Proximity Drawability

K-weak Delaunay Drawability

Strong, weak, k-weak Delaunay proximity

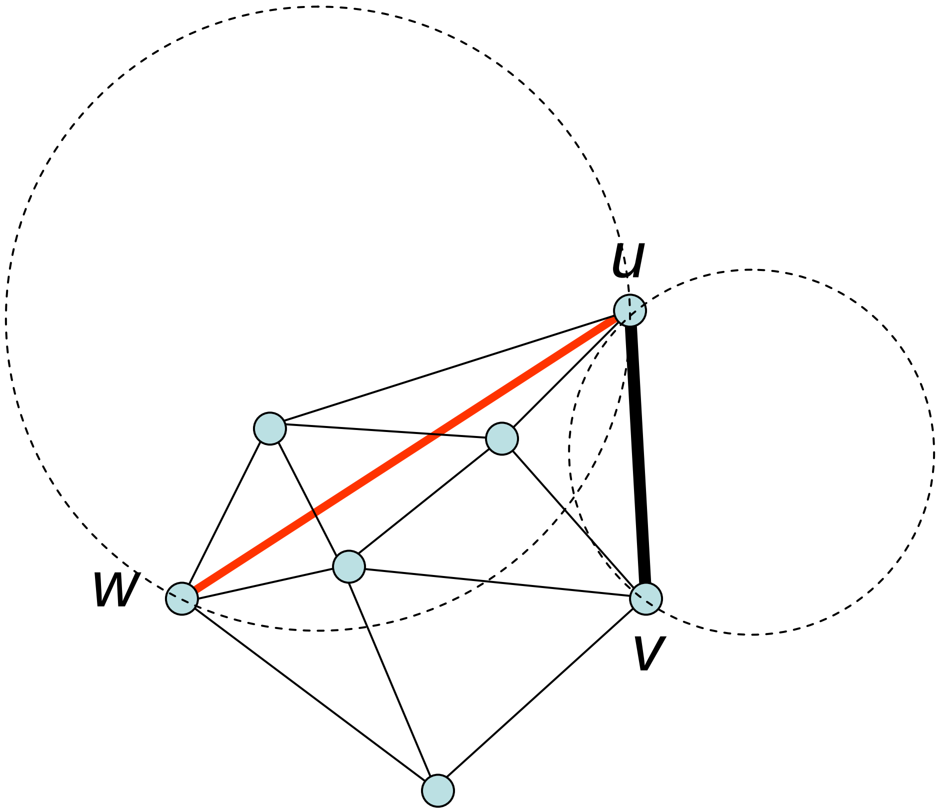
- **R(u,v)** is a disk containing u and v; it is assumed to be a closed set

- **Strong**: (u,v) $\in \Gamma \Leftrightarrow \exists$ R(u,v) that does not contain other vertices

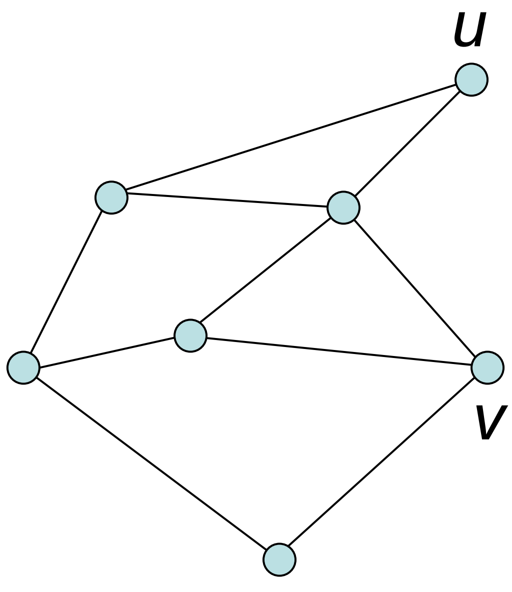
- **Weak**: (u,v) $\in \Gamma \Rightarrow \exists$ R(u,v) that does not contain other vertices

- ***k*-weak**: (u,v) $\in \Gamma \Rightarrow \exists$ R(u,v) that can only contain vertices that are at least (k+1)-hops from u and v

Strong Delaunay Drawings

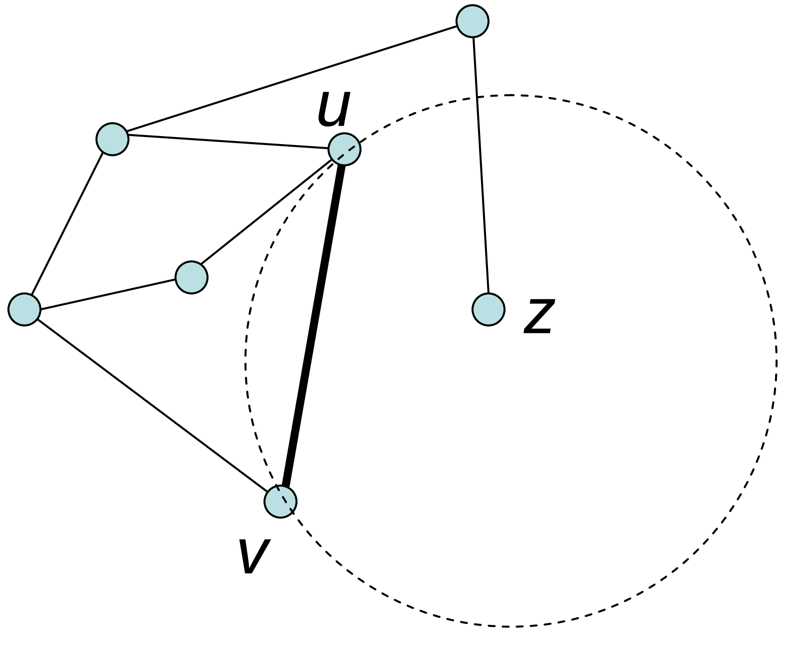


Weak Delaunay Drawings



k-weak Delaunay Drawings

$k = 2$



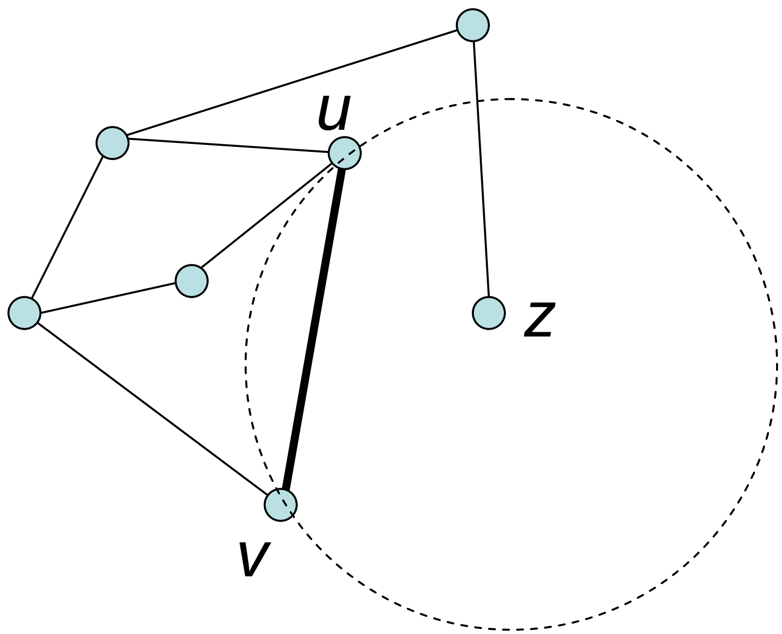
Vertex *z* has a theoretic distance from *u* and *v* greater than *k*

The general problem

- Characterize *k*-weak Delaunay drawable graphs
  - *k*-weak Delaunay drawable graphs are called D_k-drawable graphs, and the corresponding drawing is called a D_k-drawing

Preliminary observations

- A D_k-drawable graphs is also a $D_{k\text{-}1}$-drawable graphs



- This is both a D_2-drawing and a graphs is also a D_1-drawing

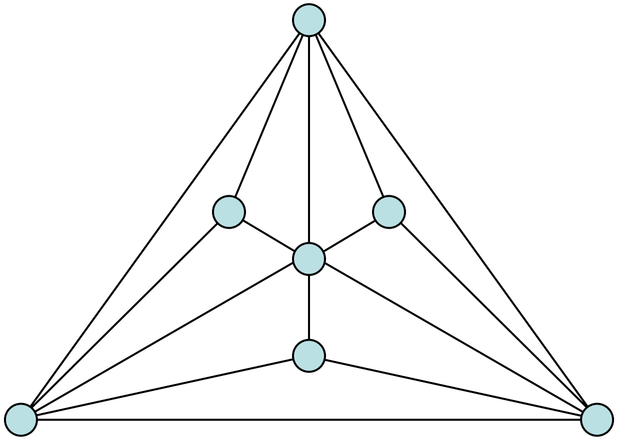
D_2-drawability: Preliminaries

- For planar graphs, the value *k*=2 seems to be particularly interesting
  - It is known that every 2-weak Delaunay drawing has a linear number of edges (Pinchasi & Smorodinsky, SoCG 2004)

- Connected outerplanar graphs are D_2-drawable
  - Consequence of a paper by Lenhart & Liotta, GD‘96

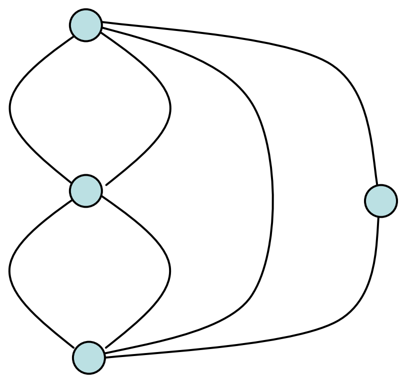
D_2-drawability: Preliminaries

- Not all planar graphs are D_2-drawable.
  - Consequence of a paper by Dillencourt, DCG'90



Specific questions

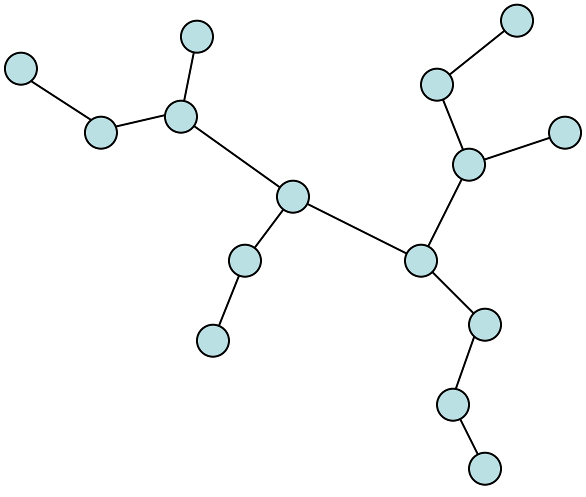
- Are two-terminal series parallel graphs D_2-drawable?
- Are bipartite planar graphs D_2-drawable?
- Variants: Values of k larger than 2, k-weak Gabriel drawability, ….



a two-terminal series parallal graph

Approximating a Minimum Spanning Tree

Minimum Spanning Tree



Minimum weight-drawability of trees

Let T be a tree. Can T be drawn as the minimum spanning tree of the points representing its vertices?

Preliminaries

- Each tree with vertex degree at most 5 can be drawn as a MST (Monma and Suri, DCG’92)
- Each tree having vertex degree greater than 6 is not drawable as a MST (Monma and Suri , DCG’92)
- For trees with maximum vertex degree 6 the problem is NP-Hard (Eades and Whitesides, Algorithmica’96)

Question

- Let T be a tree having maximum vertex degree d ($d > 5$). Compute a straight-line drawing of T such that its total edge length is at most $f(d)$ times the total edge length of the MST of the points representing the vertices
  - $f(d)$ is a function of d but it does not depend on the size of T