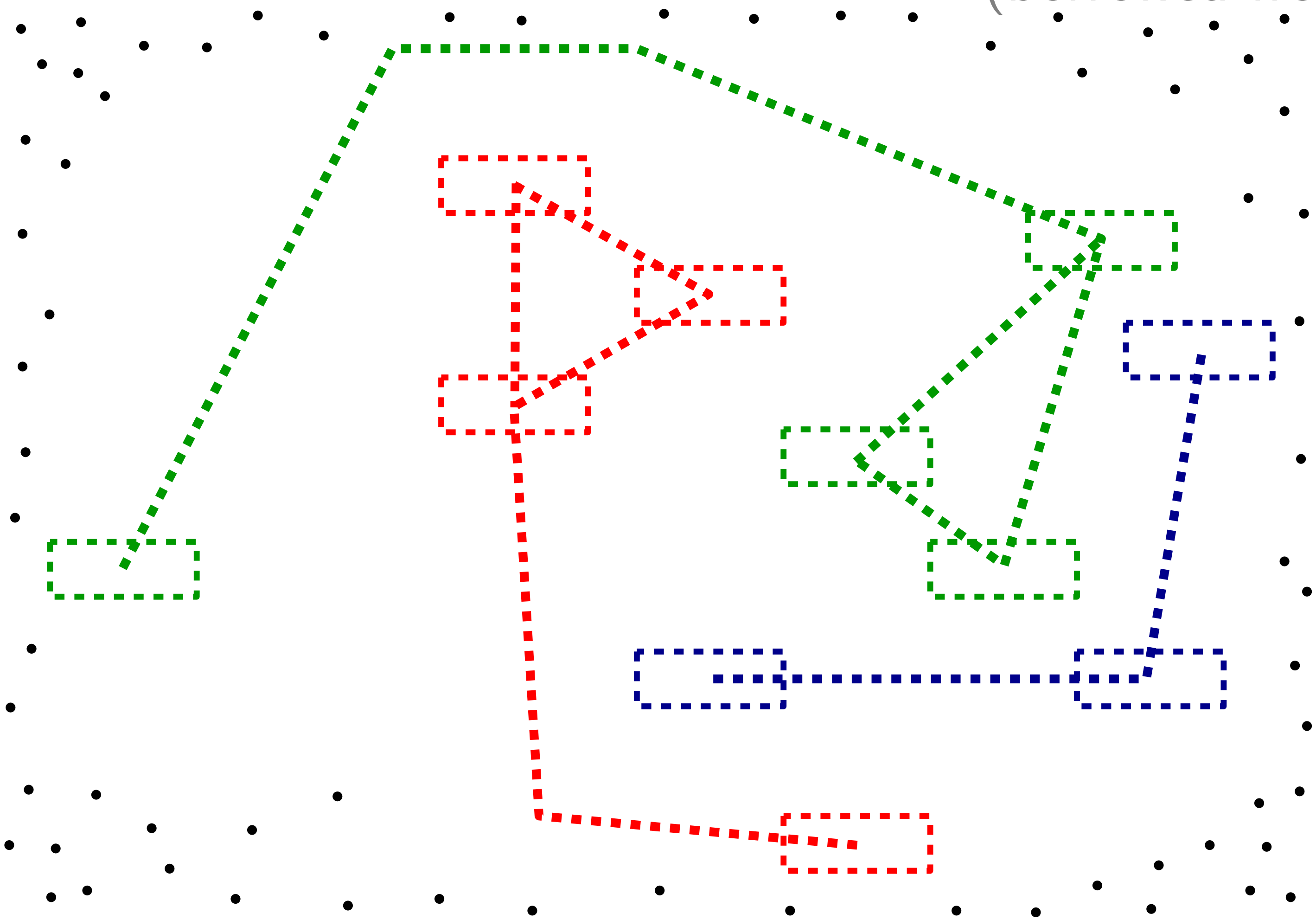
Step 3: Edge Augmentation
(optimizing visibility-based convexity)



MapSets http://gmap.cs.arizona.edu

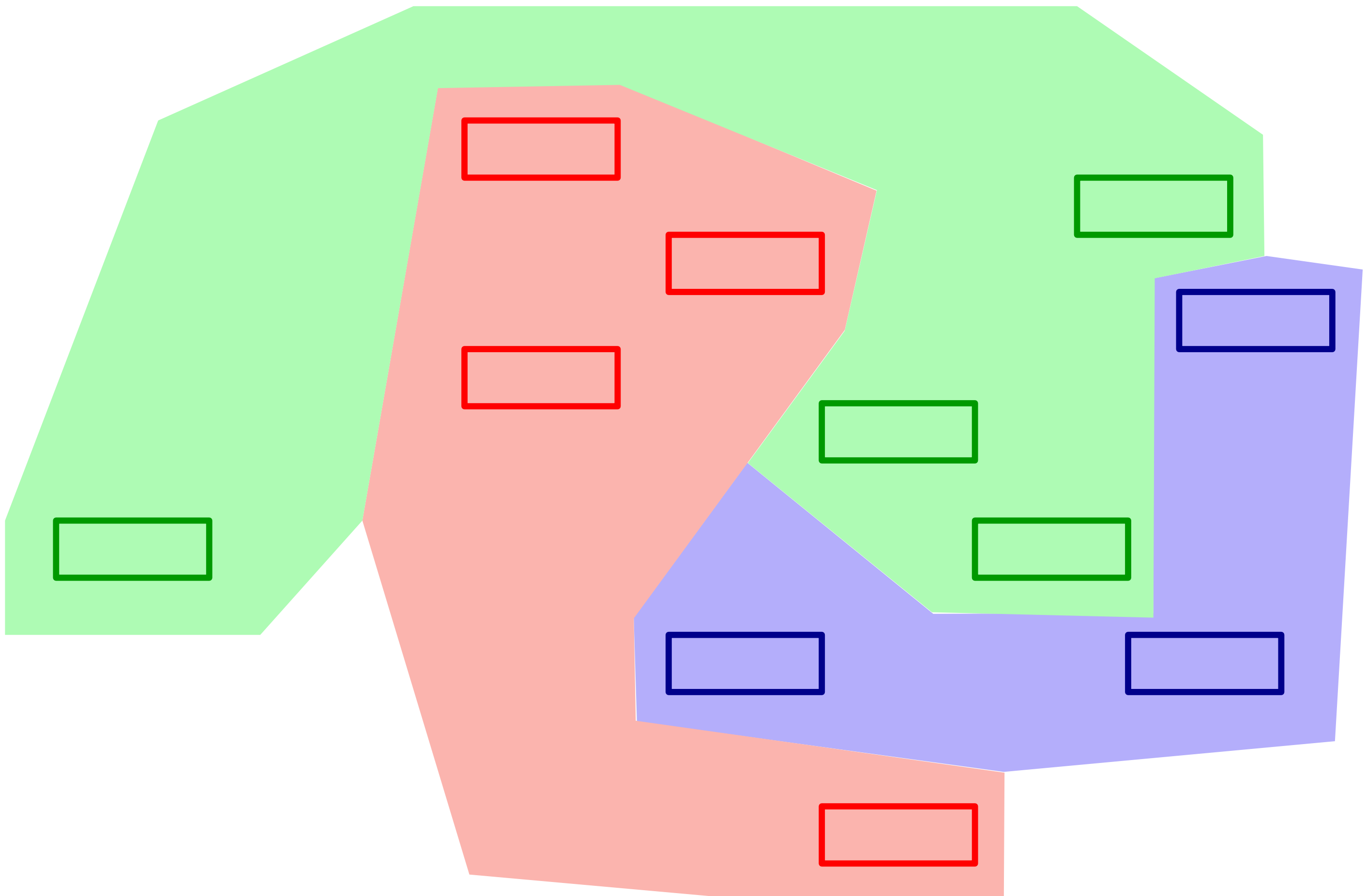
Step 4: Adding Dummy Points

(borrowed from GMap)



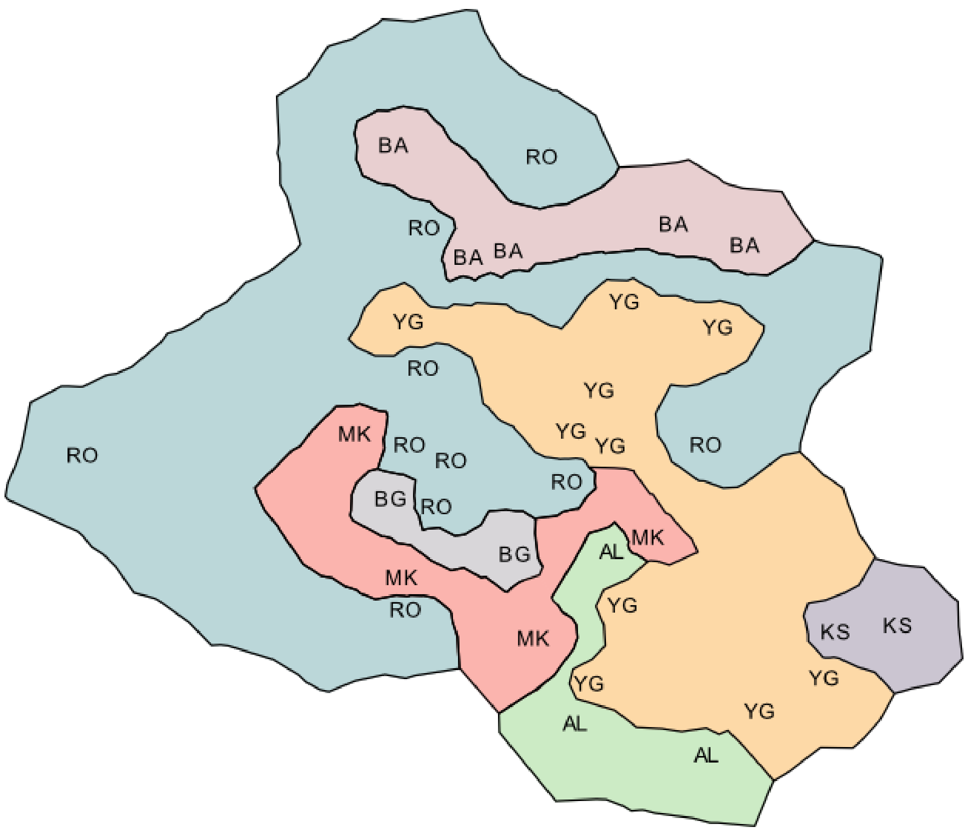
Step 5: Computing Regions

(borrowed from GMap)

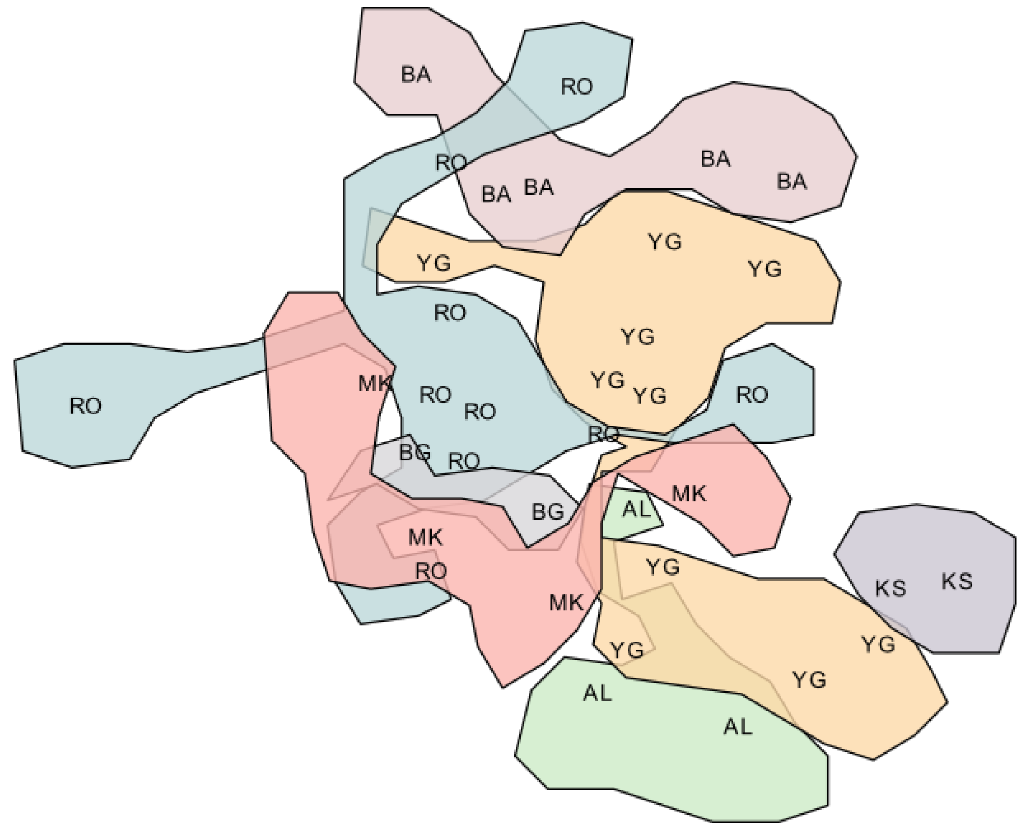


Examples

MapSets



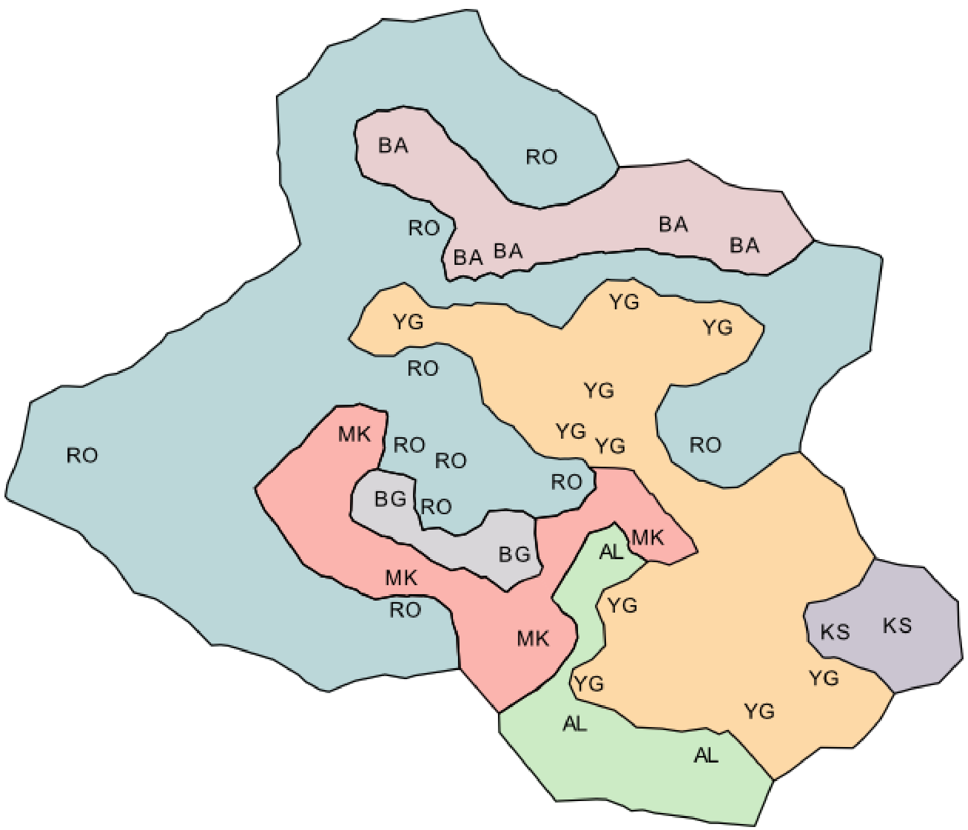
BubbleSets



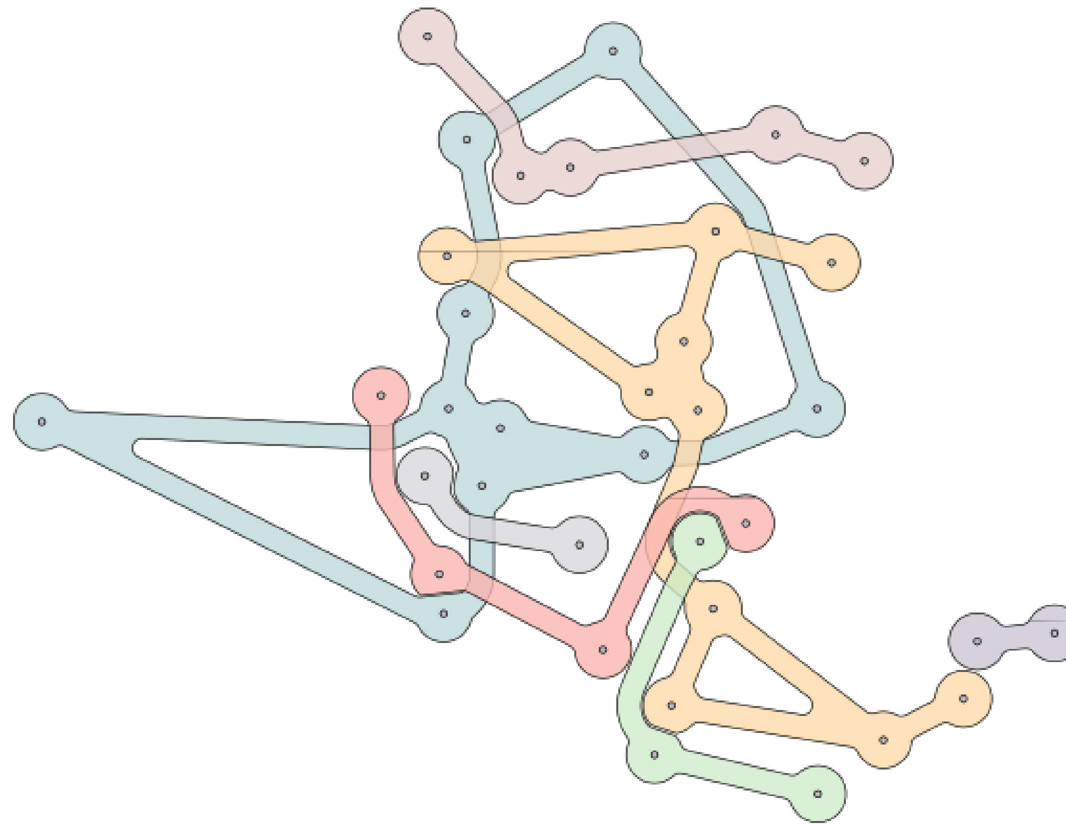
Dataset: genetic similarities between individuals in Europe
50 vertices, 7 clusters

Examples

MapSets



KelpFusion



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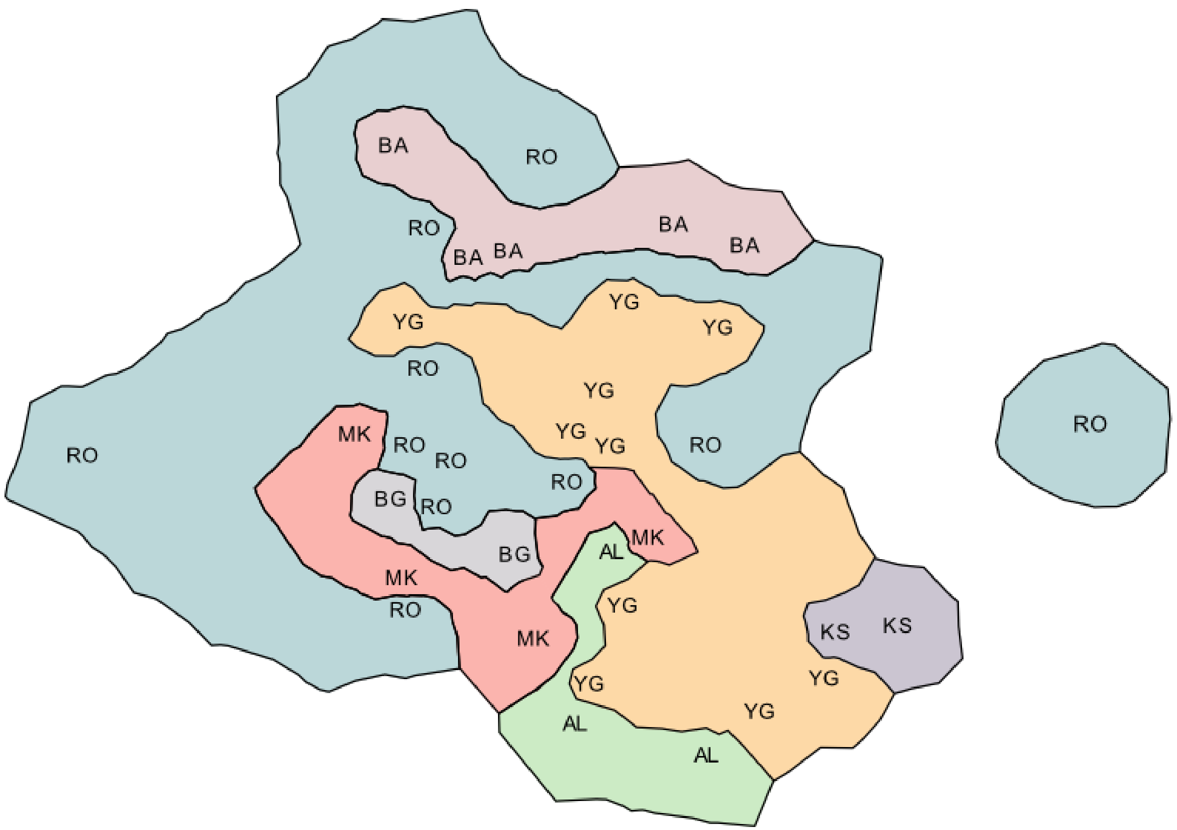
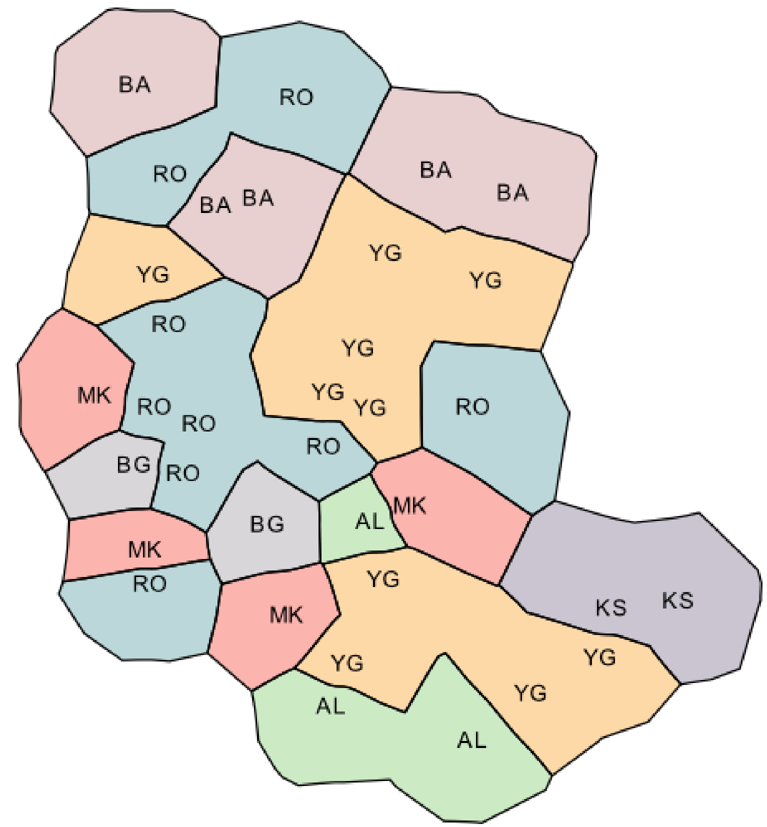
Examples

MapSets



GMap



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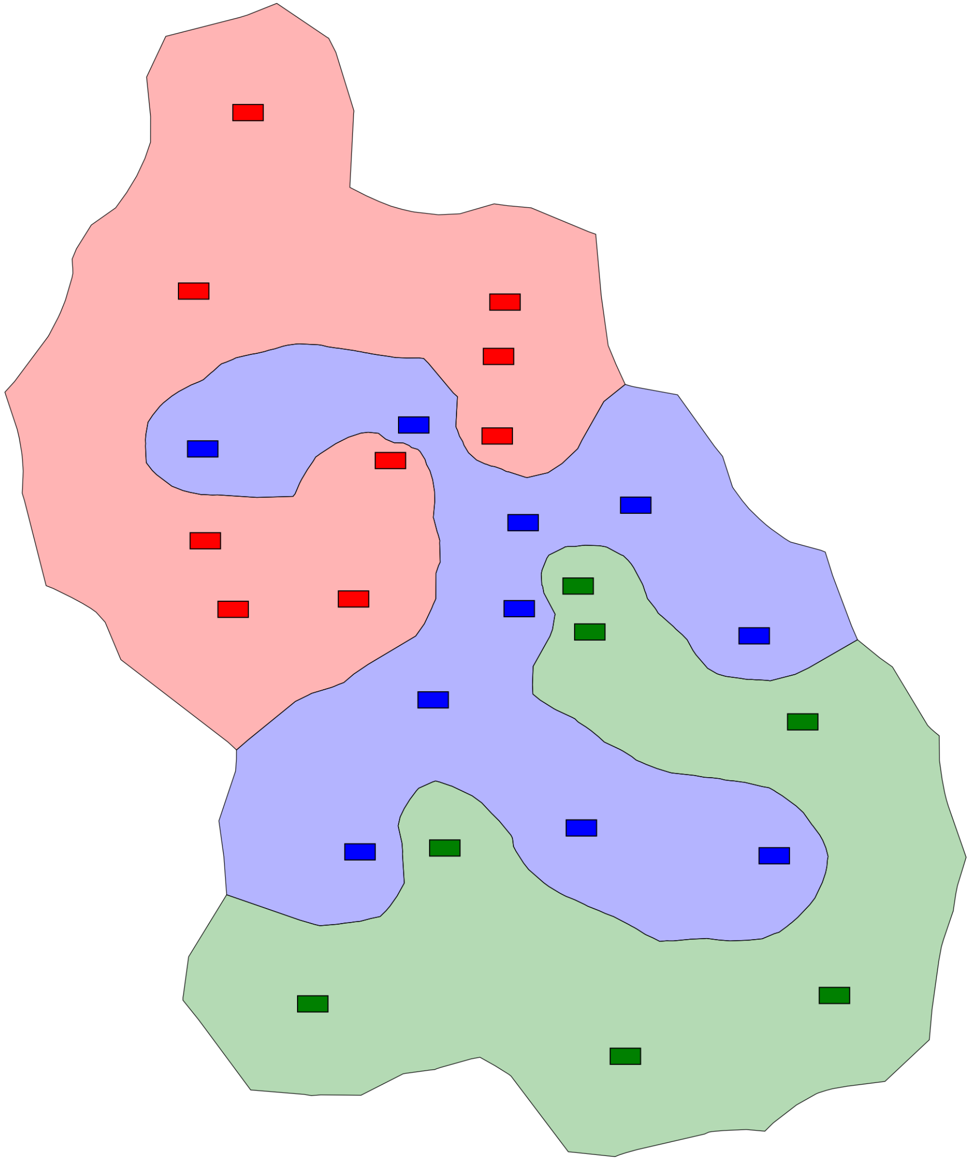
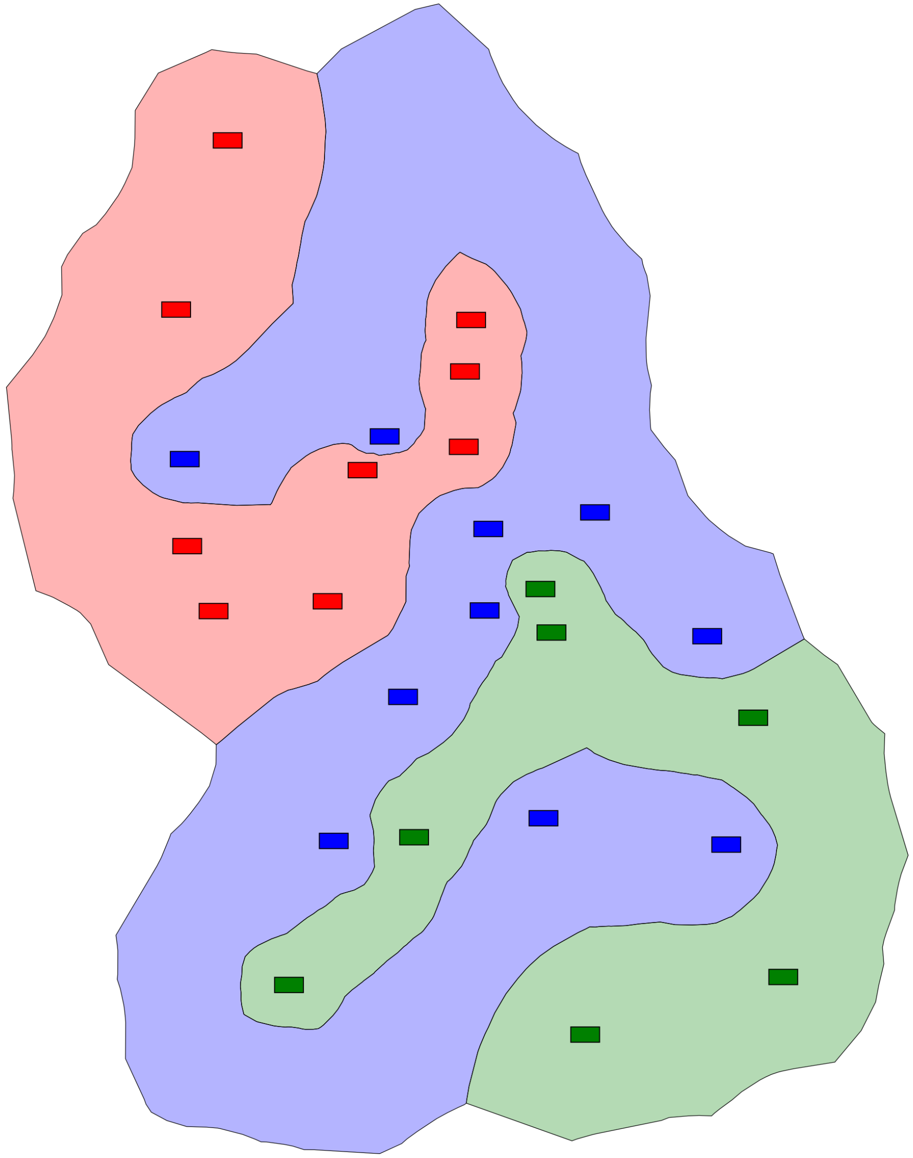
MapSets

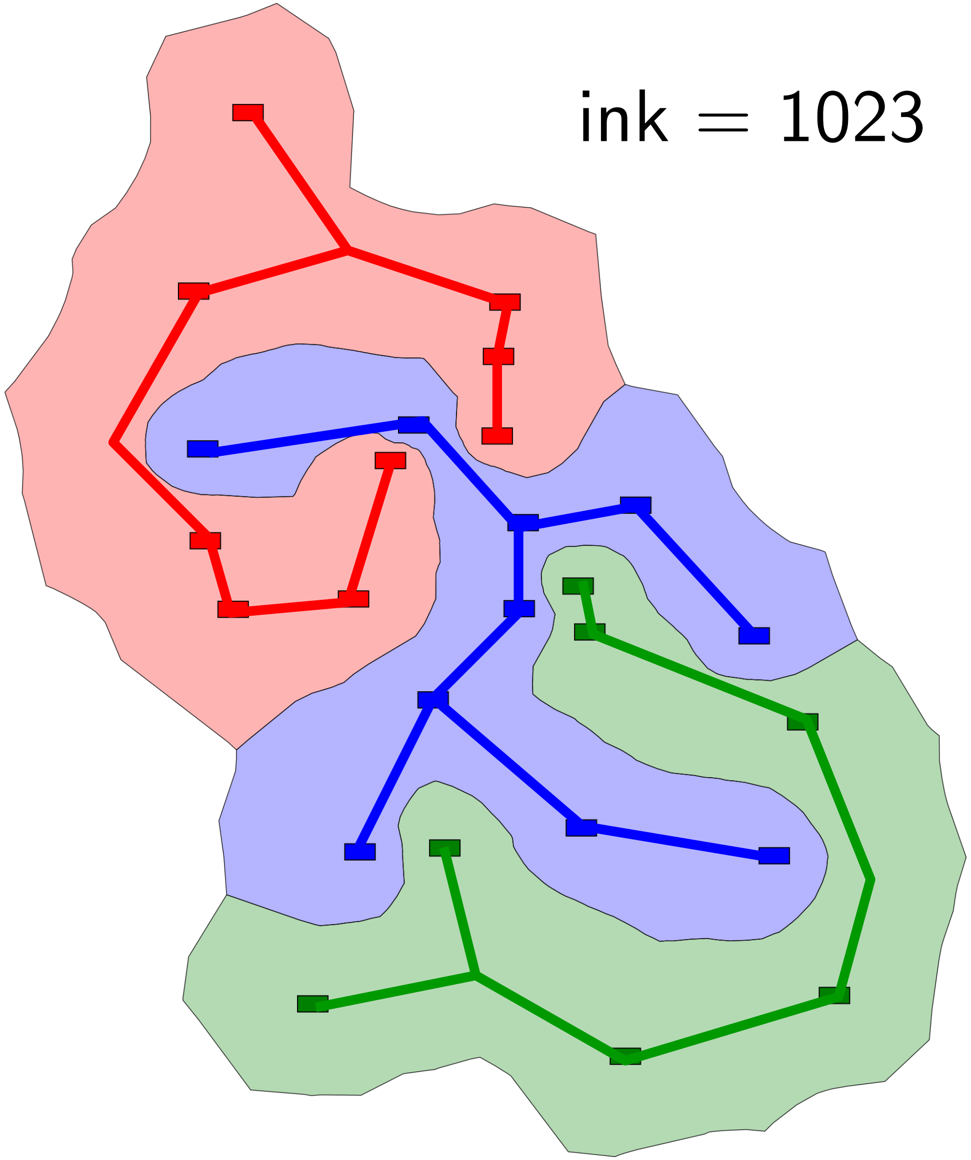


w/o optimizing ink



Examples

MapSets



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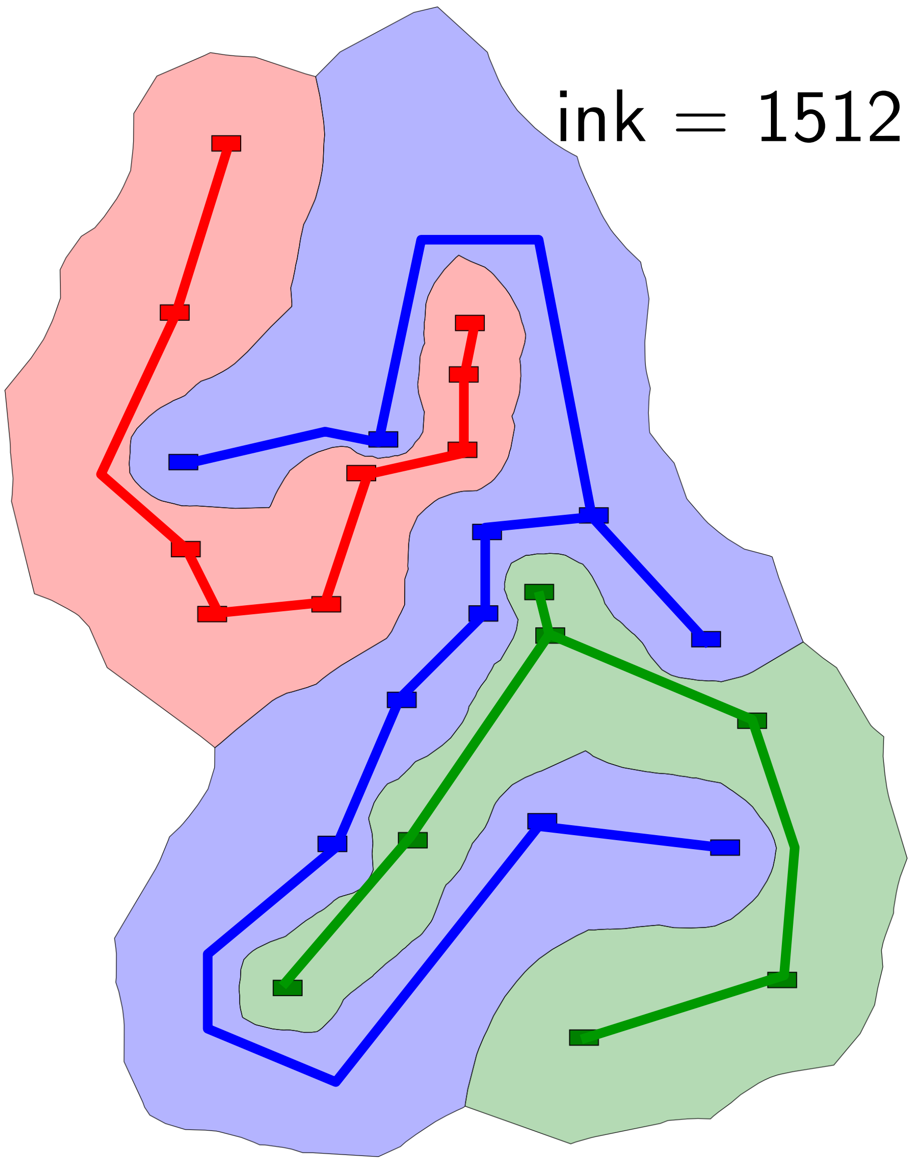
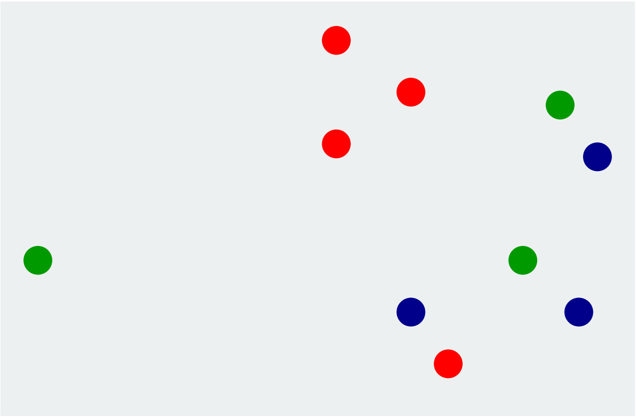


Colored (Euclidean) Spanning Trees

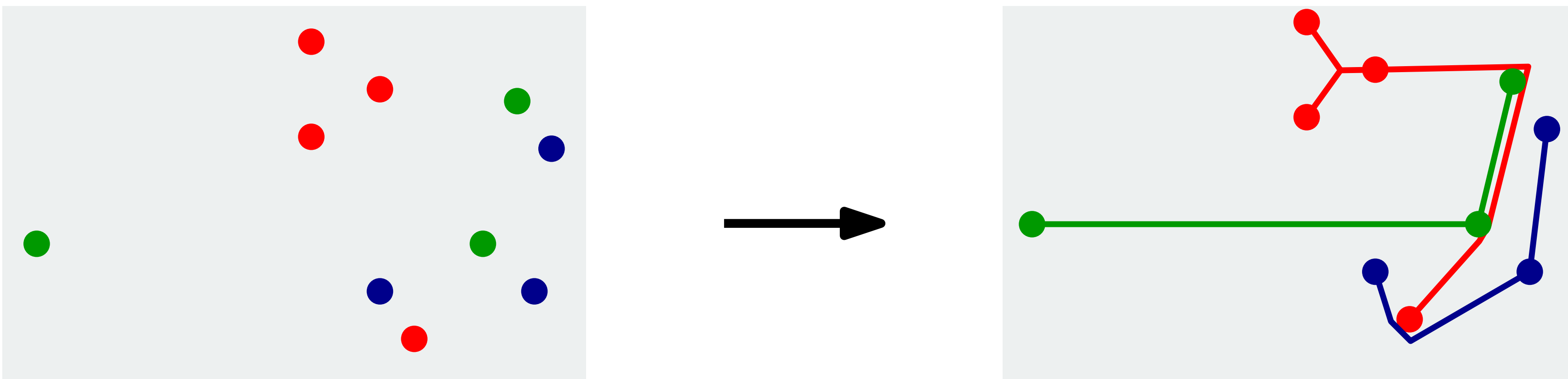
Input



k-colored point set in R^2

Colored (Euclidean) Spanning Trees

Input | **Output**

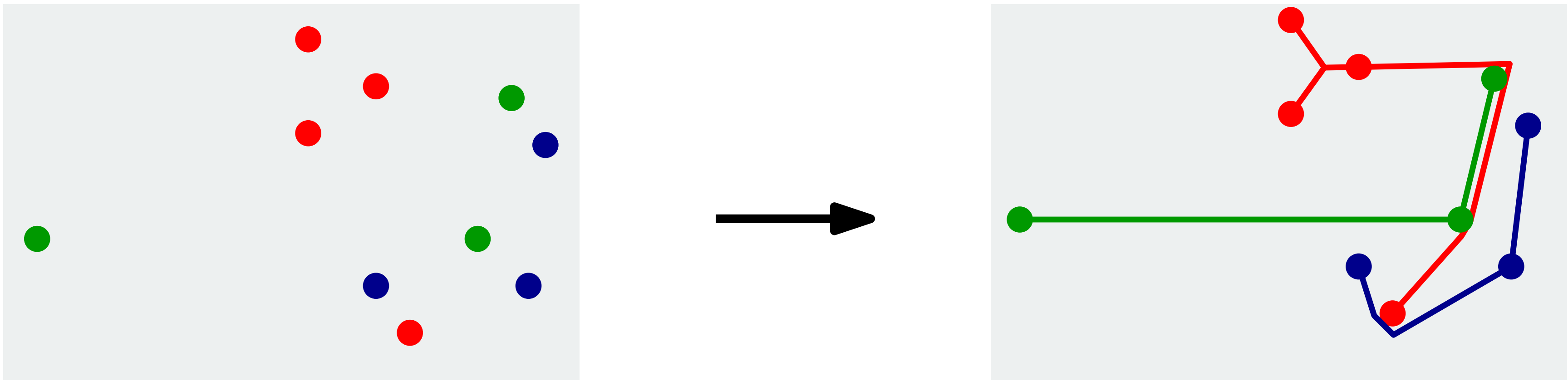


k-colored point set in R^2

k non-crossing Steiner trees

*C*olored (*E*uclidean) *S*panning *T*rees

Input | **Output**



k-colored point set in R^2 | k non-crossing Steiner trees

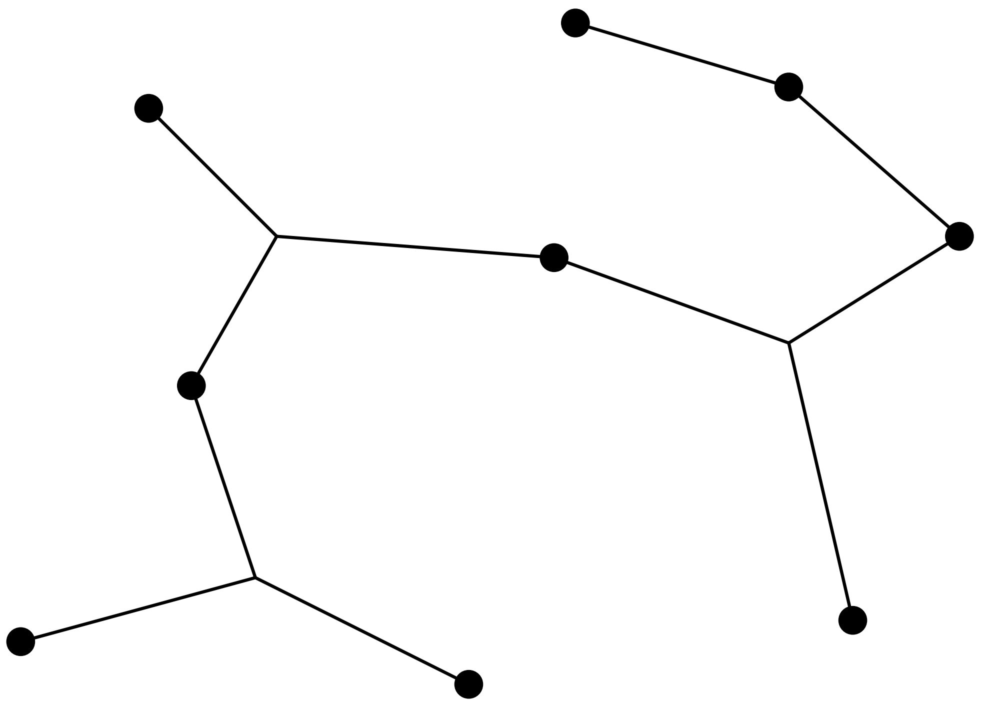
CST: Minimize total length!

*C*olored (*E*uclidean) *S*panning *T*rees

Observation 1 CST is NP-hard

*C*olored (*E*uclidean) *S*panning *T*rees

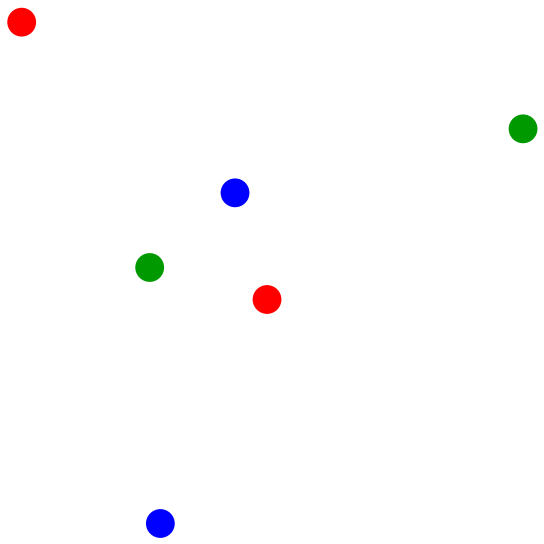
Observation 1 CST is NP-hard, even if $k = 1$



*C*olored (*E*uclidean) *S*panning *T*rees

Observation 1 CST is NP-hard, even if $k = 1$

Observation 2 CST is NP-hard, even if
– Steiner points are not allowed
– every cluster consists of *two* points [Bastert Fekete, TR'96]



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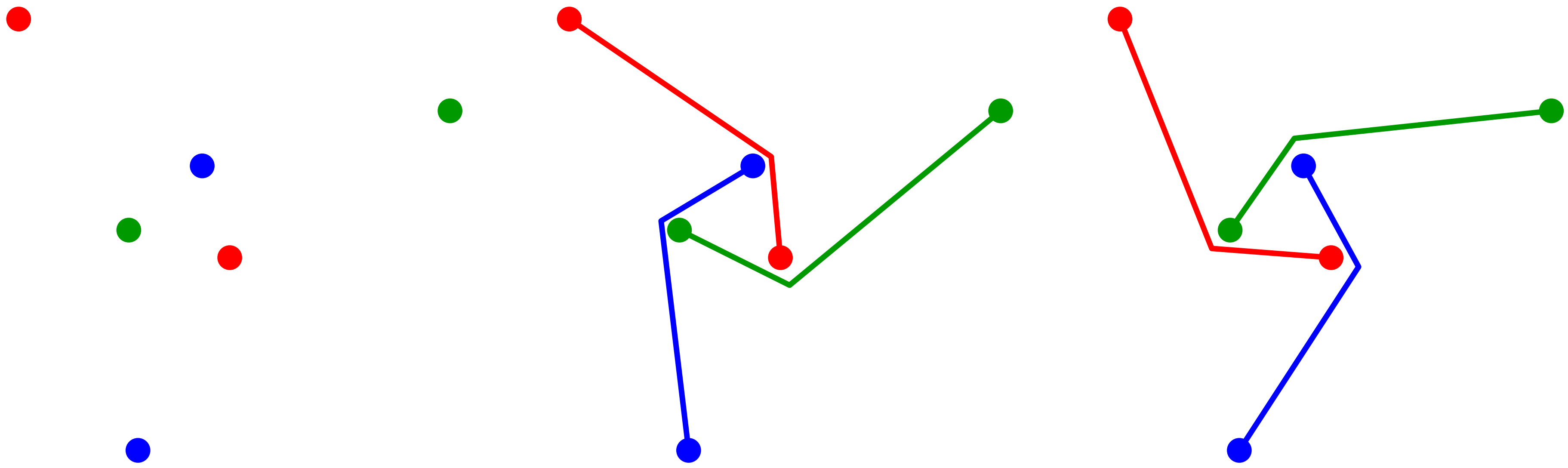

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Observation 3 CST (with $k = n/2$) is equivalent to
MIN. LENGTH EMBEDDING OF *Matchings* AT FIXED VERTEX LOCATIONS
[Chan Hoffmann Kiazyk Lubiw, GD'13]

Colored (Euclidean) Spanning Trees

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MIN. LENGTH EMBEDDING OF *Matchings* AT FIXED VERTEX LOCATIONS
[Chan Hoffmann Kiazyk Lubiw, GD'13]

Theorem (Chan et al.) CST (with $k = n/2$) admits an $O(\sqrt{k} \log k)$-approximation

*C*olored (*E*uclidean) *S*panning *T*rees

Theorem CST (with k colors) admits a $(k\rho)$-approximation

Colored (Euclidean) Spanning Trees

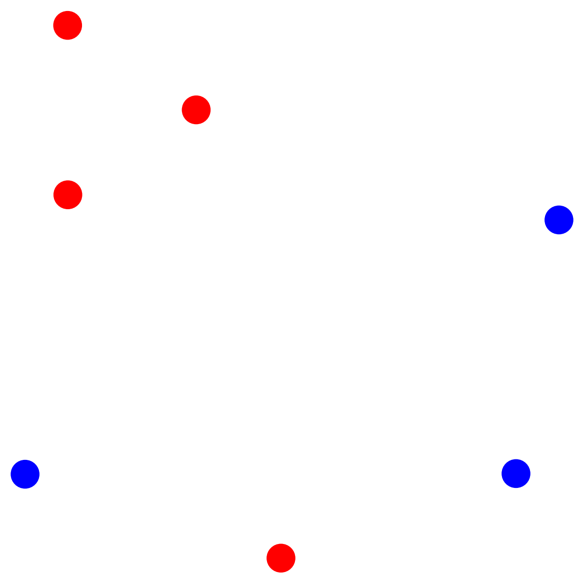
Theorem CST (with k colors) admits a $(k\rho)$-approximation

$1.15 < \rho < 1.22$
Steiner ratio, that is,
$inf\{\frac{|\text{Steiner Tree}|}{|\text{Spanning Tree}|}\}$

*C*olored (*E*uclidean) *S*panning *T*rees

Theorem CST (with k colors) admits a $(k\rho)$-approximation

Proof Algorithm ($k = 2$):



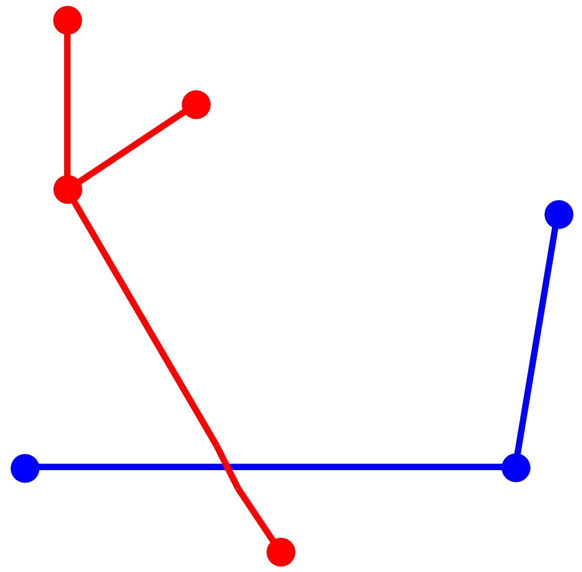
Analysis:

*C*olored (*E*uclidean) *S*panning *T*rees

Theorem CST (with k colors) admits a $(k\rho)$-approximation

Proof Algorithm ($k = 2$):
– construct red and blue minimum spanning trees

Analysis:



*C*olored (*E*uclidean) *S*panning *T*rees

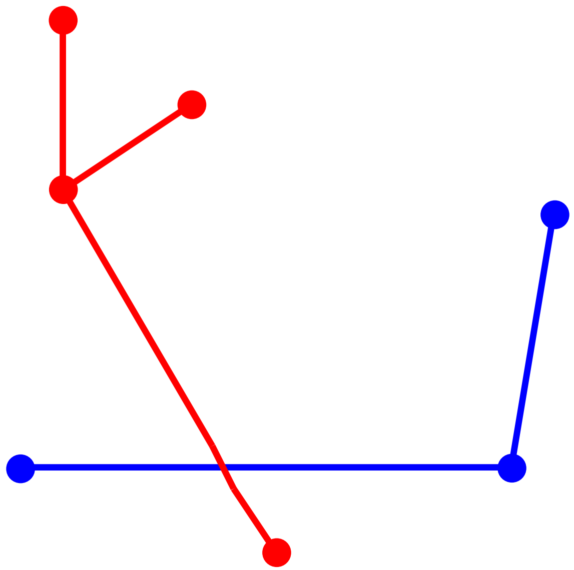
Theorem CST (with k colors) admits a $(k\rho)$-approximation

Proof Algorithm ($k = 2$):

– construct red and blue minimum spanning trees

– take the shorter one

Analysis:



Colored (Euclidean) Spanning Trees

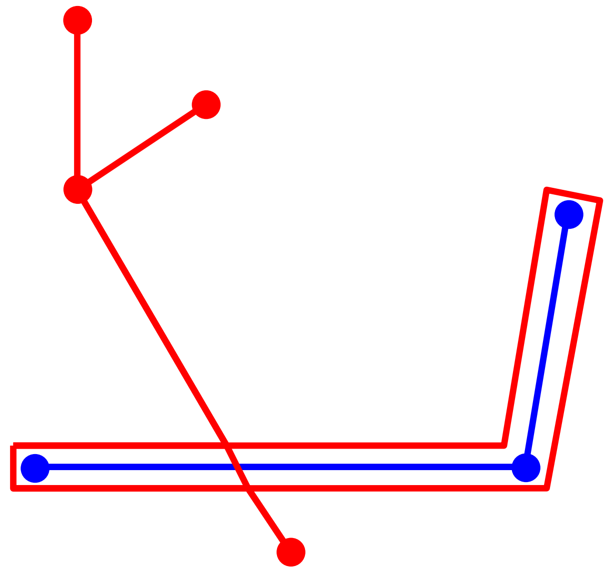
Theorem CST (with k colors) admits a $(k\rho)$-approximation

Proof Algorithm ($k = 2$):

– construct red and blue minimum spanning trees

– take the shorter one , add a "shell" around it of another color

Analysis:



*C*olored (*E*uclidean) *S*panning *T*rees

Theorem CST (with k colors) admits a $(k\rho)$-approximation

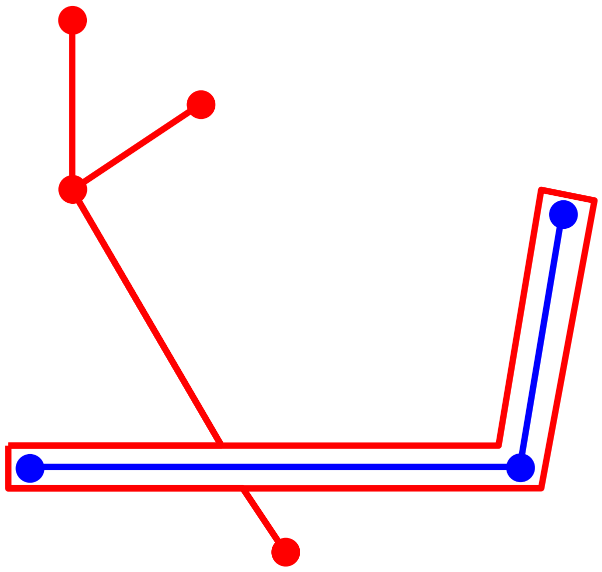
Proof Algorithm ($k = 2$):

– construct red and blue minimum spanning trees

– take the shorter one , add a “shell” around it of another color

– remove crossings

Analysis:



$\mathcal{C}$olored ($\mathcal{E}$uclidean) $\mathcal{S}$panning $\mathcal{T}$rees

Theorem CST (with k colors) admits a $(k\rho)$-approximation

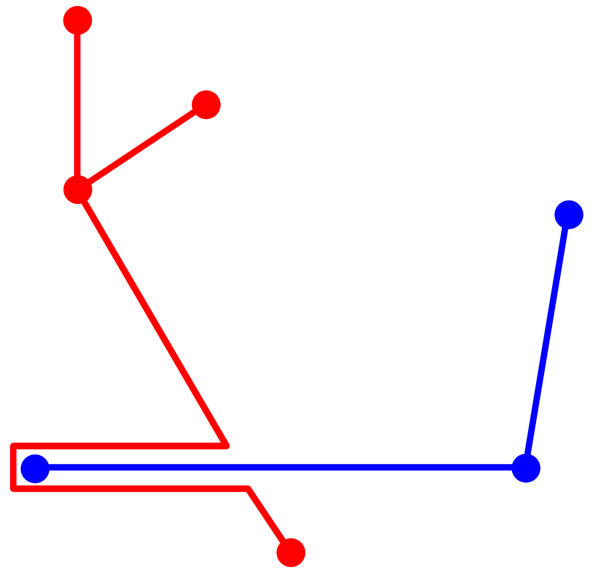
Proof Algorithm ($k = 2$):

– construct red and blue minimum spanning trees

– take the shorter one , add a "shell" around it of another color

– remove crossings and cycles

Analysis:



*C*olored (*E*uclidean) *S*panning *T*rees

Theorem CST (with k colors) admits a $(k\rho)$-approximation

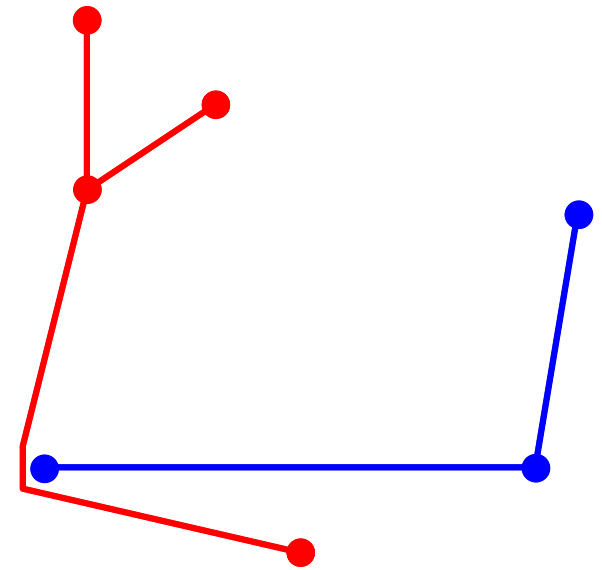
Proof Algorithm ($k = 2$):

– construct red and blue minimum spanning trees

– take the shorter one , add a “shell” around it of another color

– remove crossings and cycles, shortcut

Analysis:



Colored (Euclidean) Spanning Trees

Theorem CST (with k colors) admits a $(k\rho)$-approximation

Proof Algorithm ($k = 2$):

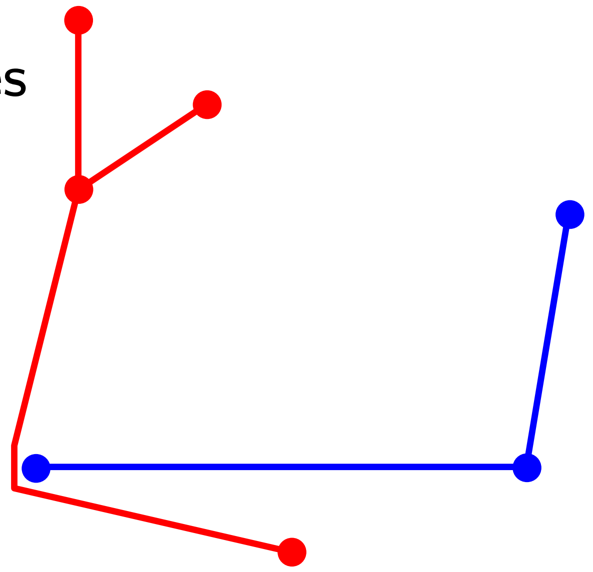
– construct red and blue minimum spanning trees

– take the shorter one , add a "shell" around it of another color

– remove crossings and cycles , shortcut

Analysis:

– let OPT_B, OPT_R be optimal non-crossing trees



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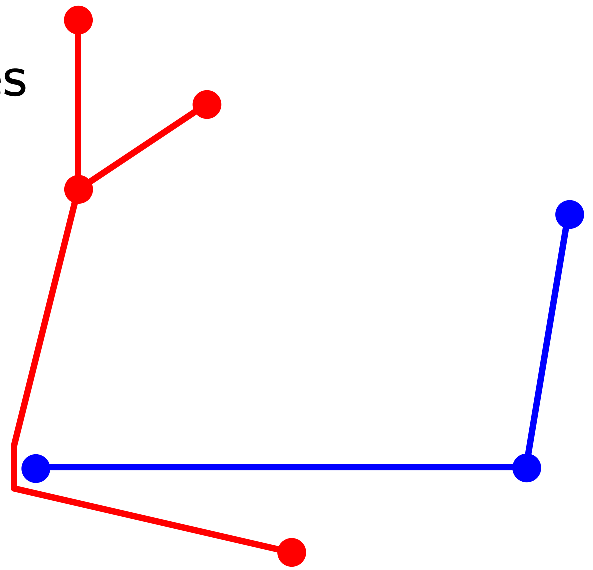

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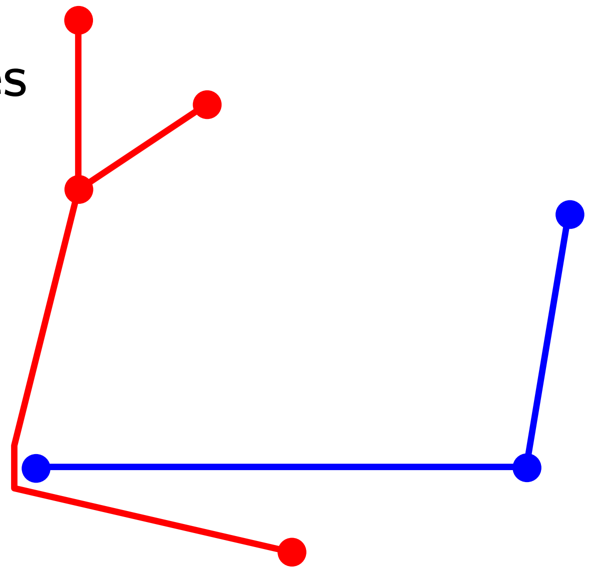

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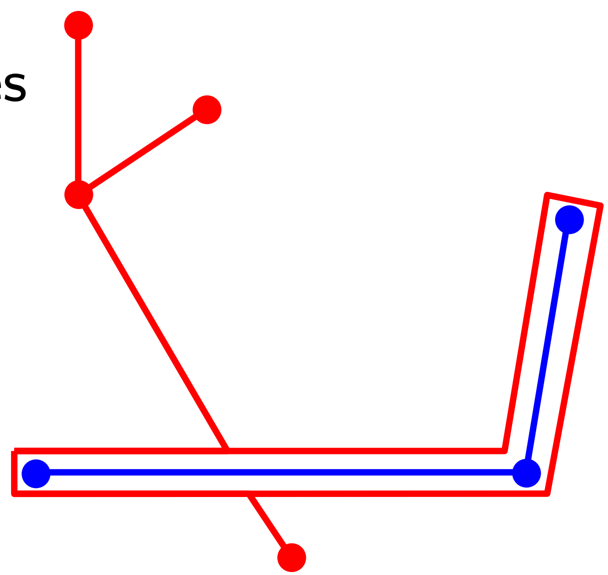

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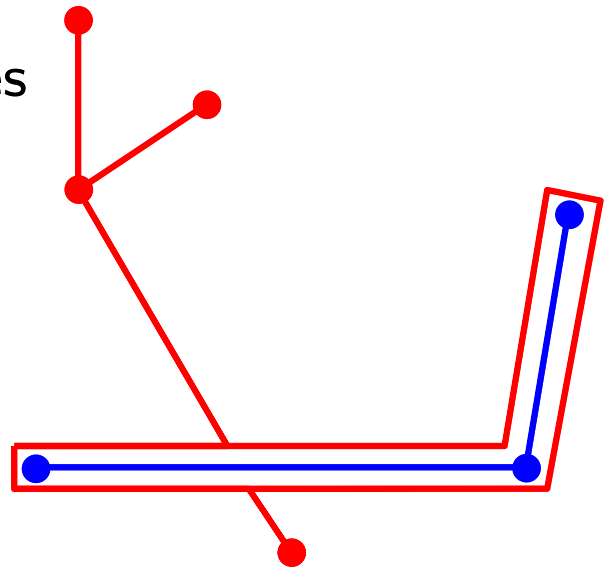

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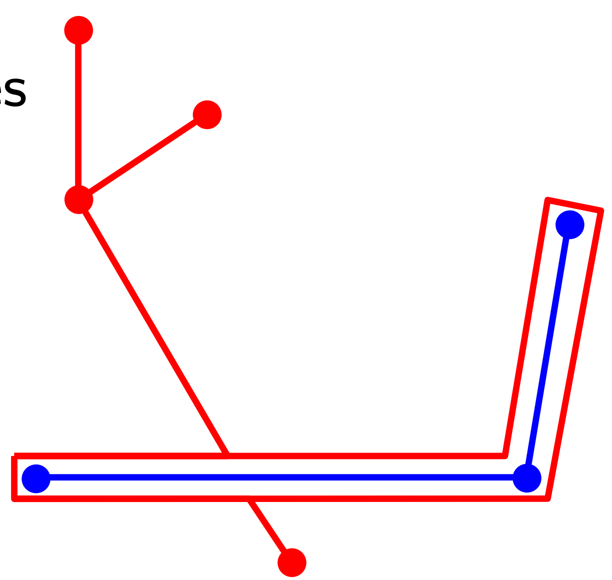

$$\frac{\text{ALG}}{\text{OPT}} \leq \frac{|\,\text{MST}_R\,| + 3|\,\text{MST}_B\,|}{|\text{Steiner Tree}_R| + |\text{Steiner Tree}_B|} \leq \rho \frac{|\,\text{MST}_R\,| + 3|\,\text{MST}_B\,|}{|\,\text{MST}_R\,| + |\,\text{MST}_B\,|}$$

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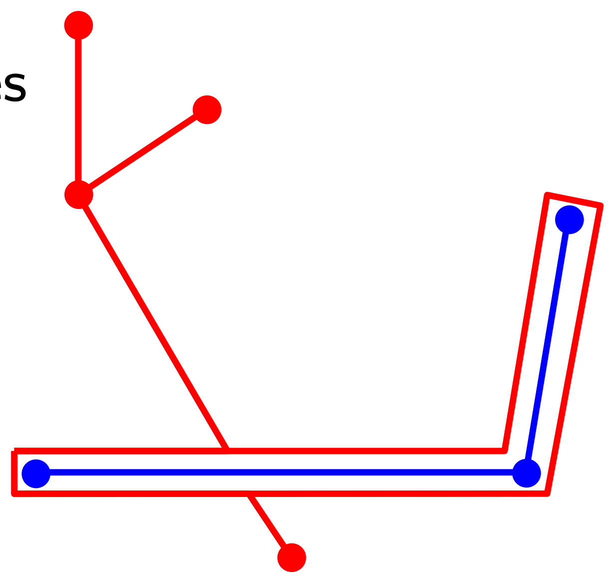

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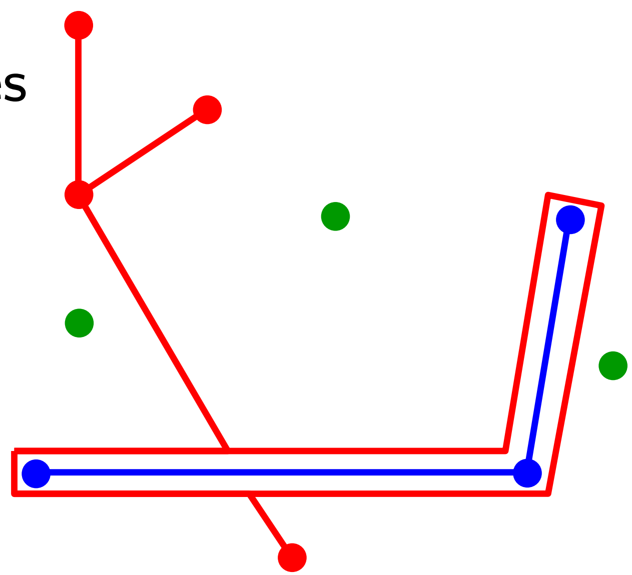

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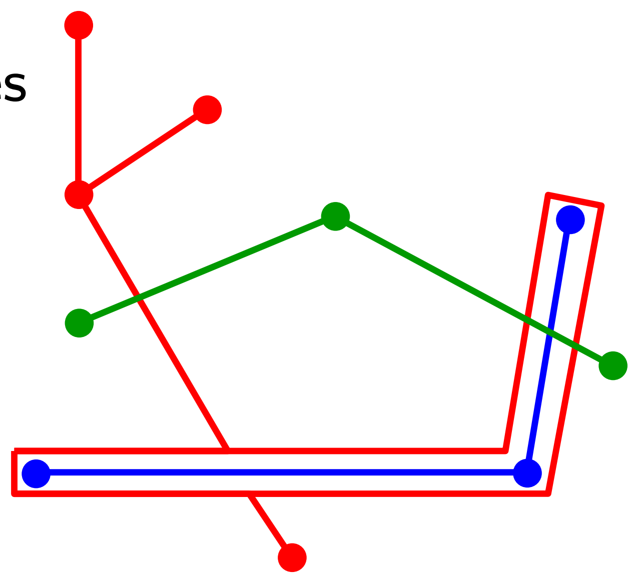

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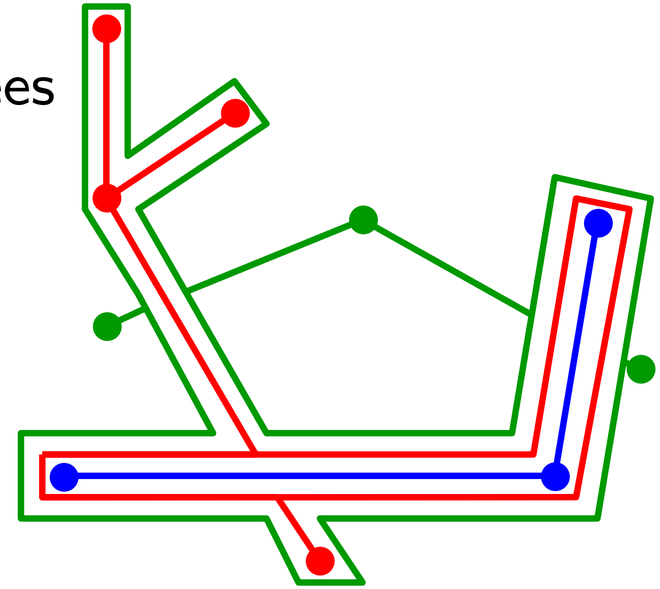

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Colored (*E*uclidean) *S*panning *T*rees

Theorem CST (with k colors) admits a $(k\rho)$-approximation

Proof Algorithm ($k = 2$): $(1 + \varepsilon)$-approx. Steiner
– construct red and blue ~~minimum spanning~~ trees
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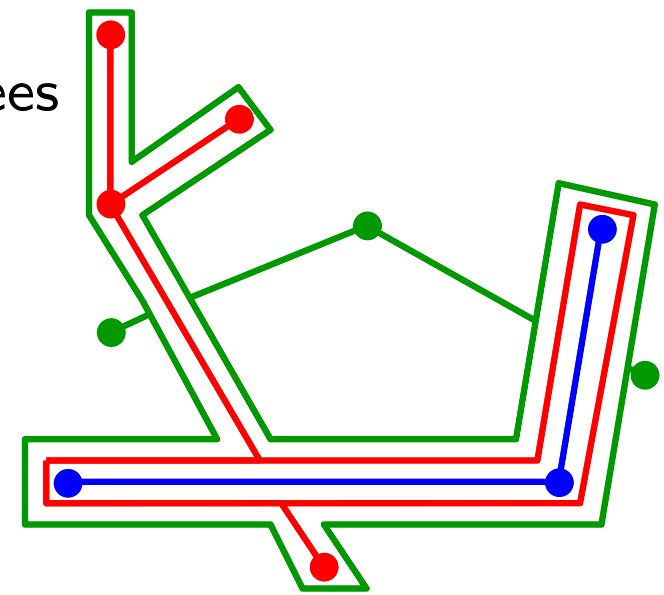

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Colored (Euclidean) Spanning Trees

$(k+\varepsilon)$

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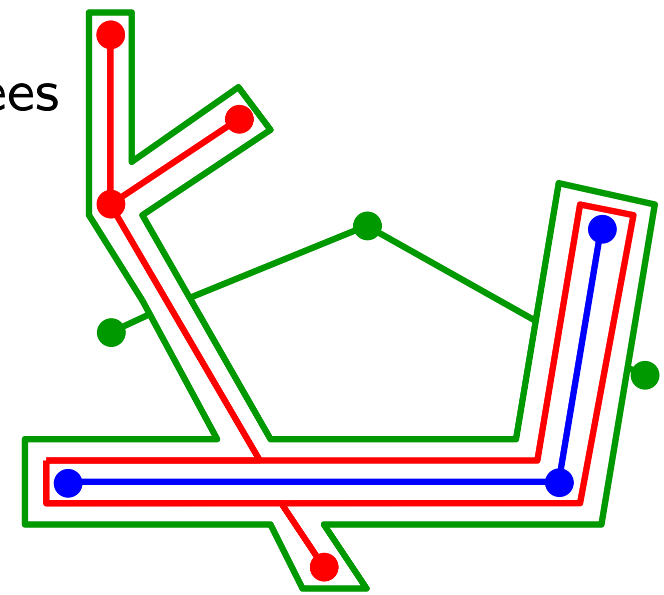

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