Column Planarity and Partial Simultaneous Geometric Embedding

Will Evans[1] Vincent Kusters[2] Maria Saumell[3] Bettina Speckmann[4]

[1]University of British Columbia

[2]ETH Zurich

[3]University of West Bohemia

[4]Technical University Eindhoven

September 25, 2014

Preliminaries

Plane straight-line embedding (PSLE) of a planar graph $G = (V, E)$:

- embed vertices as points;
- embed edges as straight-line segments;
- no two edges intersect.

Preliminaries

Plane straight-line embedding (PSLE) of a planar graph $G = (V, E)$:

- embed vertices as points;
- embed edges as straight-line segments;
- no two edges intersect.

Since a PSLE is completely determined by the drawing of its vertices, we represent a PSLE by a function $\varphi : V \to \mathbb{R}^2$.

Preliminaries

Plane straight-line embedding (PSLE) of a planar graph $G = (V, E)$:

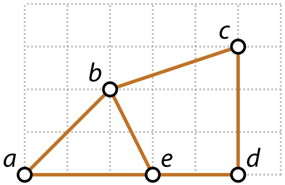
- embed vertices as points;
- embed edges as straight-line segments;
- no two edges intersect.

Since a PSLE is completely determined by the drawing of its vertices, we represent a PSLE by a function $\varphi : V \to \mathbb{R}^2$.

$$V = \{a, b, c, d, e\}$$

$$E = \{ab, ae, bc, be, cd, de\}$$

$$\begin{aligned}\varphi = \{a &\to (0,0)\\ b &\to (2,2)\\ c &\to (5,3)\\ d &\to (5,0)\\ e &\to (3,0)\}\end{aligned}$$



Column Planarity

Column Planarity is a strengthening of Unlabeled Level Planarity.

Column Planarity

Column Planarity is a strengthening of Unlabeled Level Planarity.

$G = (V, E)$ is Unlabeled Level Planar (ULP) if

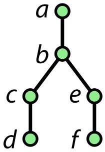
- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.

Column Planarity

Column Planarity is a strengthening of Unlabeled Level Planarity.

$G = (V, E)$ is Unlabeled Level Planar (ULP) if

- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.

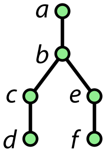


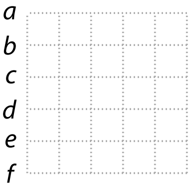
Column Planarity

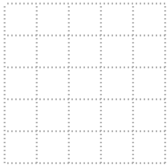
Column Planarity is a strengthening of Unlabeled Level Planarity.

$G = (V, E)$ is Unlabeled Level Planar (ULP) if

- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.





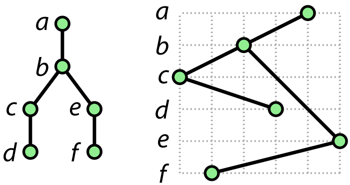


Column Planarity

Column Planarity is a strengthening of Unlabeled Level Planarity.

$G = (V, E)$ is Unlabeled Level Planar (ULP) if

- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.

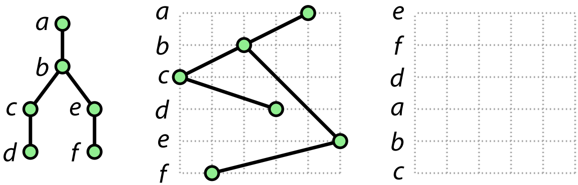


Column Planarity

Column Planarity is a strengthening of Unlabeled Level Planarity.

$G = (V, E)$ is Unlabeled Level Planar (ULP) if

- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.

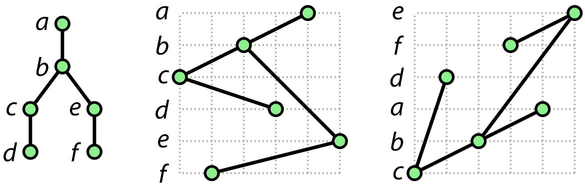


Column Planarity

Column Planarity is a strengthening of Unlabeled Level Planarity.

$G = (V, E)$ is Unlabeled Level Planar (ULP) if

- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.

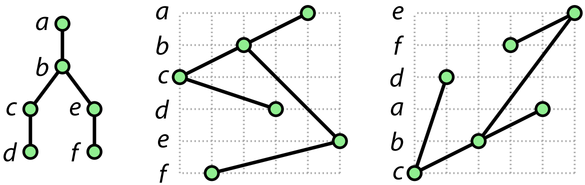


Column Planarity

Column Planarity is a strengthening of Unlabeled Level Planarity.

$G = (V, E)$ is Unlabeled Level Planar (ULP) if

- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.



Fully characterized by (Estrella-Balderrama, Fowler, and Kobourov 2007) and (Fowler and Kobourov 2008).

Column Planarity

Column Planarity is a strengthening of Unlabeled Level Planarity.

$G = (V, E)$ is Unlabeled Level Planar (ULP) if

- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.

Column Planarity

Column Planarity is a strengthening of Unlabeled Level Planarity.

$G = (V, E)$ is Unlabeled Level Planar (ULP) if

- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.

$G = (V, E)$ is Column Planar (CP) if

- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.

Column Planarity

Column Planarity is a strengthening of Unlabeled Level Planarity.

$G = (V, E)$ is Unlabeled Level Planar (ULP) if

- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.

$G = (V, E)$ is Column Planar (CP) if

- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.

Fully characterized by (Di Giacomo, Didimo, Liotta, Meijer, and Wismath 2014-09-24 10:45-11:05); they call them EAP graphs.

Column Planarity

Column Planarity is a strengthening of Unlabeled Level Planarity.

$G = (V, E)$ is Unlabeled Level Planar (ULP) if

- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.

$G = (V, E)$ is Column Planar (CP) if

- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.

Column Planarity

Column Planarity is a strengthening of Unlabeled Level Planarity.

$G = (V, E)$ is Unlabeled Level Planar (ULP) if

- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.

$G = (V, E)$ is Column Planar (CP) if

- $\exists \mathbf{x} : V \to \mathbb{R}$:
- $\forall \mathbf{y} : V \to \mathbb{R}$:
- $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ is a PLSE.

A set $R \subseteq V$ is Column Planar in $G = (V, E)$ if

- $\exists \mathbf{x} : R \to \mathbb{R}$:
- $\forall \mathbf{y} : R \to \mathbb{R}$:
- there is a PLSE φ of G with $\varphi(v) = (\mathbf{x}(v), \mathbf{y}(v))$ for all $v \in R$.

G is ULP

$\forall \mathbf{y} : V \to \mathbb{R}$

$\exists \mathbf{x} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE

G is CP

$\exists \mathbf{x} : V \to \mathbb{R}$

$\forall \mathbf{y} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE

$R \subseteq V$ is CP in G

$\exists \mathbf{x} : R \to \mathbb{R}$

$\forall \mathbf{y} : R \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$

extendable to

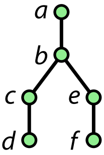
PSLE of G

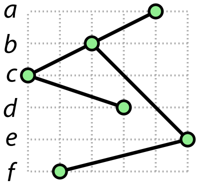
G is ULP

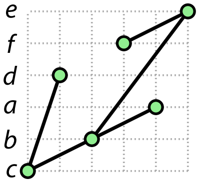
$\forall \mathbf{y} : V \to \mathbb{R}$

$\exists \mathbf{x} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE







G is CP

$\exists \mathbf{x} : V \to \mathbb{R}$

$\forall \mathbf{y} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE

$R \subseteq V$ is CP in G

$\exists \mathbf{x} : R \to \mathbb{R}$

$\forall \mathbf{y} : R \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$

extendable to

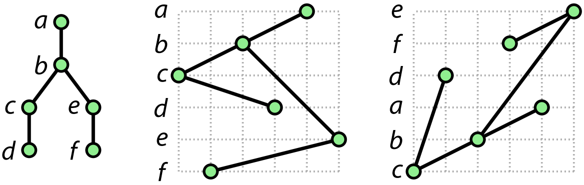
PSLE of G

G is ULP

$\forall \mathbf{y} : V \to \mathbb{R}$

$\exists \mathbf{x} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE

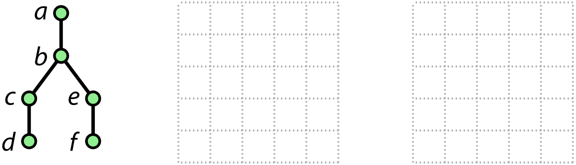


G is CP

$\exists \mathbf{x} : V \to \mathbb{R}$

$\forall \mathbf{y} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE



$R \subseteq V$ is CP in G

$\exists \mathbf{x} : R \to \mathbb{R}$

$\forall \mathbf{y} : R \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$

extendable to

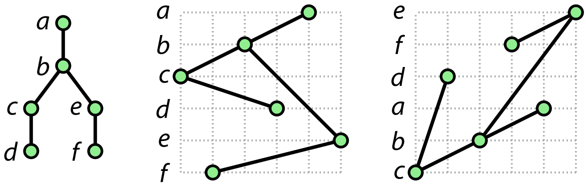
PSLE of G

G is ULP

$\forall \mathbf{y} : V \to \mathbb{R}$

$\exists \mathbf{x} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE

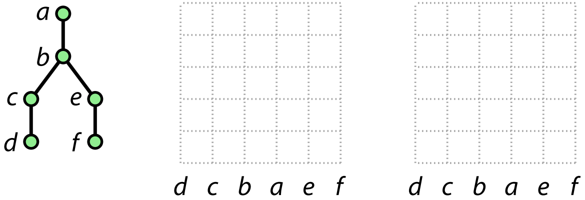


G is CP

$\exists \mathbf{x} : V \to \mathbb{R}$

$\forall \mathbf{y} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE



$R \subseteq V$ is CP in G

$\exists \mathbf{x} : R \to \mathbb{R}$

$\forall \mathbf{y} : R \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$

extendable to

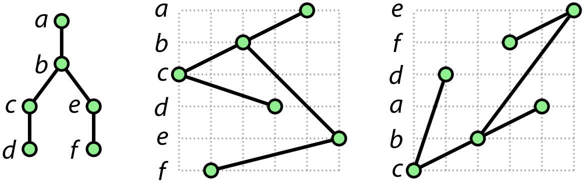
PSLE of G

G is ULP

$\forall \mathbf{y} : V \to \mathbb{R}$

$\exists \mathbf{x} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE

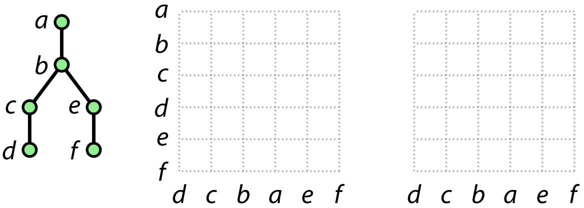


G is CP

$\exists \mathbf{x} : V \to \mathbb{R}$

$\forall \mathbf{y} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE



$R \subseteq V$ is CP in G

$\exists \mathbf{x} : R \to \mathbb{R}$

$\forall \mathbf{y} : R \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$

extendable to

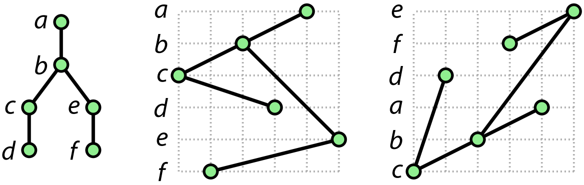
PSLE of G

G is ULP

$\forall \mathbf{y} : V \to \mathbb{R}$

$\exists \mathbf{x} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE

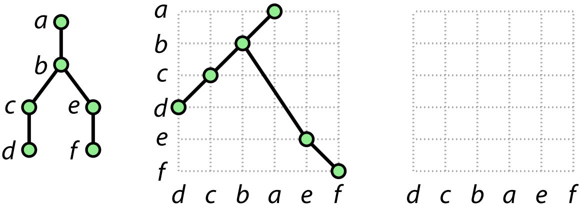


G is CP

$\exists \mathbf{x} : V \to \mathbb{R}$

$\forall \mathbf{y} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE



$R \subseteq V$ is CP in G

$\exists \mathbf{x} : R \to \mathbb{R}$

$\forall \mathbf{y} : R \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$

extendable to

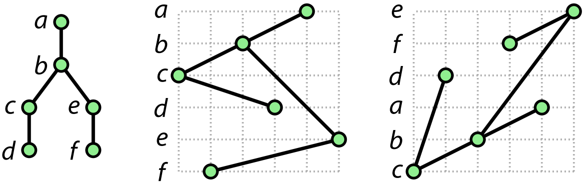
PSLE of G

G is ULP

$\forall \mathbf{y} : V \to \mathbb{R}$

$\exists \mathbf{x} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE

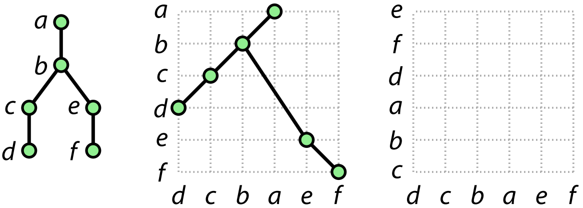


G is CP

$\exists \mathbf{x} : V \to \mathbb{R}$

$\forall \mathbf{y} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE



$R \subseteq V$ is CP in G

$\exists \mathbf{x} : R \to \mathbb{R}$

$\forall \mathbf{y} : R \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$

extendable to

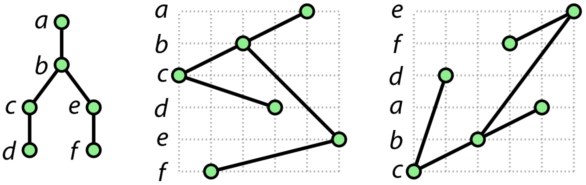
PSLE of G

G is ULP

$\forall \mathbf{y} : V \to \mathbb{R}$

$\exists \mathbf{x} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE

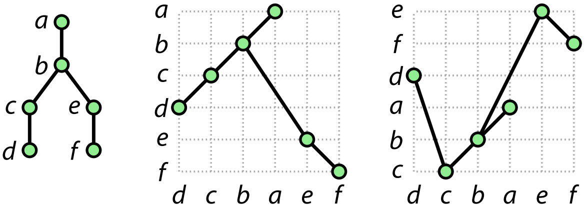


G is CP

$\exists \mathbf{x} : V \to \mathbb{R}$

$\forall \mathbf{y} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE



$R \subseteq V$ is CP in G

$\exists \mathbf{x} : R \to \mathbb{R}$

$\forall \mathbf{y} : R \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$

extendable to

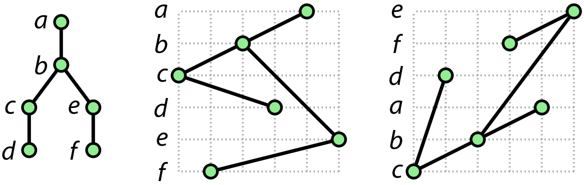
PSLE of *G*

G is ULP

$\forall \mathbf{y} : V \to \mathbb{R}$

$\exists \mathbf{x} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE

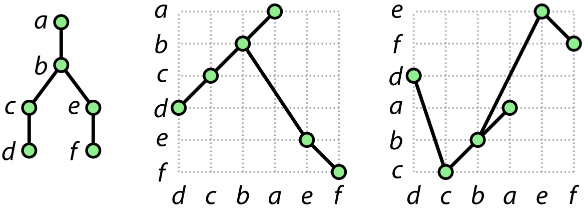


G is CP

$\exists \mathbf{x} : V \to \mathbb{R}$

$\forall \mathbf{y} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE



$R \subseteq V$ is CP in G

$\exists \mathbf{x} : R \to \mathbb{R}$

$\forall \mathbf{y} : R \to \mathbb{R}$

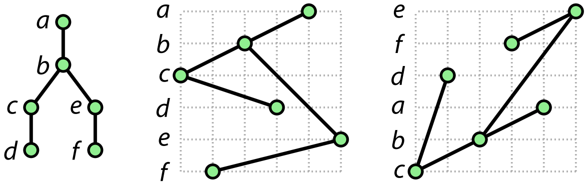
$(\mathbf{x}, \mathbf{y})$ extendable to PSLE of G

G is ULP

$\forall \mathbf{y} : V \to \mathbb{R}$

$\exists \mathbf{x} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE

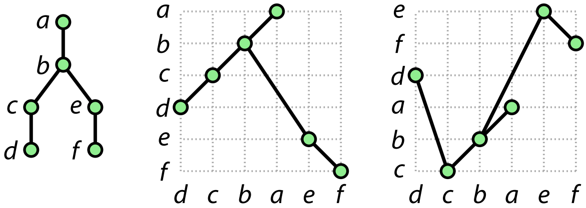


G is CP

$\exists \mathbf{x} : V \to \mathbb{R}$

$\forall \mathbf{y} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE



$R \subseteq V$ is CP in G

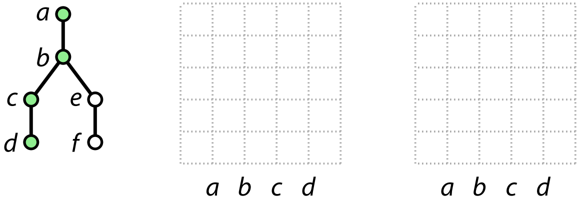
$\exists \mathbf{x} : R \to \mathbb{R}$

$\forall \mathbf{y} : R \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$

extendable to

PSLE of G

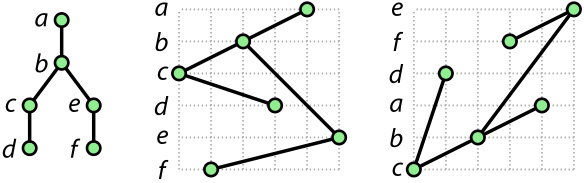


G is ULP

$\forall \mathbf{y} : V \to \mathbb{R}$

$\exists \mathbf{x} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE

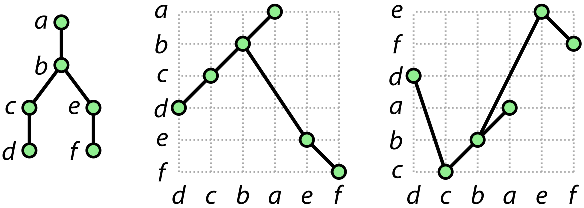


G is CP

$\exists \mathbf{x} : V \to \mathbb{R}$

$\forall \mathbf{y} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE



$R \subseteq V$ is CP in G

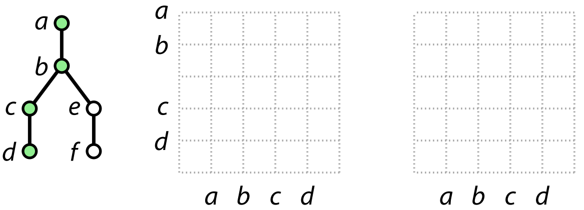
$\exists \mathbf{x} : R \to \mathbb{R}$

$\forall \mathbf{y} : R \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$

extendable to

PSLE of G

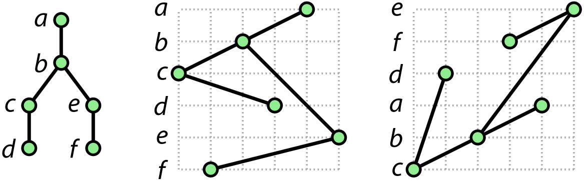


G is ULP

$\forall \mathbf{y} : V \to \mathbb{R}$

$\exists \mathbf{x} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE

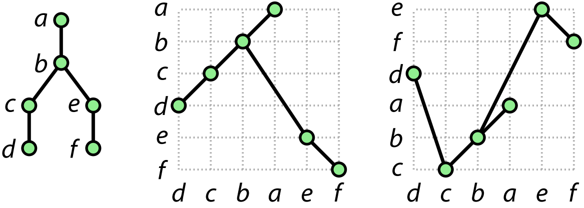


G is CP

$\exists \mathbf{x} : V \to \mathbb{R}$

$\forall \mathbf{y} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE



$R \subseteq V$ is CP in *G*

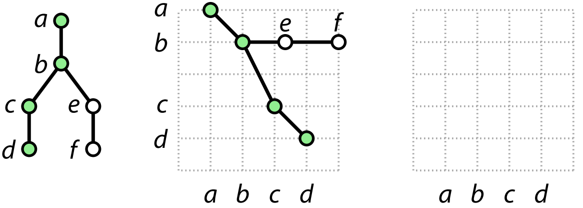
$\exists \mathbf{x} : R \to \mathbb{R}$

$\forall \mathbf{y} : R \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$

extendable to

PSLE of *G*

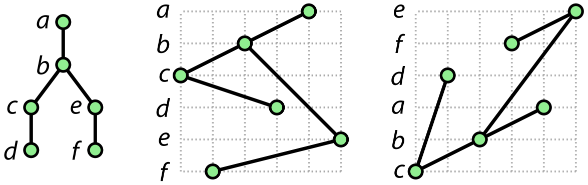


G is ULP

$\forall \mathbf{y} : V \to \mathbb{R}$

$\exists \mathbf{x} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE

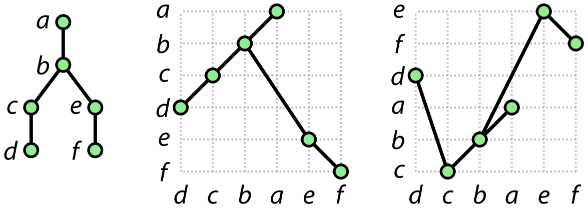


G is CP

$\exists \mathbf{x} : V \to \mathbb{R}$

$\forall \mathbf{y} : V \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$ is PSLE



$R \subseteq V$ is CP in G

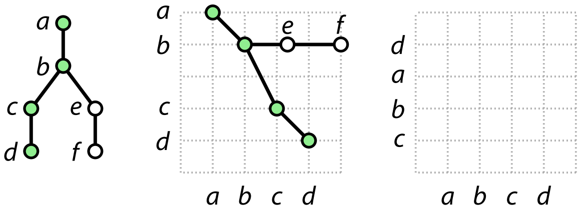
$\exists \mathbf{x} : R \to \mathbb{R}$

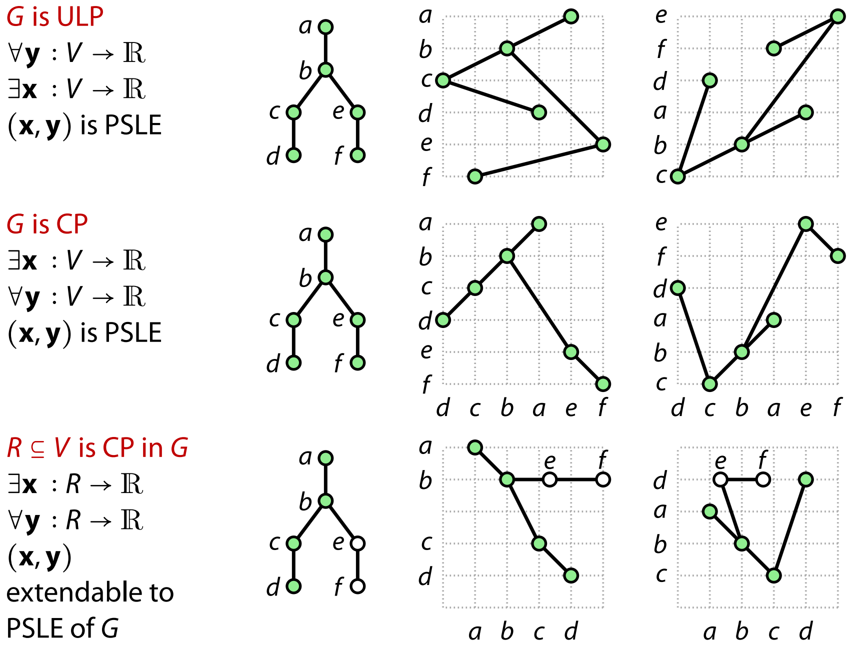
$\forall \mathbf{y} : R \to \mathbb{R}$

$(\mathbf{x}, \mathbf{y})$

extendable to

PSLE of G




G is ULP
$\forall \mathbf{y} : V \to \mathbb{R}$
$\exists \mathbf{x} : V \to \mathbb{R}$
$(\mathbf{x}, \mathbf{y})$ is PSLE
G is CP
$\exists \mathbf{x} : V \to \mathbb{R}$
$\forall \mathbf{y} : V \to \mathbb{R}$
$(\mathbf{x}, \mathbf{y})$ is PSLE
$R \subseteq V$ is CP in G
$\exists \mathbf{x} : R \to \mathbb{R}$
$\forall \mathbf{y} : R \to \mathbb{R}$
$(\mathbf{x}, \mathbf{y})$
extendable to
PSLE of G

Results

Theorem

Every tree has a column planar subset of at least $\frac{14}{17}n$ *vertices.*

Results

Theorem

Every tree has a column planar subset of at least $\frac{14}{17}n$ vertices.

Theorem

There exists a family of trees where every column planar subset has at most $(\frac{5}{6}+\epsilon)n$ vertices.

Results

Theorem

Every tree has a column planar subset of at least $\frac{14}{17}n$ vertices.

Theorem

There exists a family of trees where every column planar subset has at most $(\frac{5}{6}+\epsilon)n$ vertices.

Corollary

Every two trees on a set of n vertices admit an $\frac{11}{17}n$-partial SGE.

Results

Theorem

Every tree has a column planar subset of at least $\frac{14}{17}n$ vertices.

Proof.

1. Define when a $R \subseteq V$ in a tree $T = (V, E)$ is nice.
2. Every tree has a nice R with $|R| \geq \frac{14}{17}n$. [sketch]
3. R is nice $\implies$ R is column planar in T. [skip]

$\square$

Theorem

There exists a family of trees where every column planar subset has at most $(\frac{5}{6} + \epsilon)n$ vertices.

Corollary

Every two trees on a set of n vertices admit an $\frac{11}{17}n$-partial SGE.

Results

Theorem

Every tree has a column planar subset of at least $\frac{14}{17}n \approx 0.82n$ vertices.

Proof.

1. Define when a $R \subseteq V$ in a tree $T = (V, E)$ is nice.
2. Every tree has a nice R with $|R| \geq \frac{14}{17}n$. [sketch]
3. R is nice $\implies$ R is column planar in T. [skip]

$\square$

Theorem

There exists a family of trees where every column planar subset has at most $(\frac{5}{6} + \epsilon)n \approx (0.83 + \epsilon)n$ vertices.

Corollary

Every two trees on a set of n vertices admit an $\frac{11}{17}n$-partial SGE.

Results

Theorem

Every tree has a column planar subset of at least $\frac{14}{17}n \approx 0.82n$ vertices.

Proof.

1. Define when a $R \subseteq V$ in a tree $T = (V, E)$ is nice.
2. Every tree has a nice R with $|R| \geq \frac{14}{17}n$. [sketch]
3. R is nice $\implies$ R is column planar in T. [skip]

□

Theorem

There exists a family of trees where every column planar subset has at most $(\frac{5}{6} + \epsilon)n \approx (0.83 + \epsilon)n$ vertices.

Corollary

Every two trees on a set of n vertices admit an $\frac{11}{17}n$-partial SGE.

Niceness

Let $T = (V, E)$ be a rooted tree and $R \subseteq V$. A vertex $v \in R$ is heavy if it has a child in R.

R is nice if for all $v \in R$:

Niceness

Let $T = (V, E)$ be a rooted tree and $R \subseteq V$. A vertex $v \in R$ is heavy if it has a child in R.

R is nice if for all $v \in R$:

- $p(v) \in R$: v has ≤ 2 nonleaf children in R (≤ 1 is heavy);

Niceness

Let $T = (V, E)$ be a rooted tree and $R \subseteq V$. A vertex $v \in R$ is heavy if it has a child in R.

R is nice if for all $v \in R$:

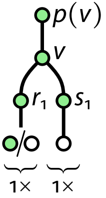
- $p(v) \in R$: v has ≤ 2 nonleaf children in R (≤ 1 is heavy);
- $p(v) \notin R$: v has ≤ 4 nonleaf children in R (≤ 2 are heavy);

Niceness

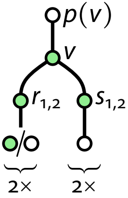
Let $T = (V, E)$ be a rooted tree and $R \subseteq V$. A vertex $v \in R$ is heavy if it has a child in R.

R is nice if for all $v \in R$:

- $p(v) \in R$: v has ≤ 2 nonleaf children in R (≤ 1 is heavy);
- $p(v) \notin R$: v has ≤ 4 nonleaf children in R (≤ 2 are heavy);



or

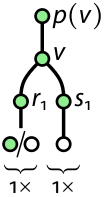


Niceness

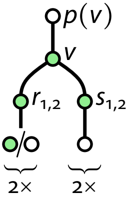
Let $T = (V, E)$ be a rooted tree and $R \subseteq V$. A vertex $v \in R$ is heavy if it has a child in R.

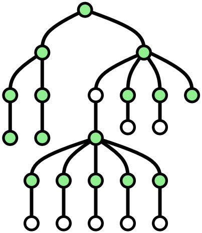
R is nice if for all $v \in R$:

- $p(v) \in R$: v has ≤ 2 nonleaf children in R (≤ 1 is heavy);
- $p(v) \notin R$: v has ≤ 4 nonleaf children in R (≤ 2 are heavy);



or



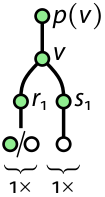


Niceness

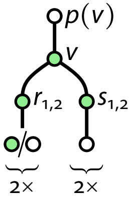
Let $T = (V, E)$ be a rooted tree and $R \subseteq V$. A vertex $v \in R$ is heavy if it has a child in R.

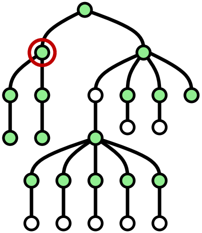
R is nice if for all $v \in R$:

- $p(v) \in R$: v has ≤ 2 nonleaf children in R (≤ 1 is heavy);
- $p(v) \notin R$: v has ≤ 4 nonleaf children in R (≤ 2 are heavy);



or



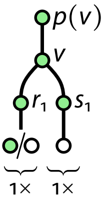


Niceness

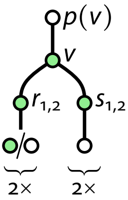
Let $T = (V, E)$ be a rooted tree and $R \subseteq V$. A vertex $v \in R$ is heavy if it has a child in R.

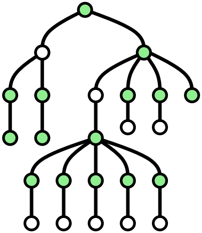
R is nice if for all $v \in R$:

- $p(v) \in R$: v has ≤ 2 nonleaf children in R (≤ 1 is heavy);
- $p(v) \notin R$: v has ≤ 4 nonleaf children in R (≤ 2 are heavy);



or



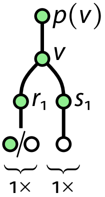


Niceness

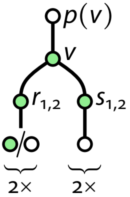
Let $T = (V, E)$ be a rooted tree and $R \subseteq V$. A vertex $v \in R$ is heavy if it has a child in R.

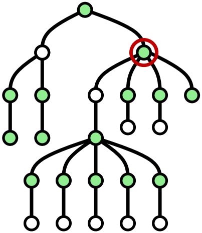
R is nice if for all $v \in R$:

- $p(v) \in R$: v has ≤ 2 nonleaf children in R (≤ 1 is heavy);
- $p(v) \notin R$: v has ≤ 4 nonleaf children in R (≤ 2 are heavy);



or



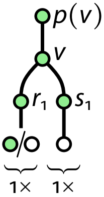


Niceness

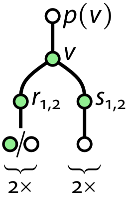
Let $T = (V, E)$ be a rooted tree and $R \subseteq V$. A vertex $v \in R$ is heavy if it has a child in R.

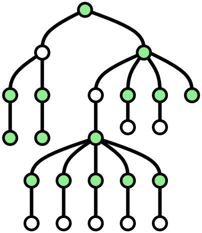
R is nice if for all $v \in R$:

- $p(v) \in R$: v has ≤ 2 nonleaf children in R (≤ 1 is heavy);
- $p(v) \notin R$: v has ≤ 4 nonleaf children in R (≤ 2 are heavy);



or



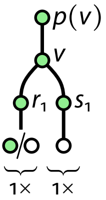


Niceness

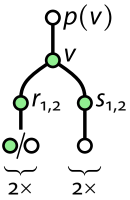
Let $T = (V, E)$ be a rooted tree and $R \subseteq V$. A vertex $v \in R$ is heavy if it has a child in R.

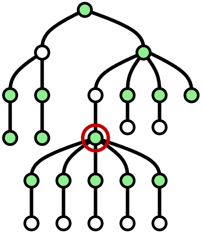
R is nice if for all $v \in R$:

- $p(v) \in R$: v has ≤ 2 nonleaf children in R (≤ 1 is heavy);
- $p(v) \notin R$: v has ≤ 4 nonleaf children in R (≤ 2 are heavy);



or



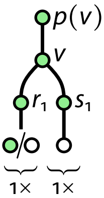


Niceness

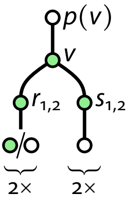
Let $T = (V, E)$ be a rooted tree and $R \subseteq V$. A vertex $v \in R$ is heavy if it has a child in R.

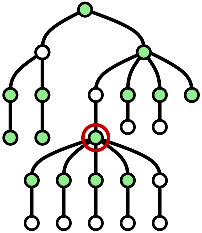
R is nice if for all $v \in R$:

- $p(v) \in R$: v has ≤ 2 nonleaf children in R (≤ 1 is heavy);
- $p(v) \notin R$: v has ≤ 4 nonleaf children in R (≤ 2 are heavy);



or



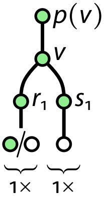


Niceness

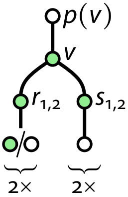
Let $T = (V, E)$ be a rooted tree and $R \subseteq V$. A vertex $v \in R$ is heavy if it has a child in R.

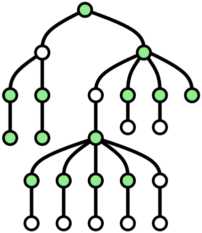
R is nice if for all $v \in R$:

- $p(v) \in R$: v has ≤ 2 nonleaf children in R (≤ 1 is heavy);
- $p(v) \notin R$: v has ≤ 4 nonleaf children in R (≤ 2 are heavy);



or





Results

Theorem

Every tree has a column planar subset of at least $\frac{14}{17}n \approx 0.82n$ vertices.

Proof.

1. Define when a $R \subseteq V$ in a tree $T = (V, E)$ is nice.
2. Every tree has a nice R with $|R| \geq \frac{14}{17}n$. [sketch]
3. R is nice $\implies$ R is column planar in T. [skip]

$\square$

Theorem

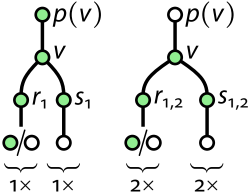
There exists a family of trees where every column planar subset has at most $(\frac{5}{6} + \epsilon)n \approx (0.83 + \epsilon)n$ vertices.

Corollary

Every two trees on a set of n vertices admit an $\frac{11}{17}n$-partial SGE.

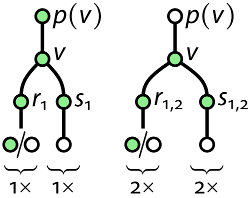
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.

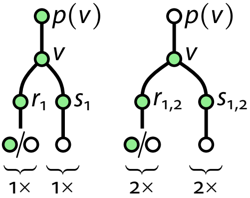


Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17}n$.

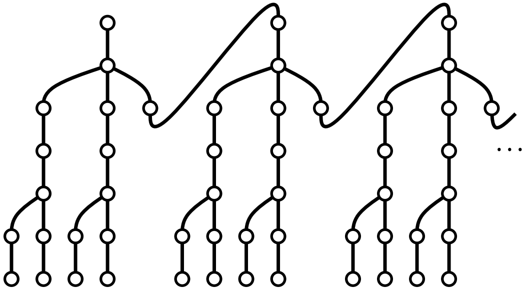
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



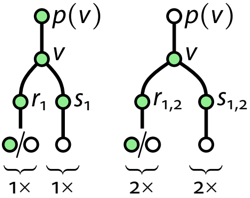
Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17}n$.



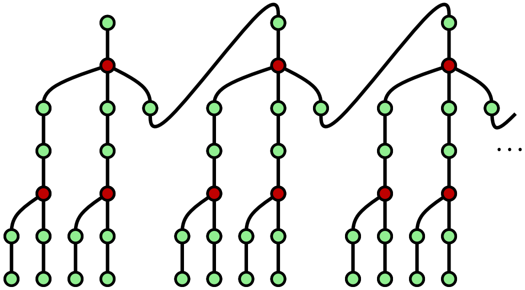
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



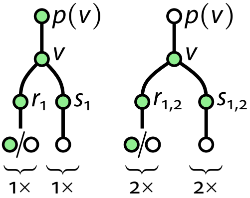
Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17}n$.



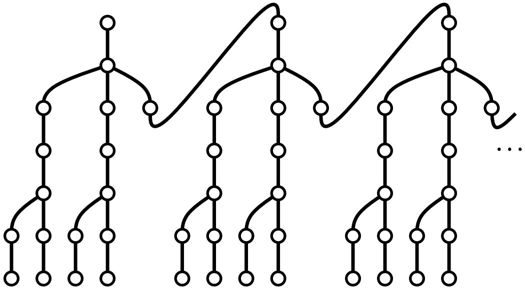
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



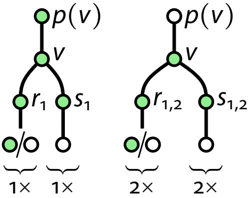
Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17} n$.



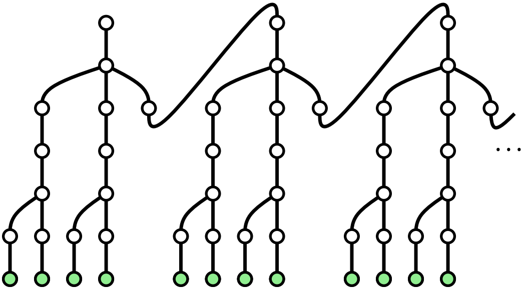
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17}n$.

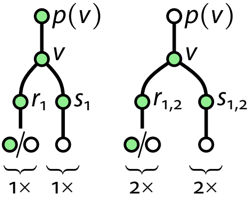


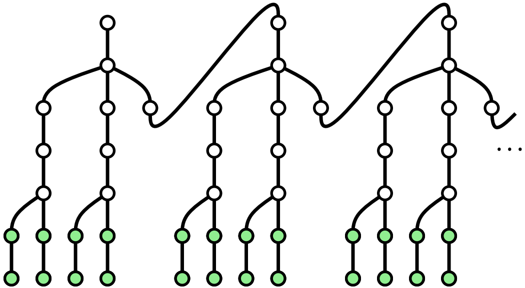
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.

Lemma

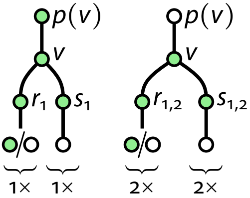
Every tree has a nice R with $|R| \geq \frac{14}{17}n$.





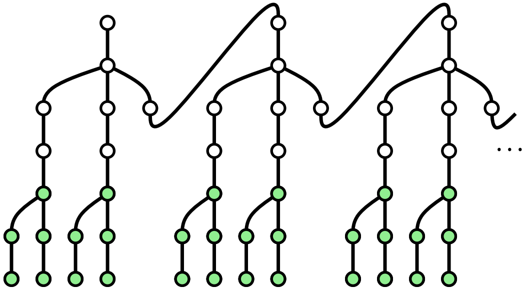
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



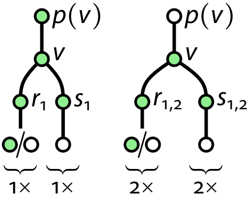
Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17}n$.



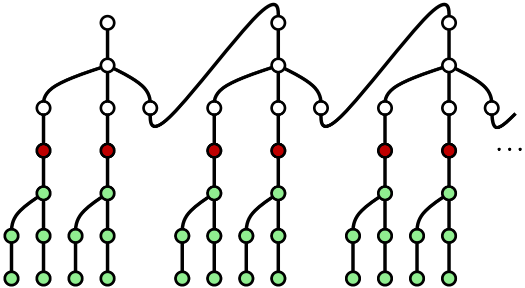
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



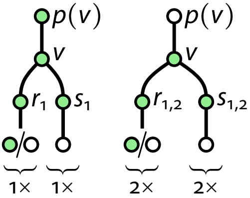
Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17}n$.



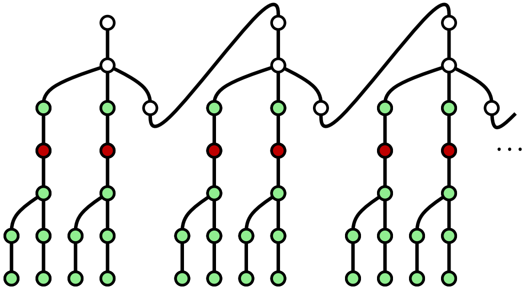
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



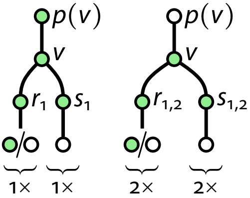
Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17}n$.



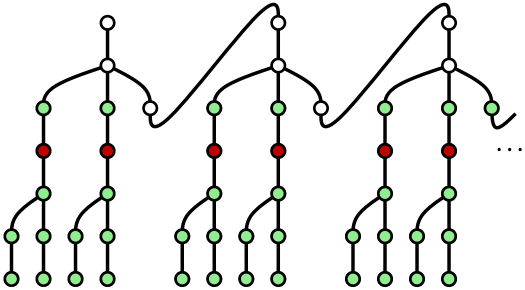
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



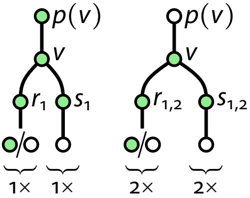
Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17}n$.



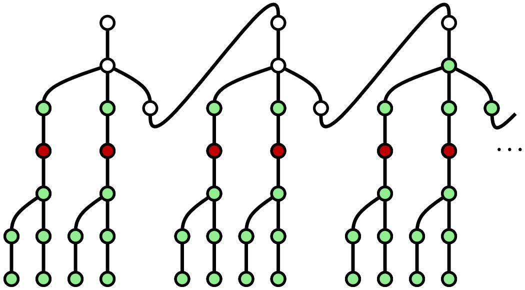
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



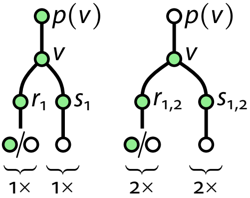
Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17}n$.



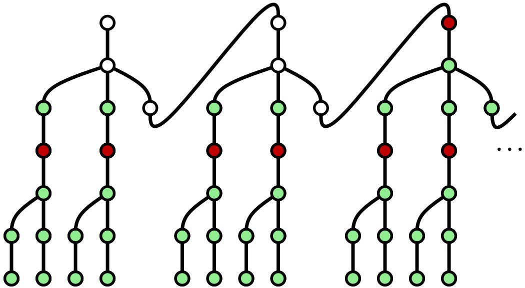
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



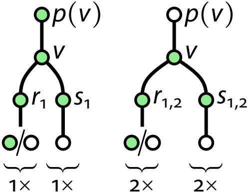
Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17} n$.



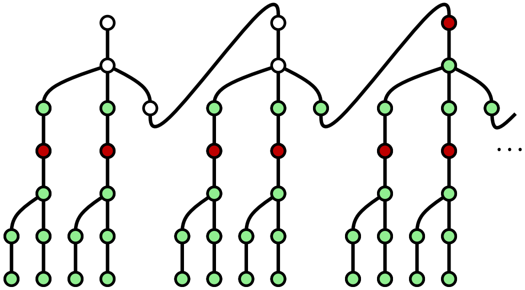
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



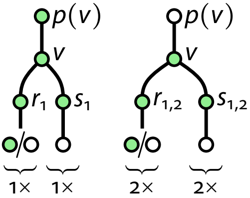
Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17}n$.



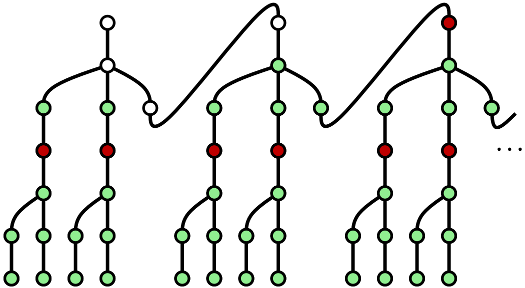
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



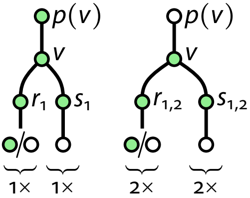
Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17}n$.



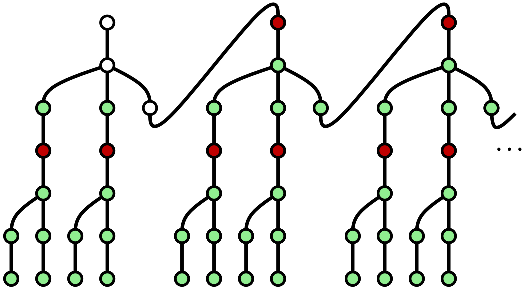
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



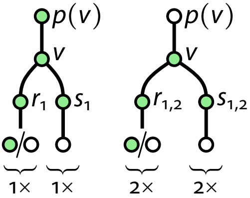
Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17} n$.



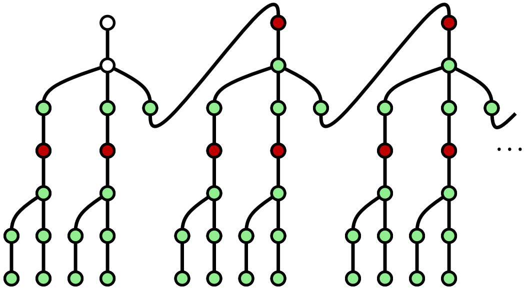
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



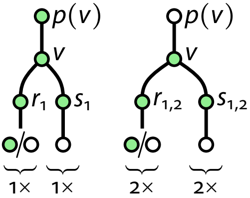
Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17} n$.



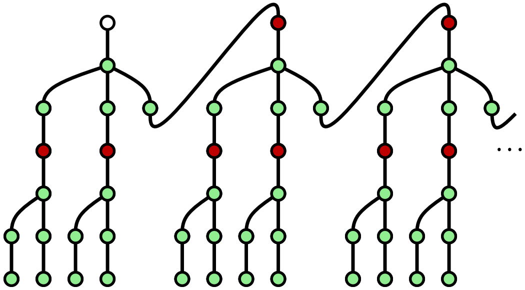
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



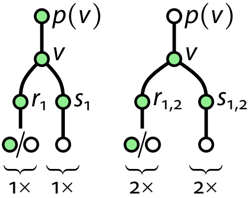
Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17} n$.



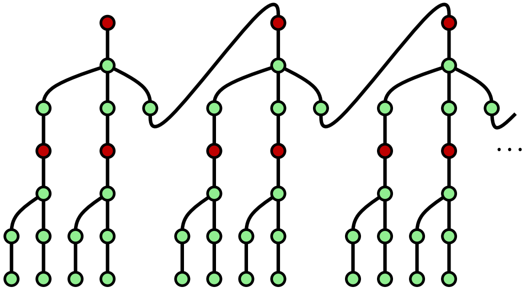
Computing a nice R

Algorithm 1: take all leaves and unary nodes.
Algorithm 2: construct R greedily/bottom-up.



Lemma

Every tree has a nice R with $|R| \geq \frac{14}{17}n$.



Results

Theorem

Every tree has a column planar subset of at least $\frac{14}{17}n \approx 0.82n$ vertices.

Proof.

1. Define when a $R \subseteq V$ in a tree $T = (V, E)$ is nice.
2. Every tree has a nice R with $|R| \geq \frac{14}{17}n$. [sketch]
3. R is nice $\implies$ R is column planar in T. [skip]

$\square$

Theorem

There exists a family of trees where every column planar subset has at most $(\frac{5}{6} + \epsilon)n \approx (0.83 + \epsilon)n$ vertices.

Corollary

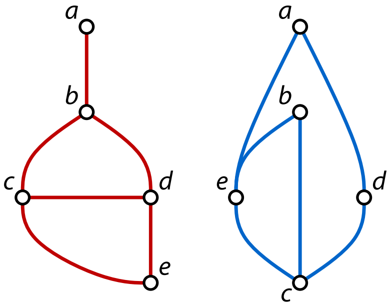
Every two trees on a set of n vertices admit an $\frac{11}{17}n$-partial SGE.

Partial Simultaneous Geometric Embedding

A simultaneous geometric embedding (SGE) of $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ is a pair of PSLEs φ_1 of G_1 and φ_2 of G_2 with $\varphi_1(v) = \varphi_2(v)$ for all $v \in V$.

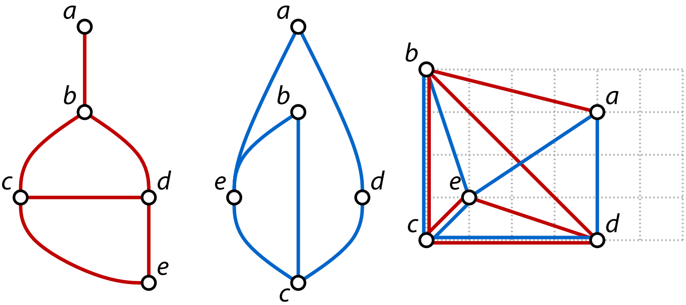
Partial Simultaneous Geometric Embedding

A simultaneous geometric embedding (SGE) of $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ is a pair of PSLEs φ_1 of G_1 and φ_2 of G_2 with $\varphi_1(v) = \varphi_2(v)$ for all $v \in V$.



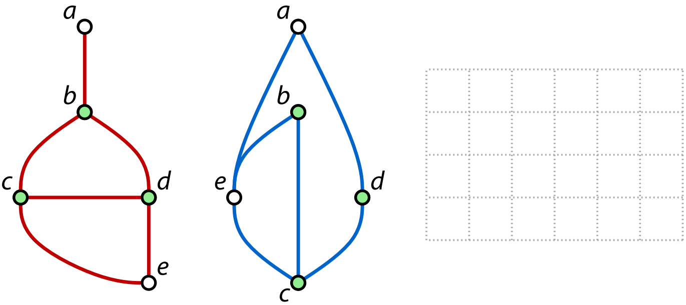
Partial Simultaneous Geometric Embedding

A simultaneous geometric embedding (SGE) of $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ is a pair of PSLEs φ_1 of G_1 and φ_2 of G_2 with $\varphi_1(v) = \varphi_2(v)$ for all $v \in V$.



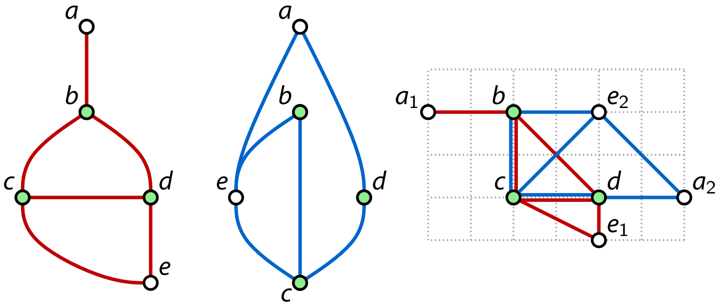
Partial Simultaneous Geometric Embedding

A k-partial simultaneous geometric embedding (k-partial SGE) of $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ is a pair of PSLEs φ_1 of G_1 and φ_2 of G_2 with $\varphi_1(v) = \varphi_2(v)$ ~~for all $v \in V$~~ for all $v \in X \subseteq V$ with $|X| = k$.



Partial Simultaneous Geometric Embedding

A *k*-partial simultaneous geometric embedding (*k*-partial SGE) of $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ is a pair of PSLEs φ_1 of G_1 and φ_2 of G_2 with $\varphi_1(v) = \varphi_2(v)$ ~~for all $v \in V$~~ for all $v \in X \subseteq V$ with $|X| = k$.



Partial Simultaneous Geometric Embedding

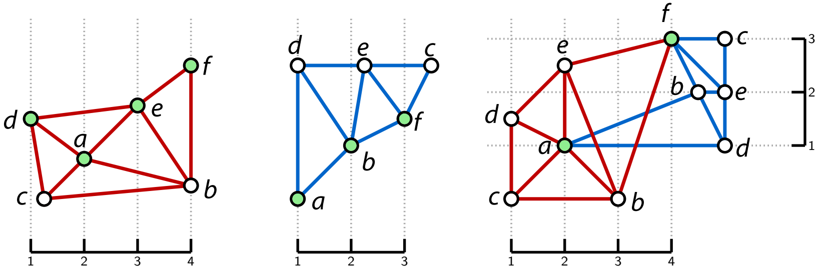
Lemma

Let $G_i = (V, E_i)$ and R_i column planar in G_i for $i = 1, 2$.
Then G_1 and G_2 admit a $(|R_1| + |R_2| - n)$-partial SGE.

Partial Simultaneous Geometric Embedding

Lemma

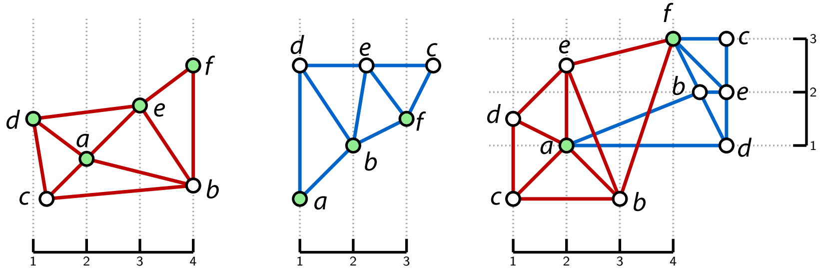
Let $G_i = (V, E_i)$ and R_i column planar in G_i for $i = 1, 2$.
Then G_1 and G_2 admit a $(|R_1| + |R_2| - n)$-partial SGE.



Partial Simultaneous Geometric Embedding

Lemma

Let $G_i = (V, E_i)$ and R_i column planar in G_i for $i = 1, 2$.
Then G_1 and G_2 admit a $(|R_1| + |R_2| - n)$-partial SGE.



Corollary

Every two trees on a set of n vertices admit an $\frac{11}{17}n$-partial SGE.

Future work

- Is polynomial area sufficient?
- Close the gap between the lower and upper bound for trees.
- Other classes of graphs?

Future work

- Is polynomial area sufficient?
- Close the gap between the lower and upper bound for trees.
- Other classes of graphs?

Thanks!