Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Drawing Graphs within Restricted Area

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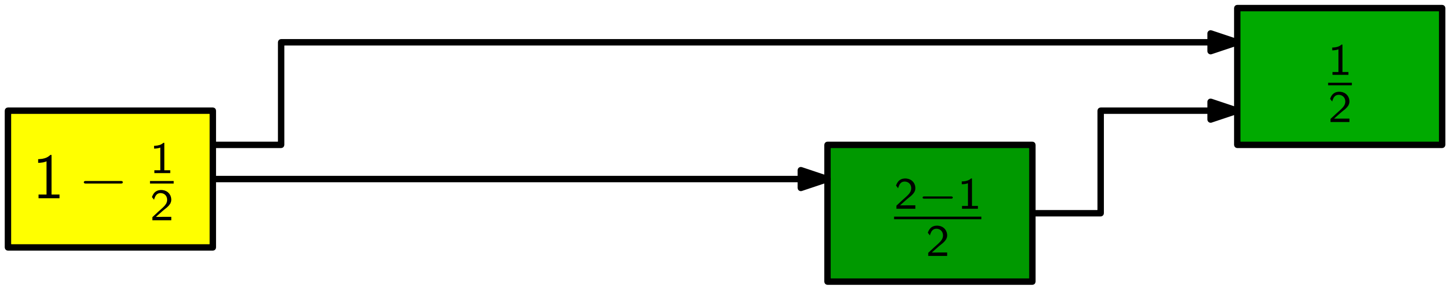
Department of Computer Science
University of California, Santa Barbara

Joint work with Maximilian Aulbach,
Julian Schuhmann, and Alexander Wolff

Motivation – Calculation Graphs

- directed graph of calculation steps



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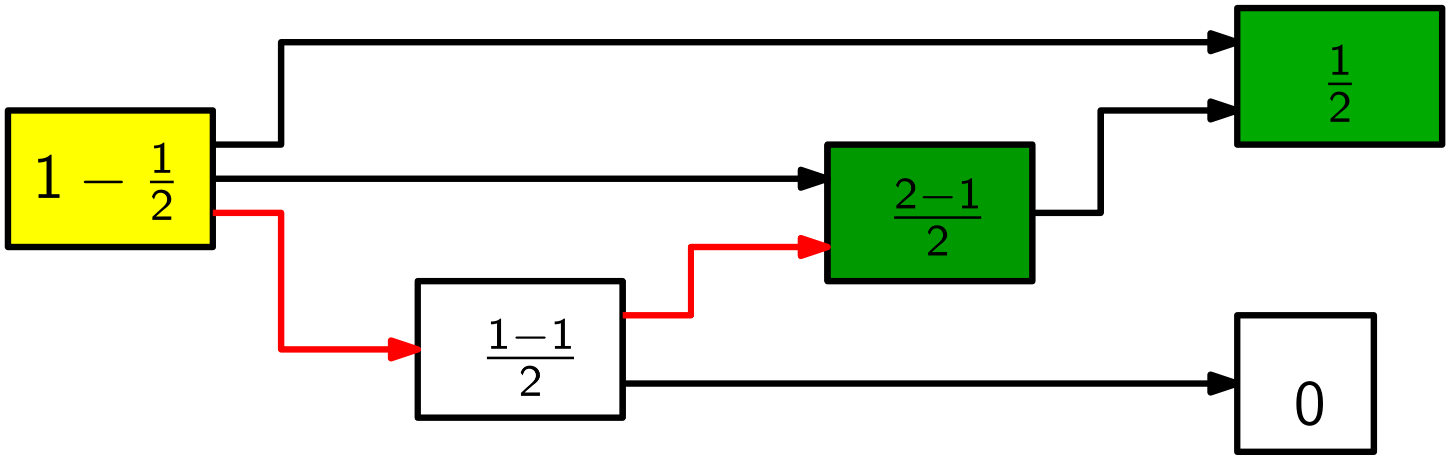

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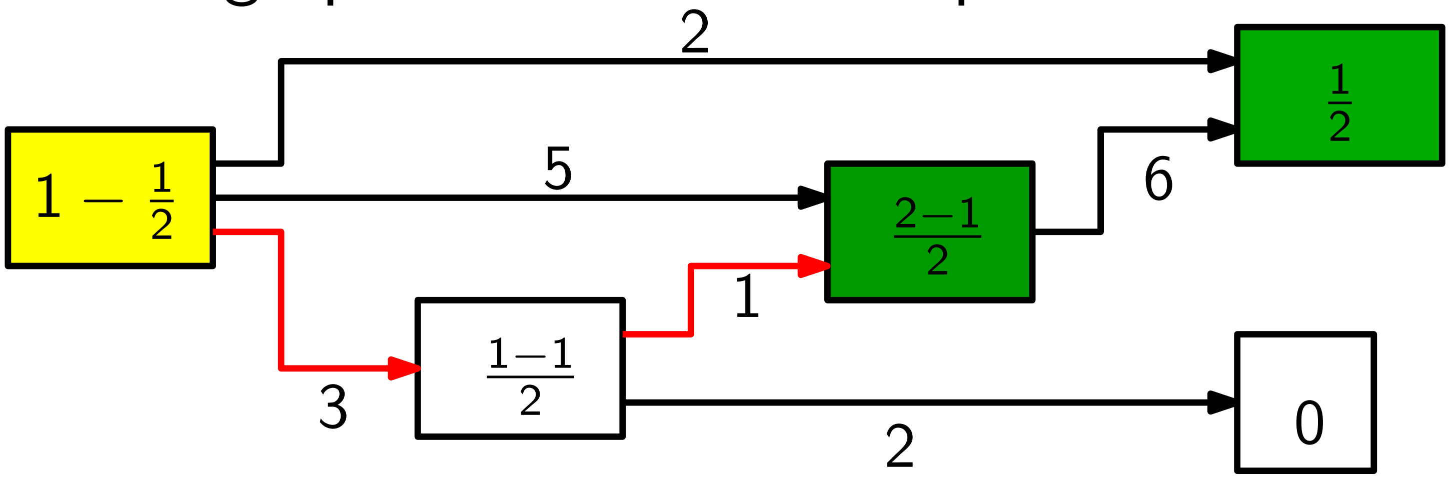

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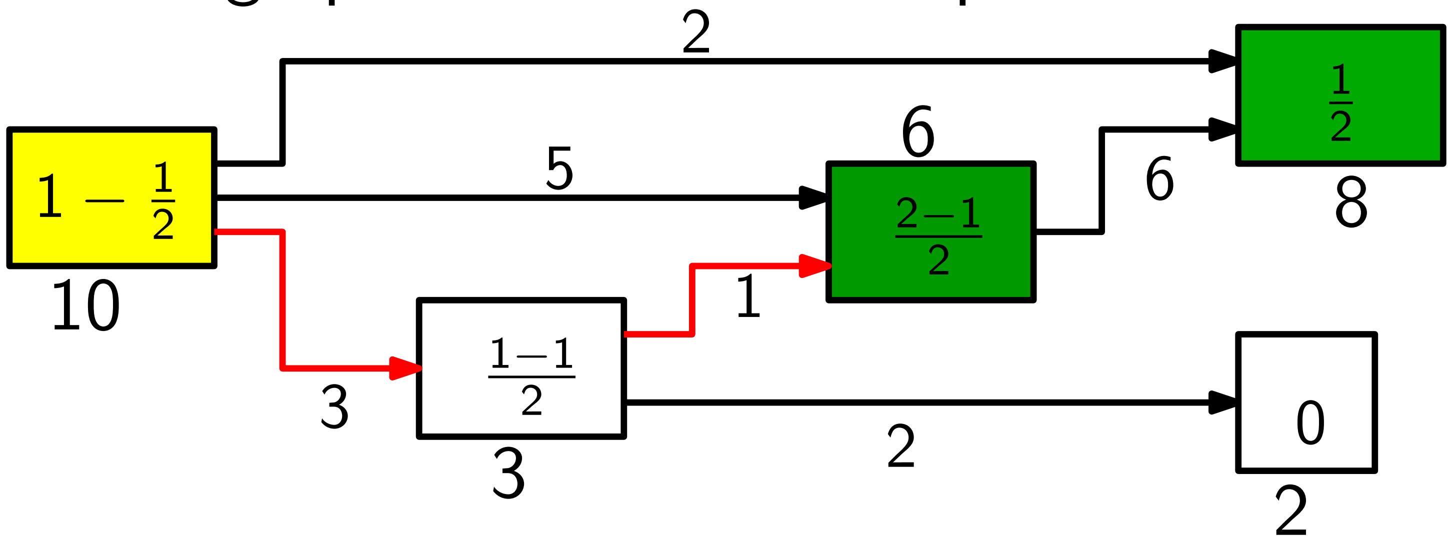

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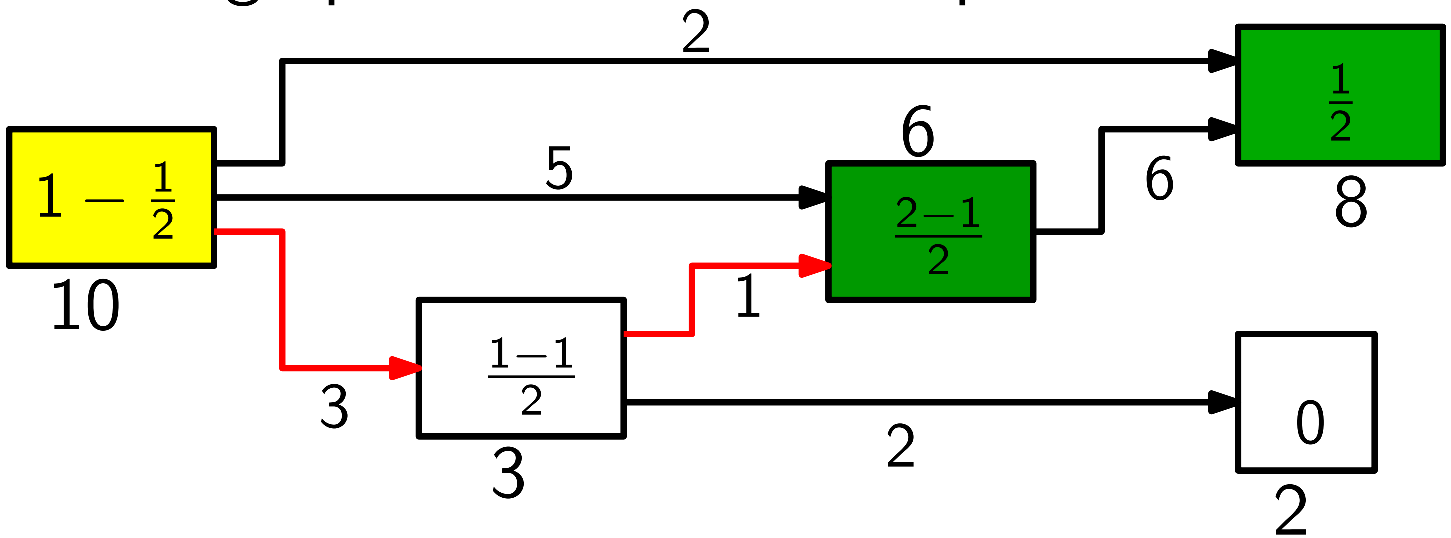

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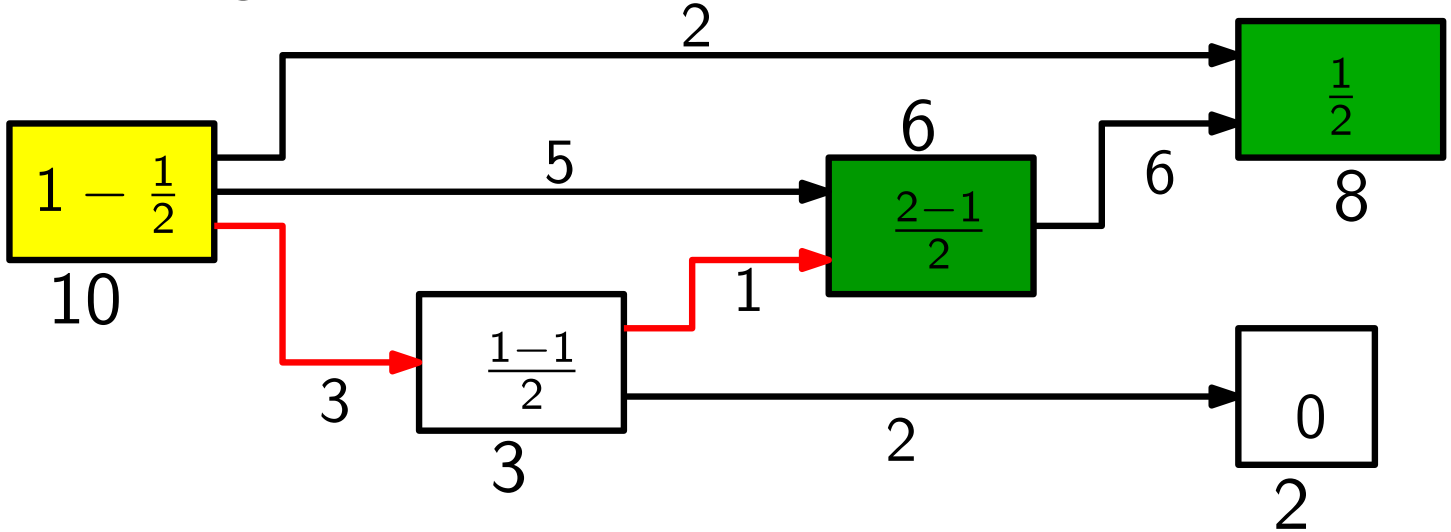

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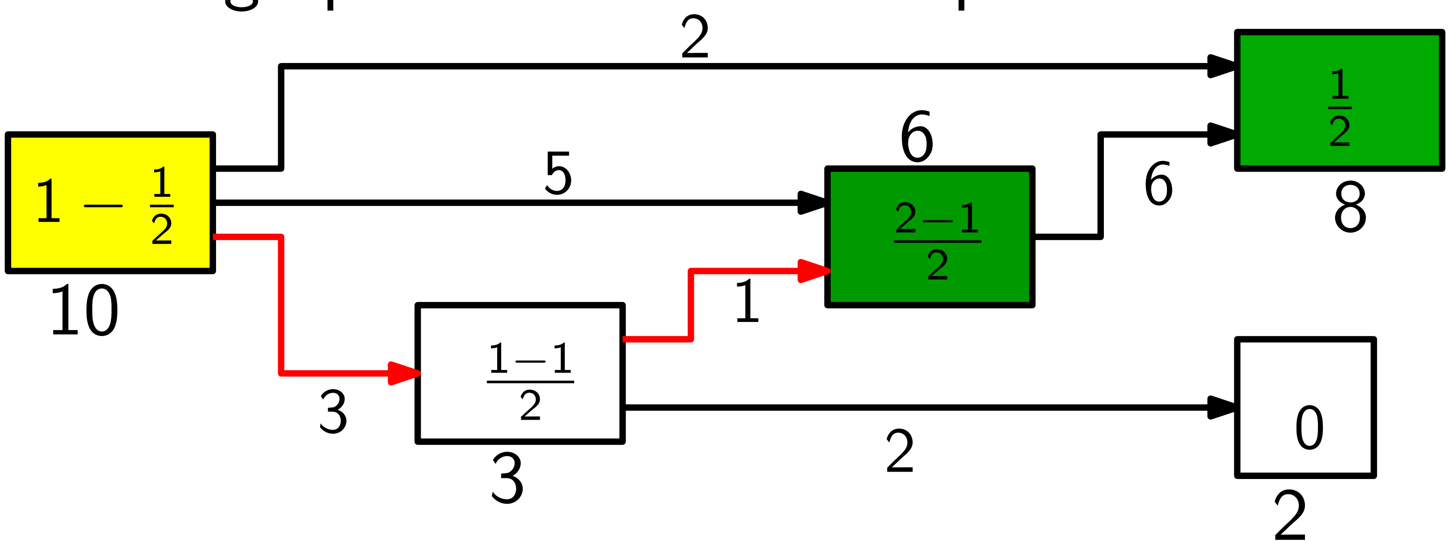

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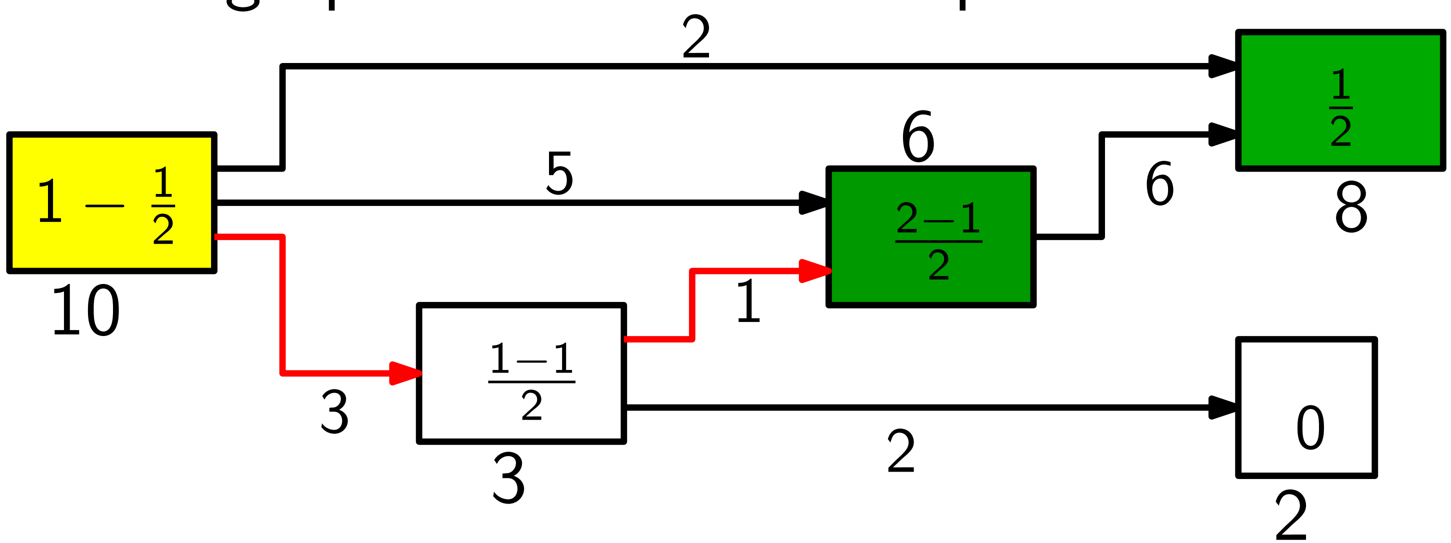

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NP-hard

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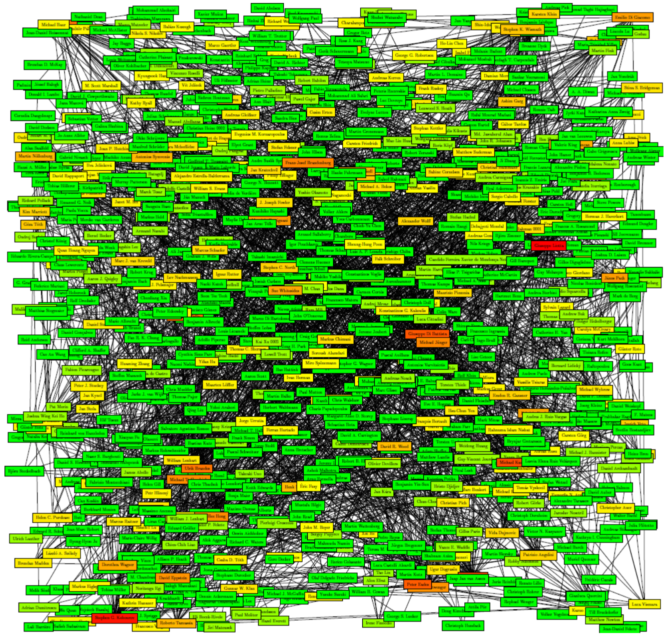

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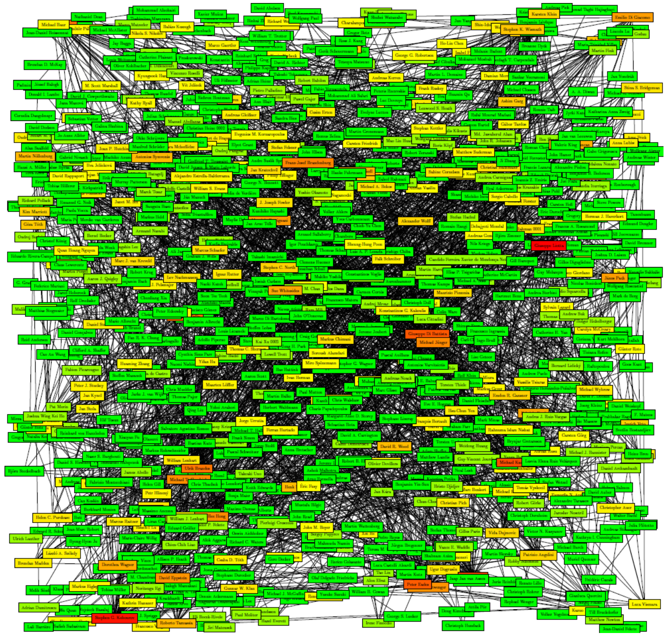

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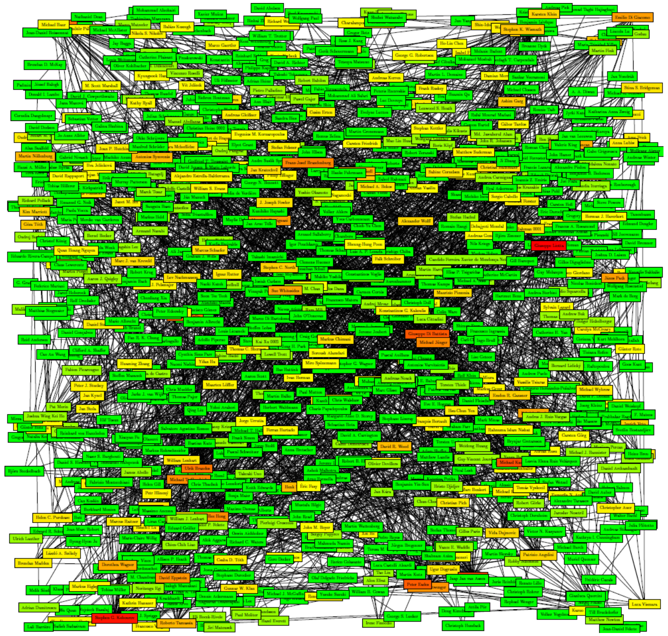

Formally:

Input: – weighted graph $G = (V, E)$
– vertex sizes $w(v)$ and $h(v)$ for each $v \in V$
– prescribed drawing area of width W and height H

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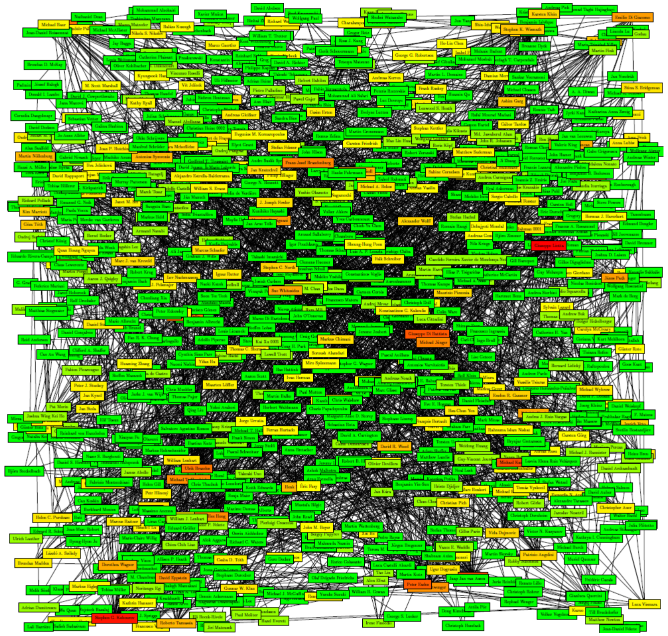

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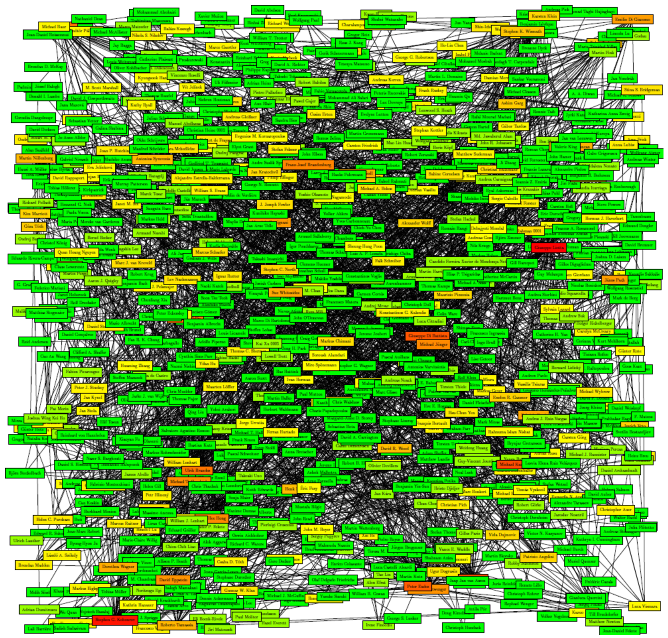

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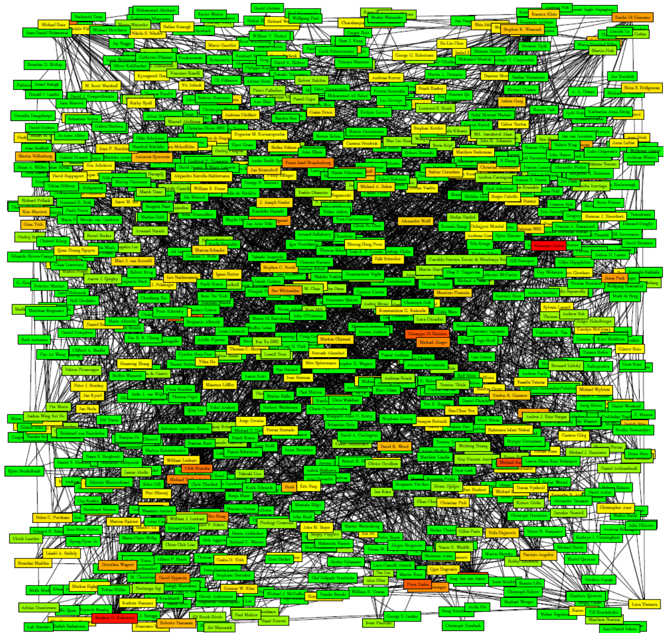

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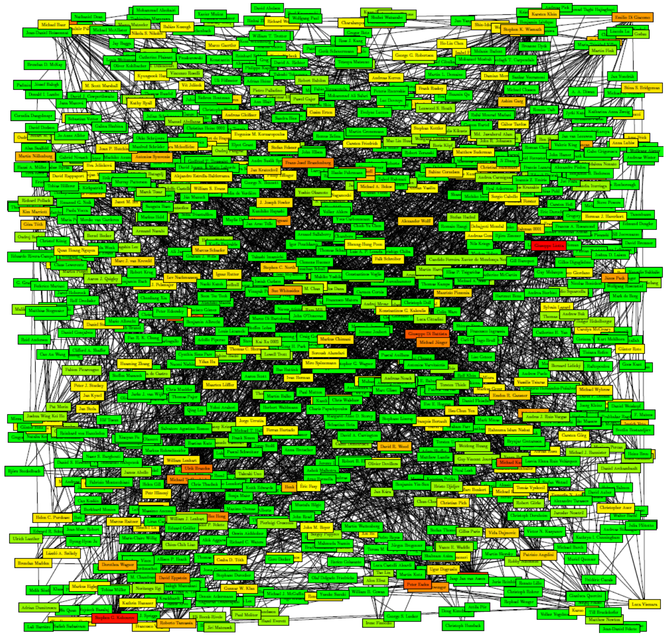

We present heuristics for
– calculation graphs
– straight-line drawings of
general weighted graphs

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Related Work

- [Fruchterman, Reingold 1991]:
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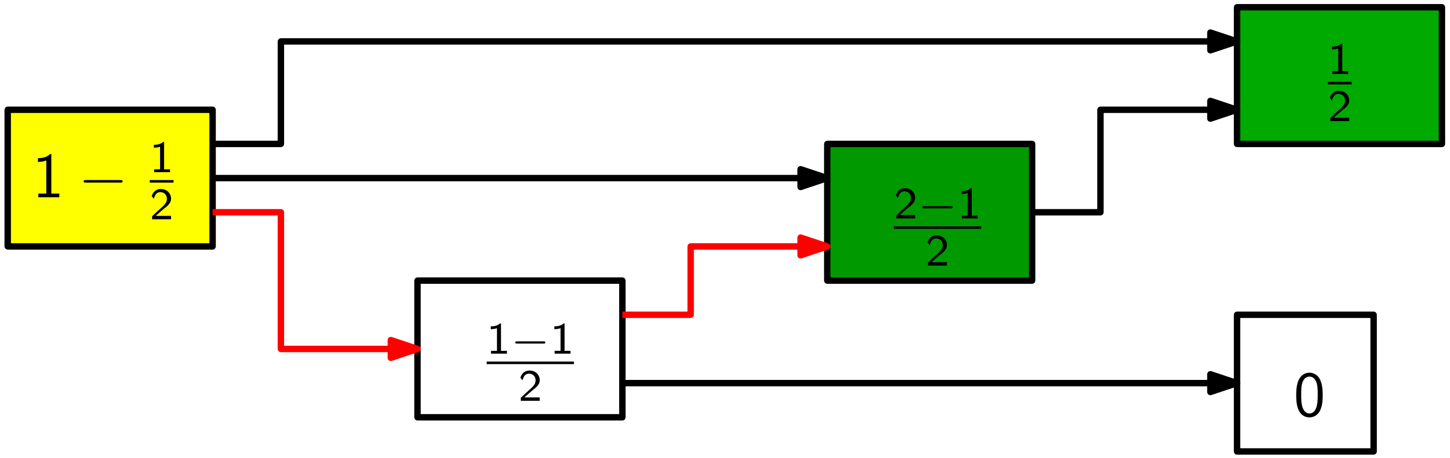
- interactive methods for graph exploration:
  – [Dwyer et al. 2008]: overview+detail
  – [Da Lozzo et al. 2011]: graph exploration on smartphone

Drawing Calculation Graphs



- hierarchical drawing (left to right)
- start vertex left
- orthogonal edges

Calculation Graphs – Our Approach

- based on Sugiyama framework
- extra phases for removing vertices/edges

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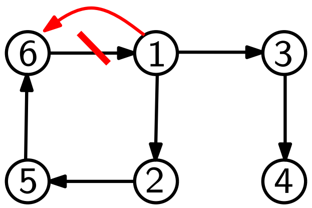

breaking cycles

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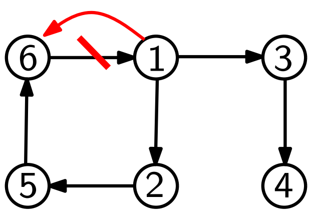

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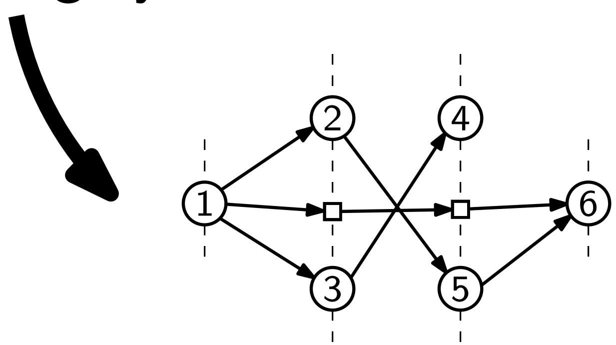

layer assignment

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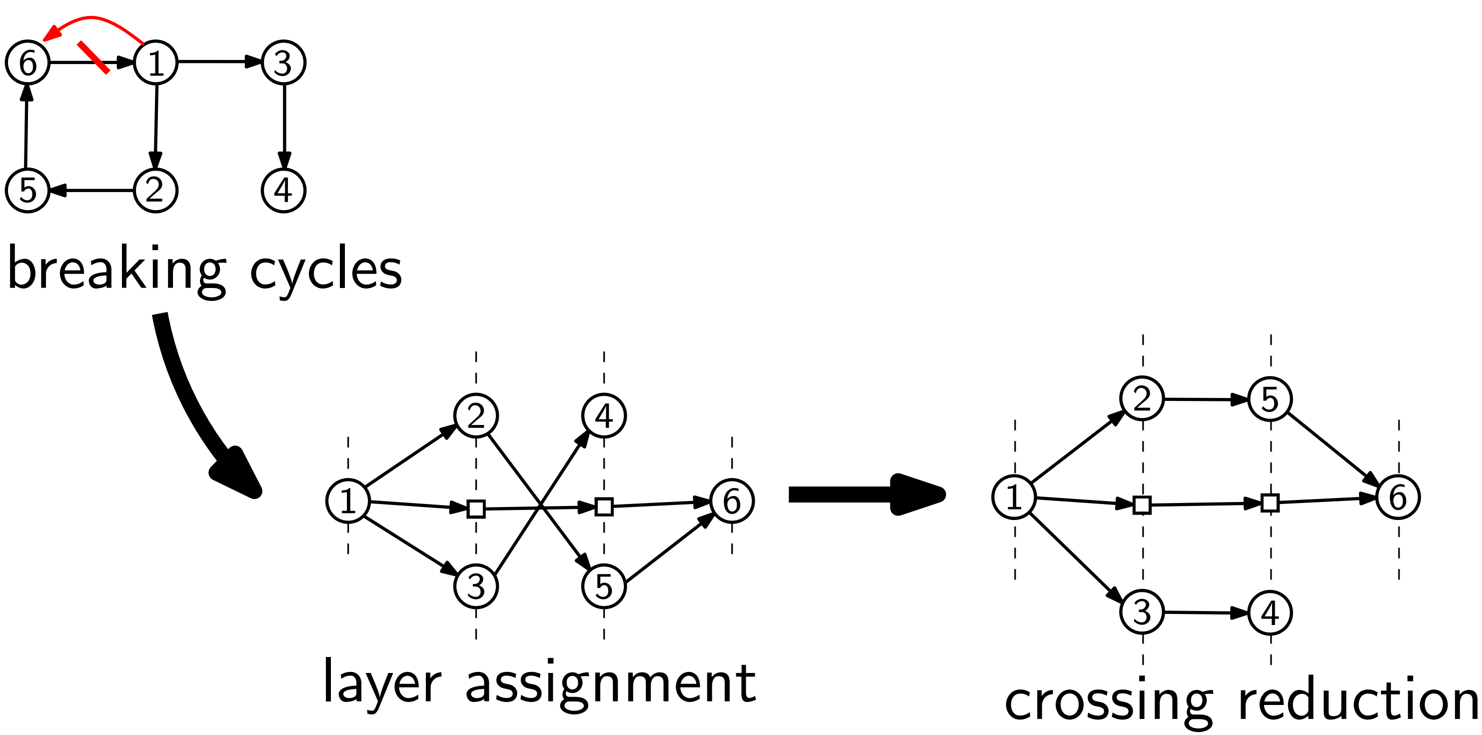

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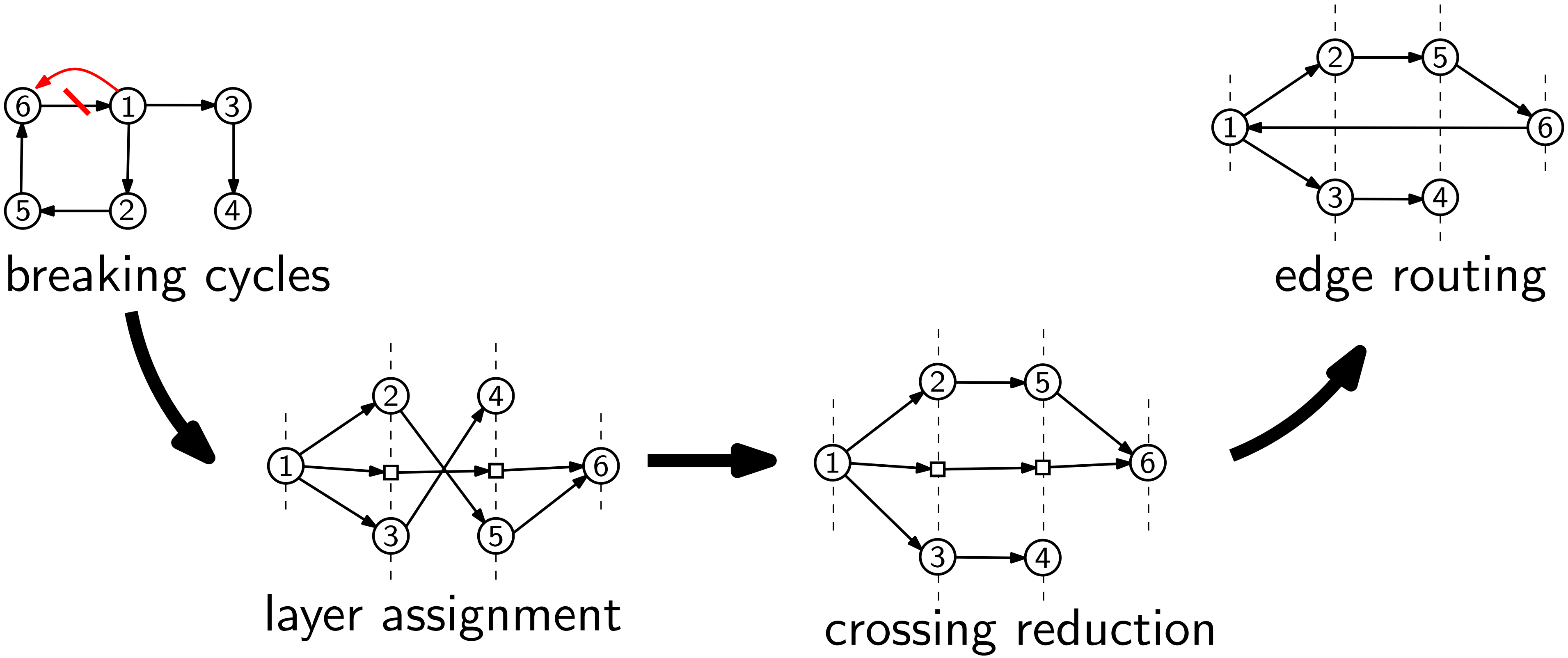

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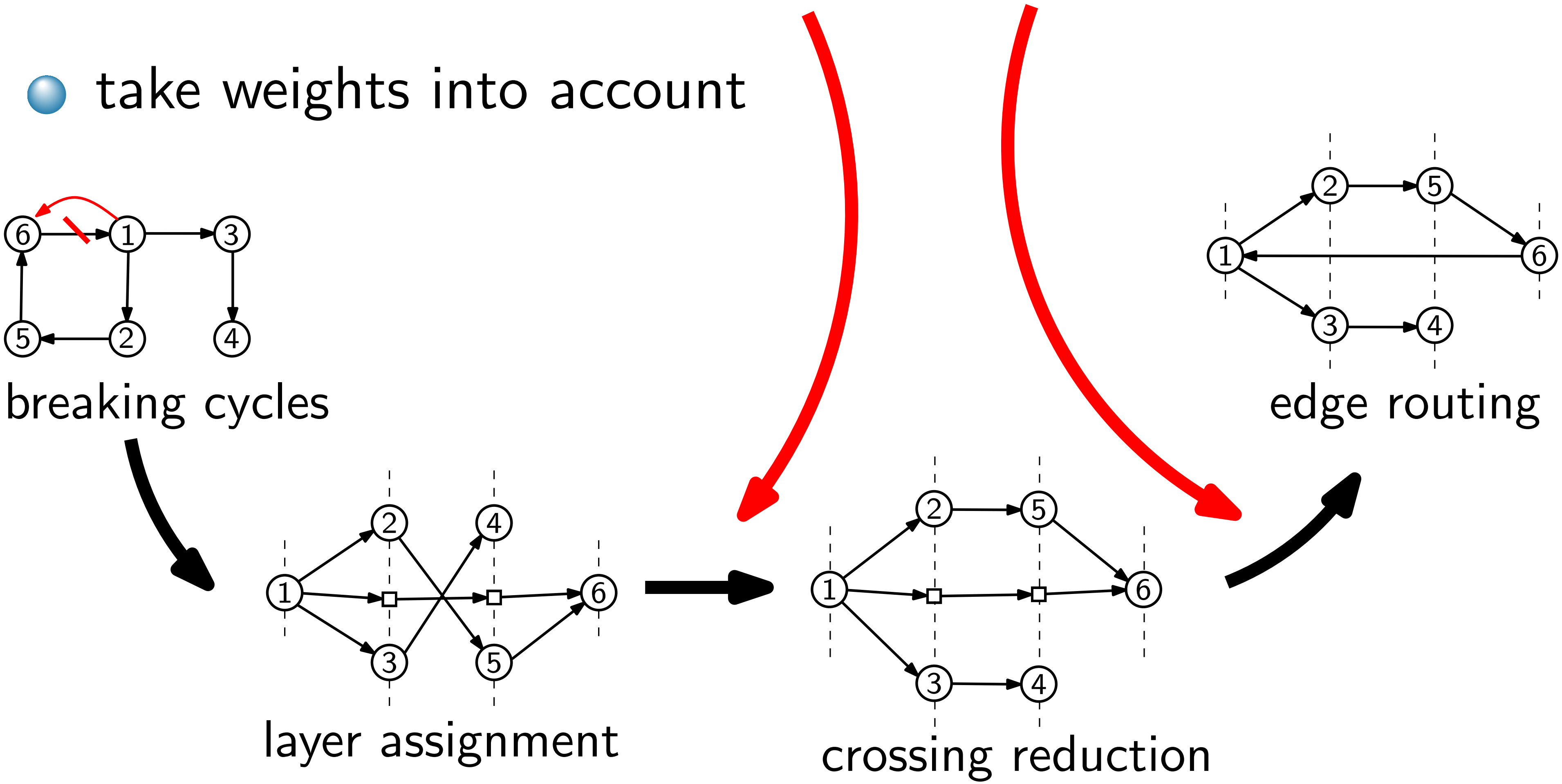

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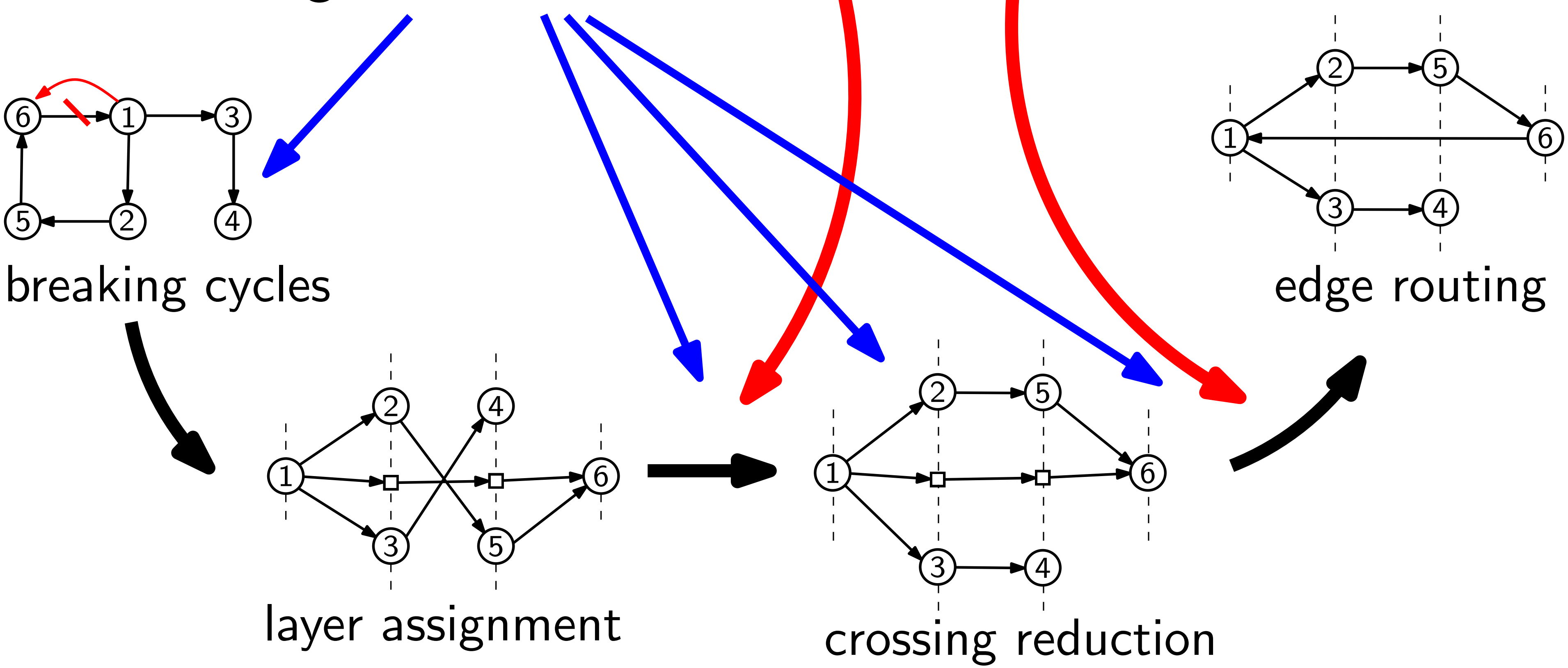

Our Adjustments

- breaking cycles: minimize **weight** of reverted edges
  can afford ILP solution

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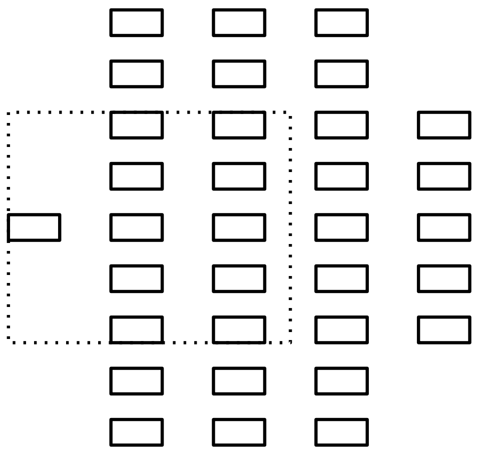

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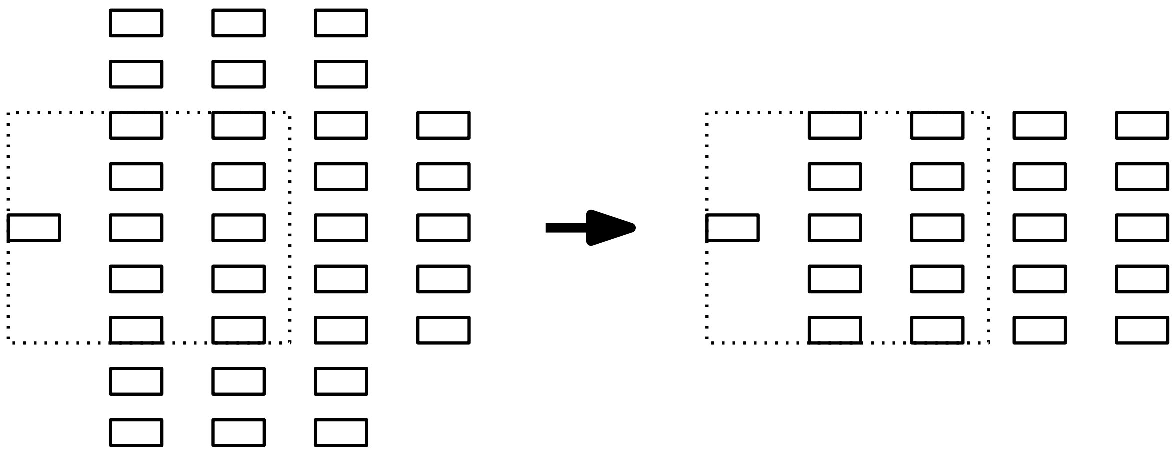

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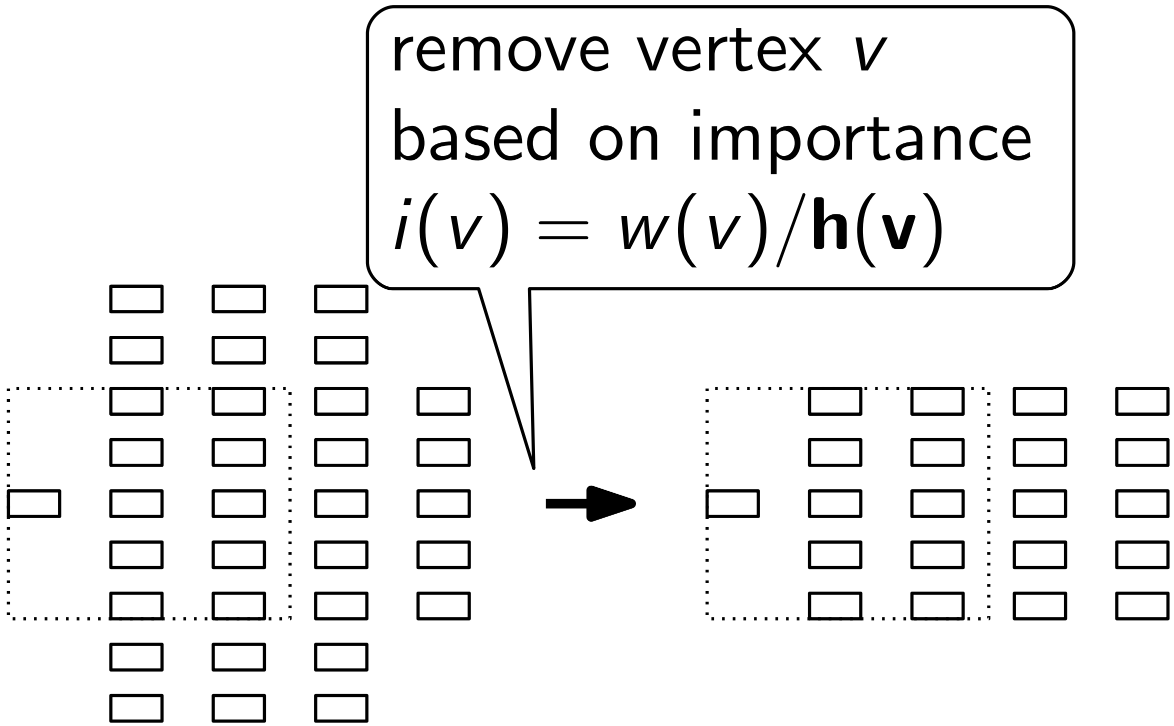

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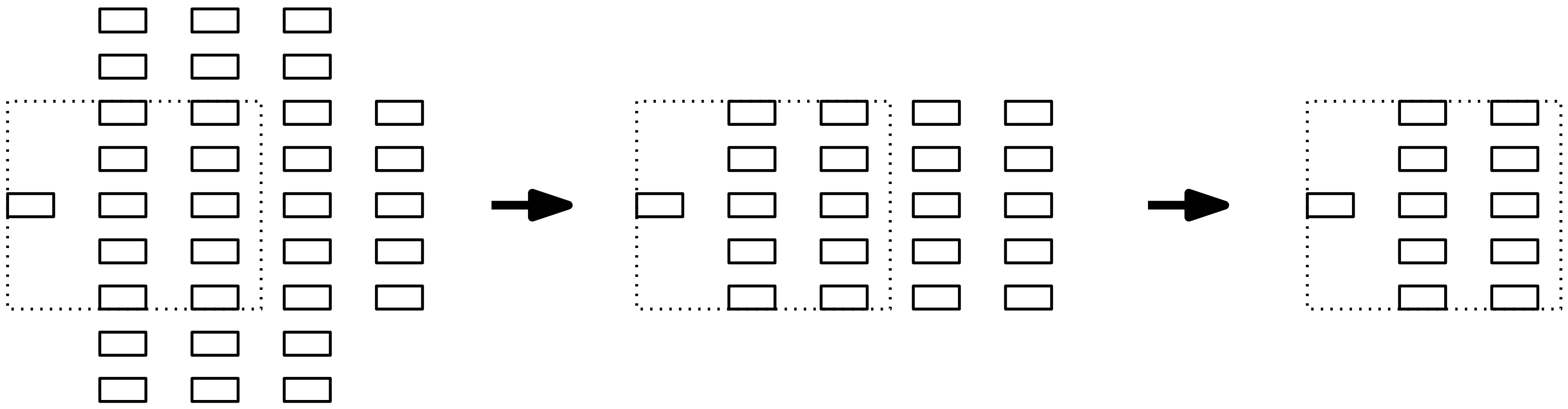

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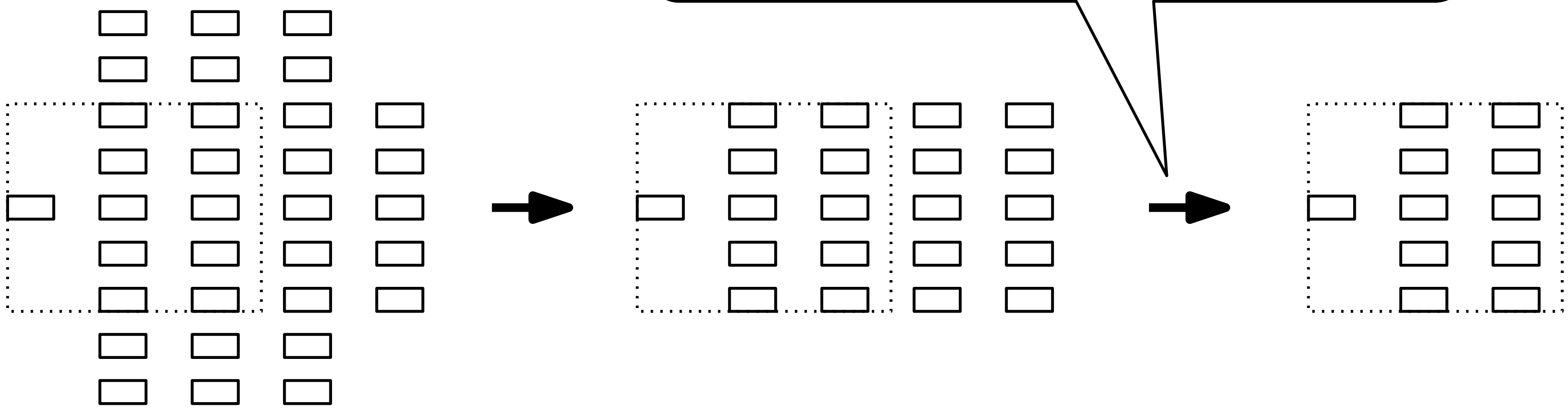
remove layer L based on importance
$i(L) = \sum_{v \in L} w(v)/\text{width}(L)$



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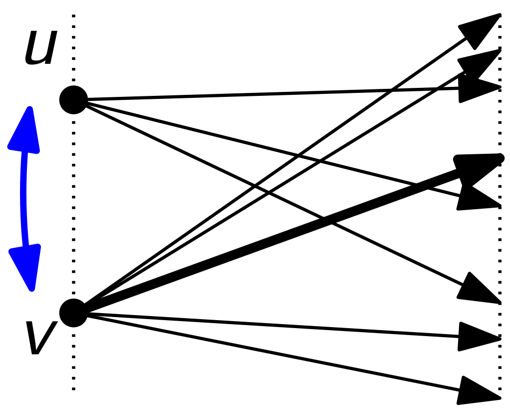

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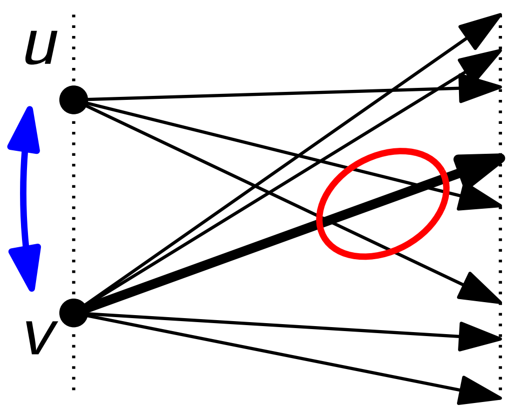

higher cost for crossing heavy edge

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extension: try to reinsert removed
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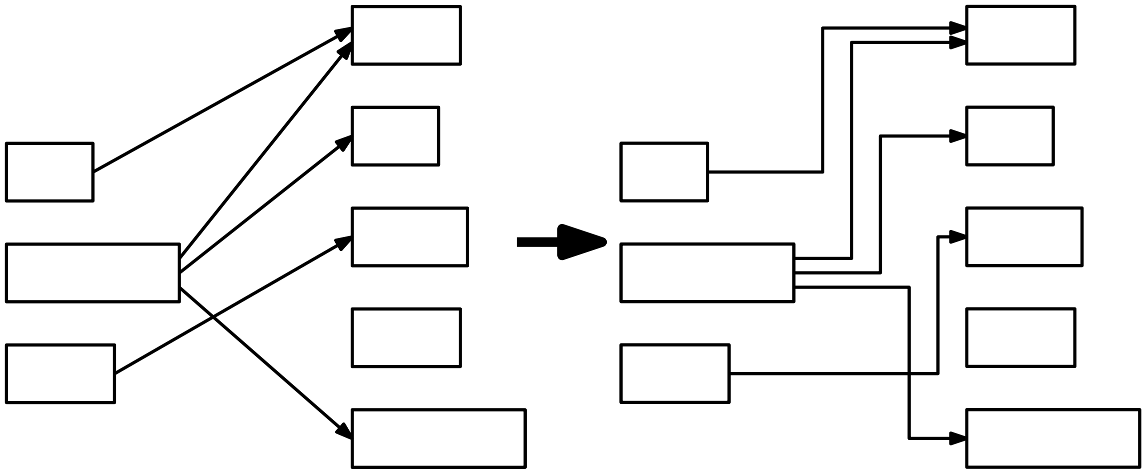
- place vertical segments between consecutive layers



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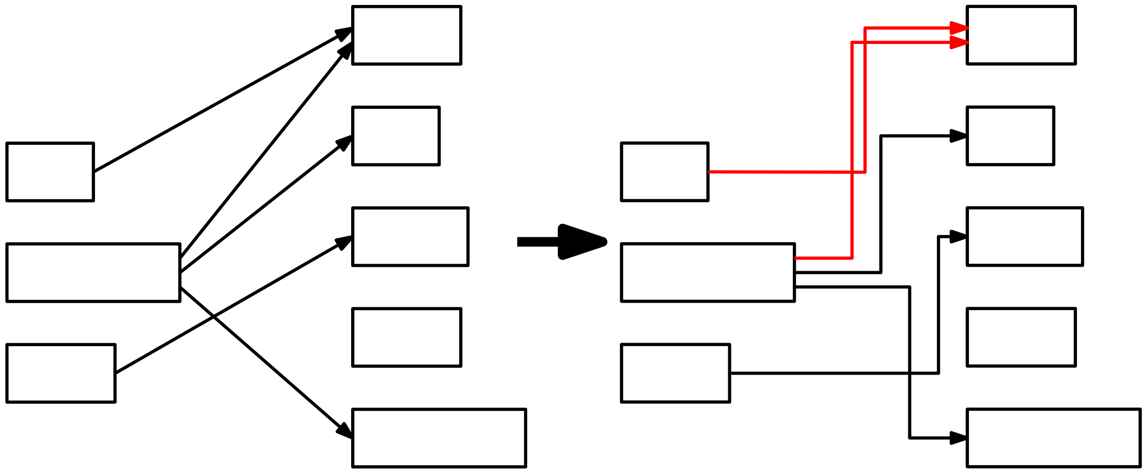

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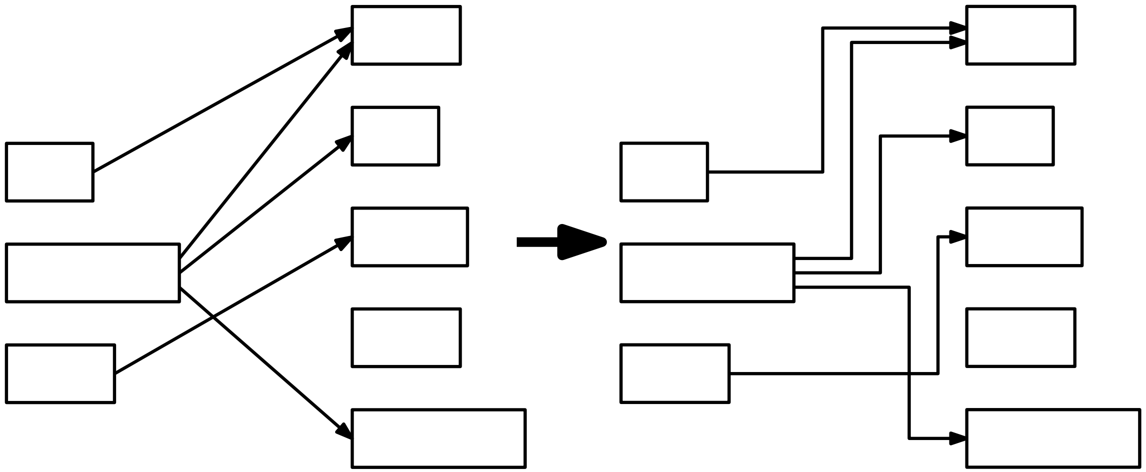

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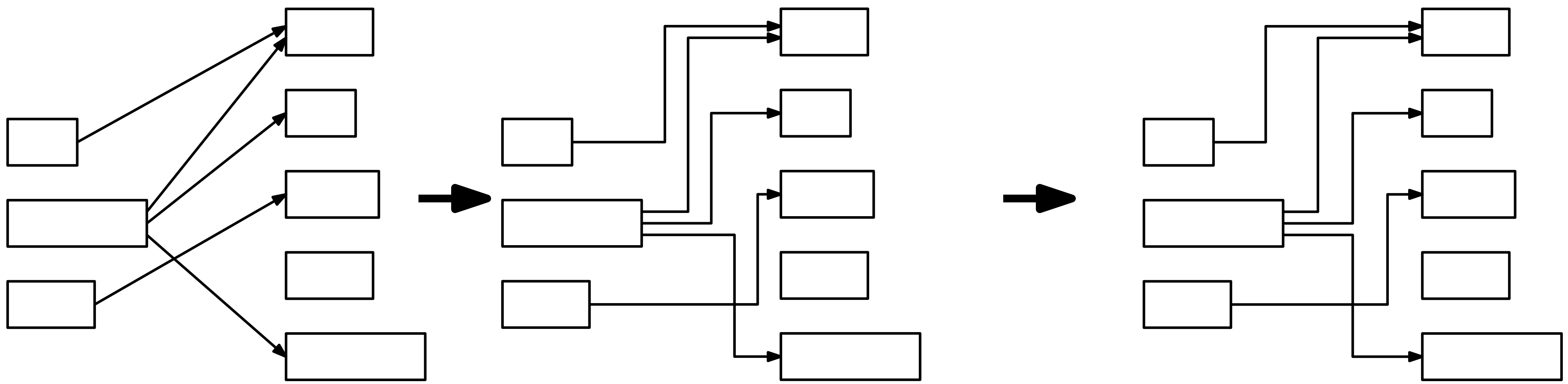

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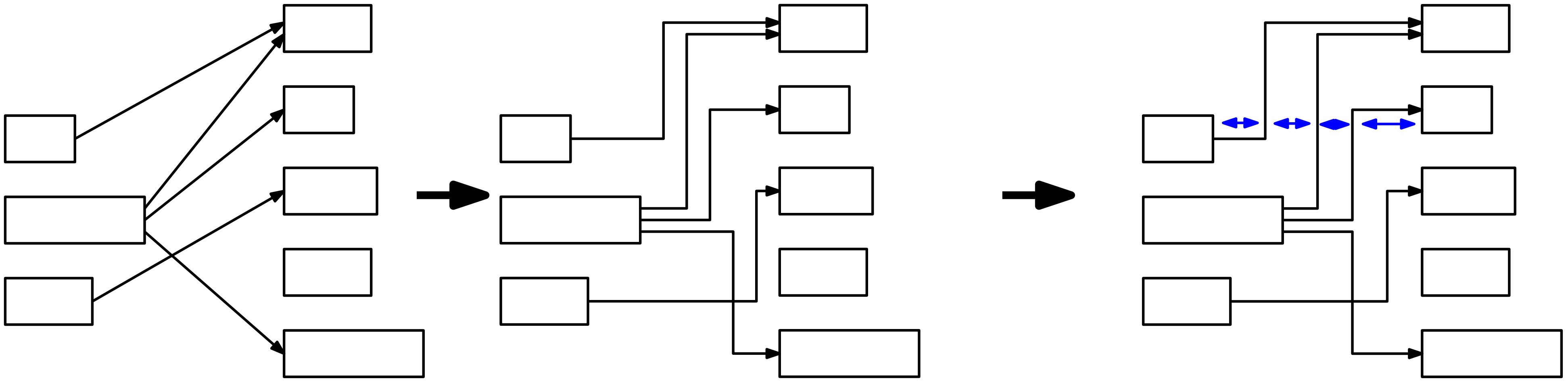

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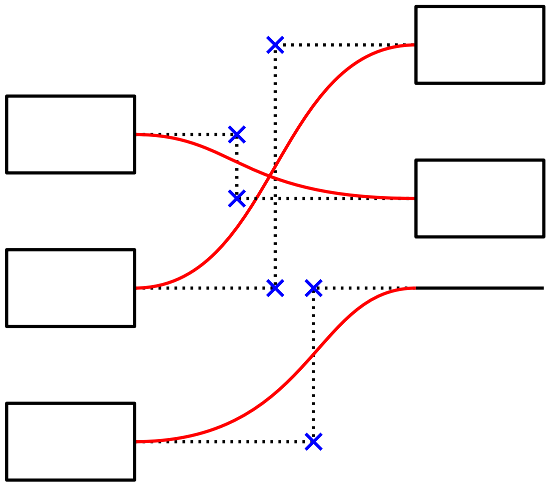
Extension: Bézier Curves



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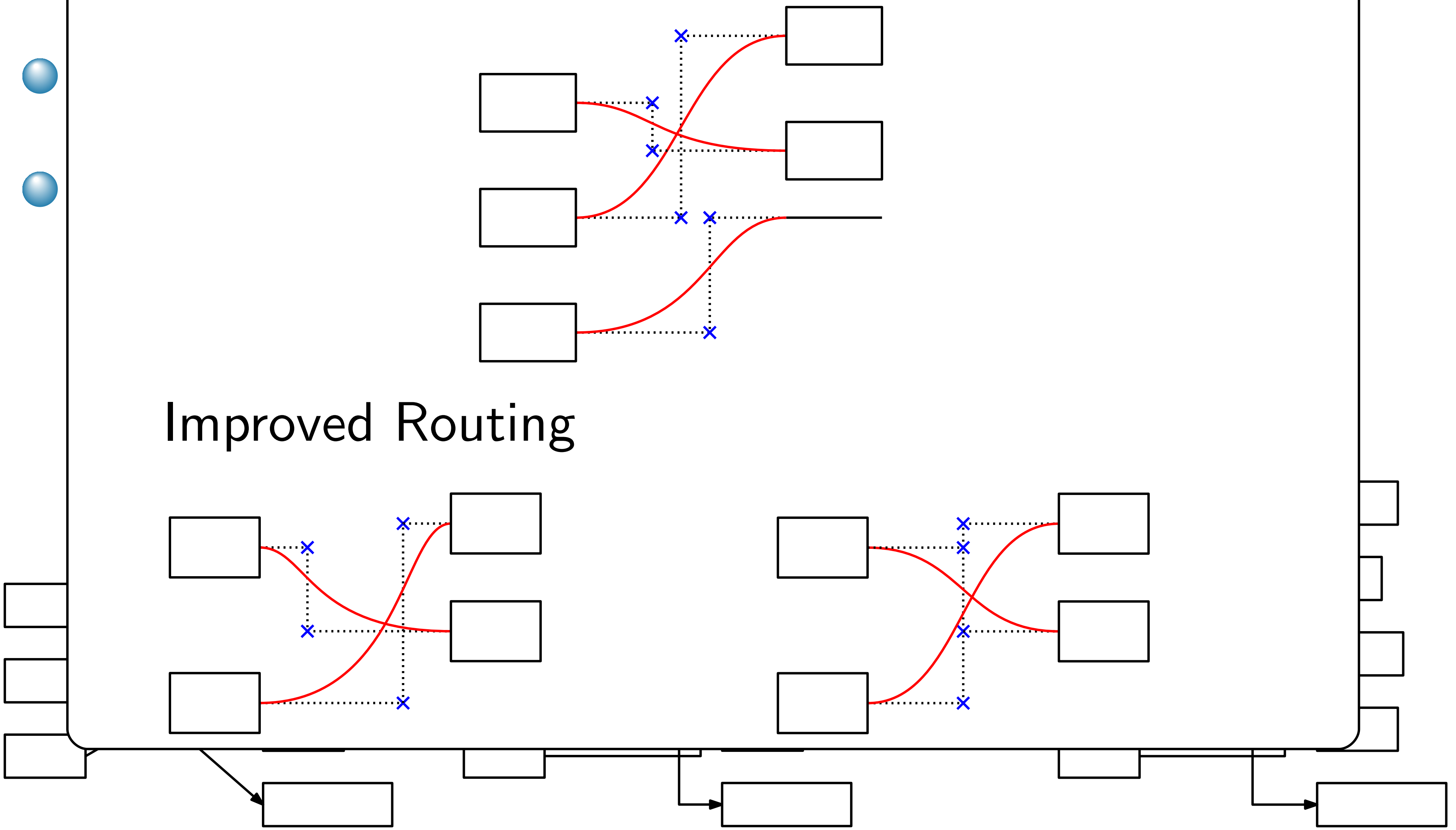
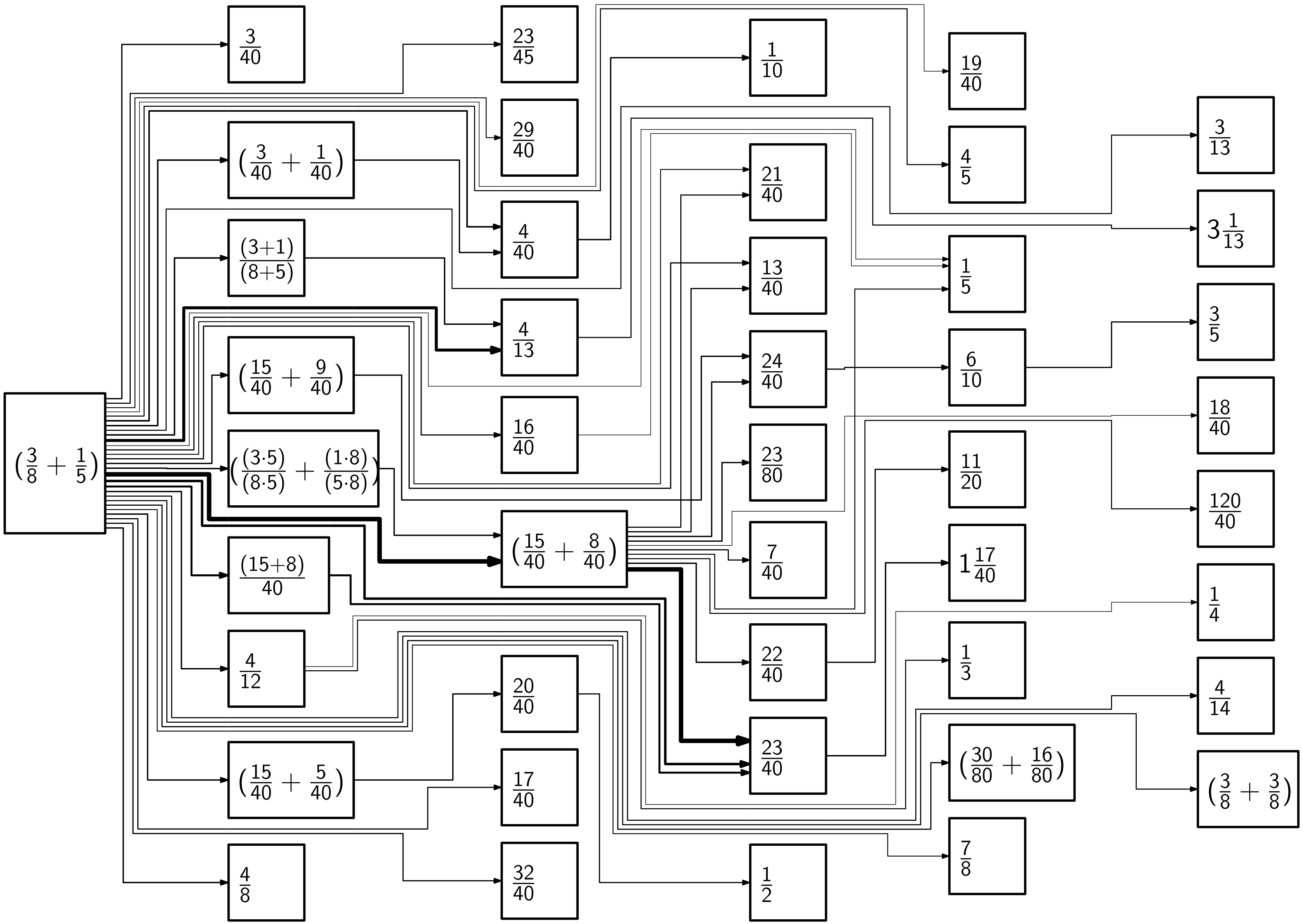


Example



38 of 358 vertices (10.6 %); 91.1 % of the vertex weight

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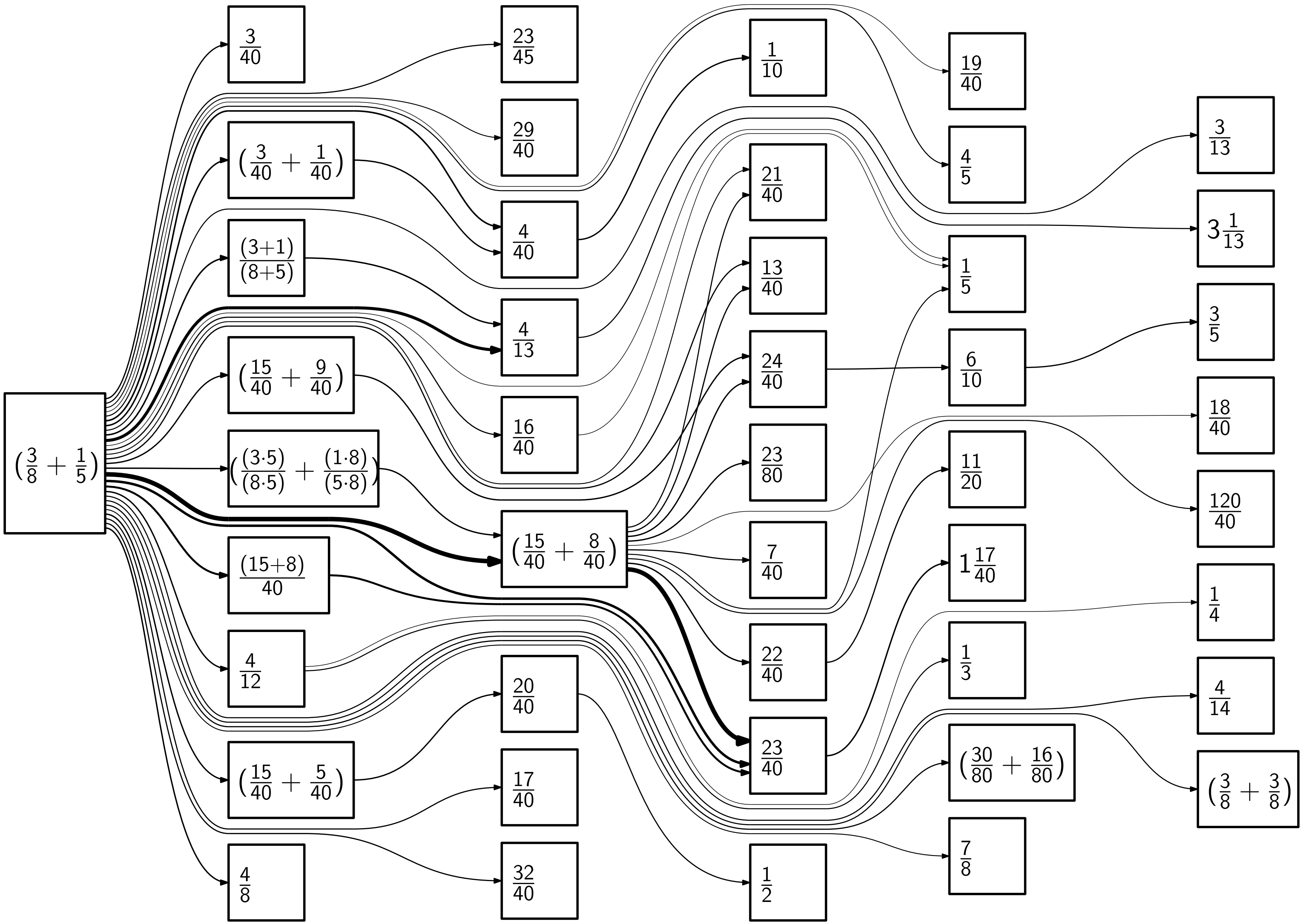

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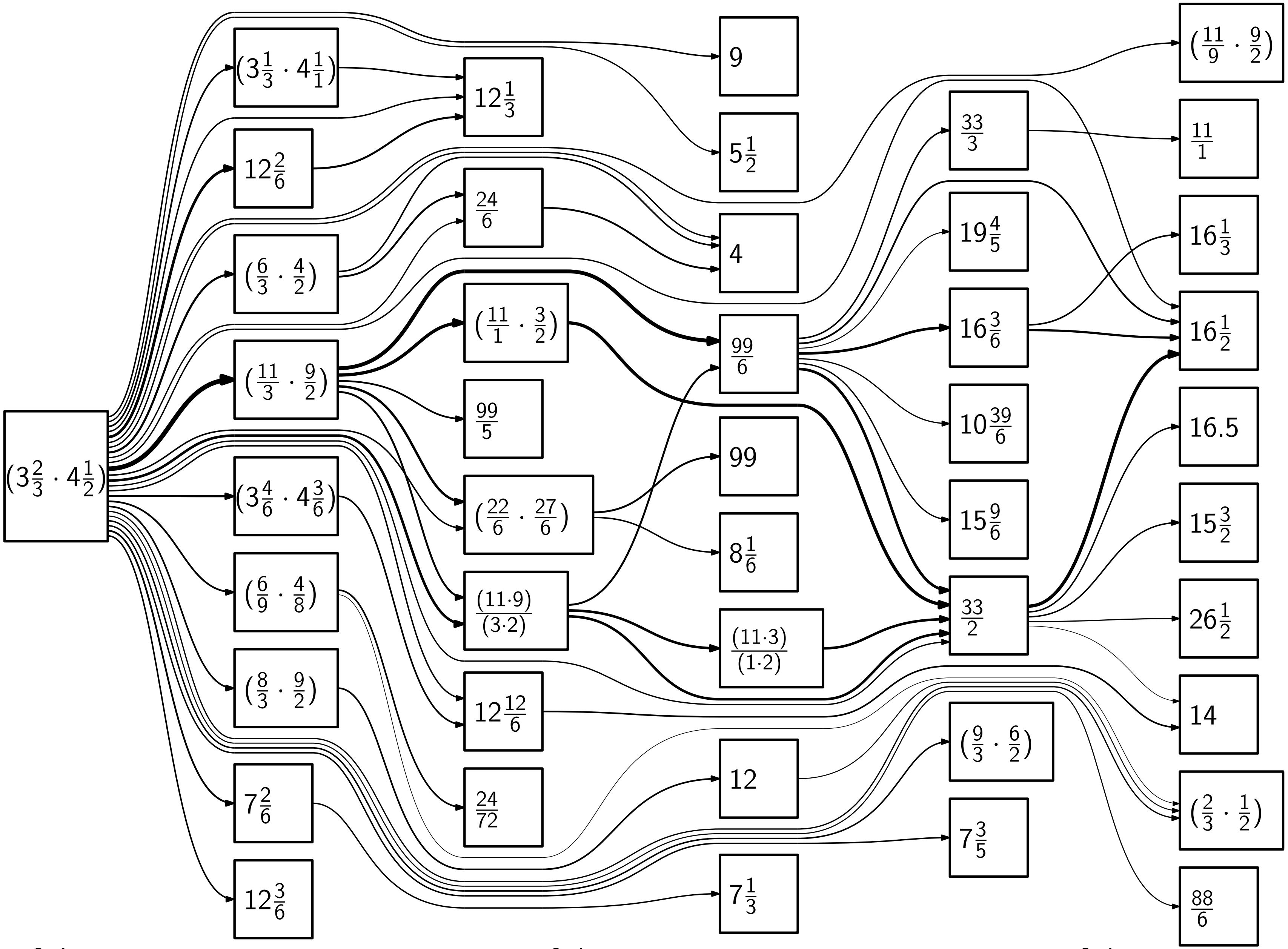
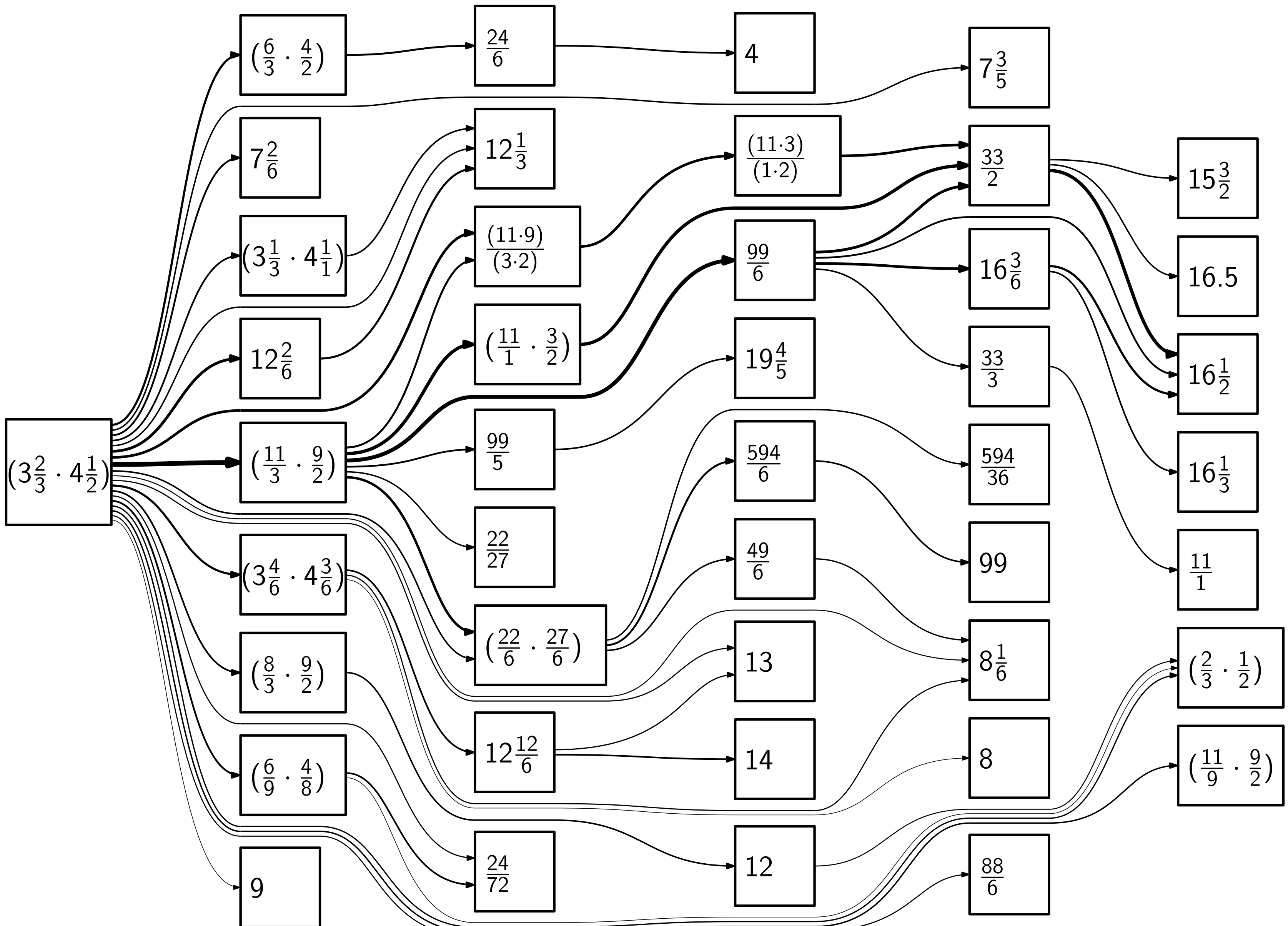
Example 2



4.7 % of 1031 vert.; 75.9 % vertex weight; 74.1 % edge weight

Example 2 – Planar Output



4.3 % of 1031 vert.; 76.6 % vertex weight; 70.2 % edge weight

Remarks

- runtime for graph with 1031 vertices and 1549 edges to A4 paper: $\approx 3s$

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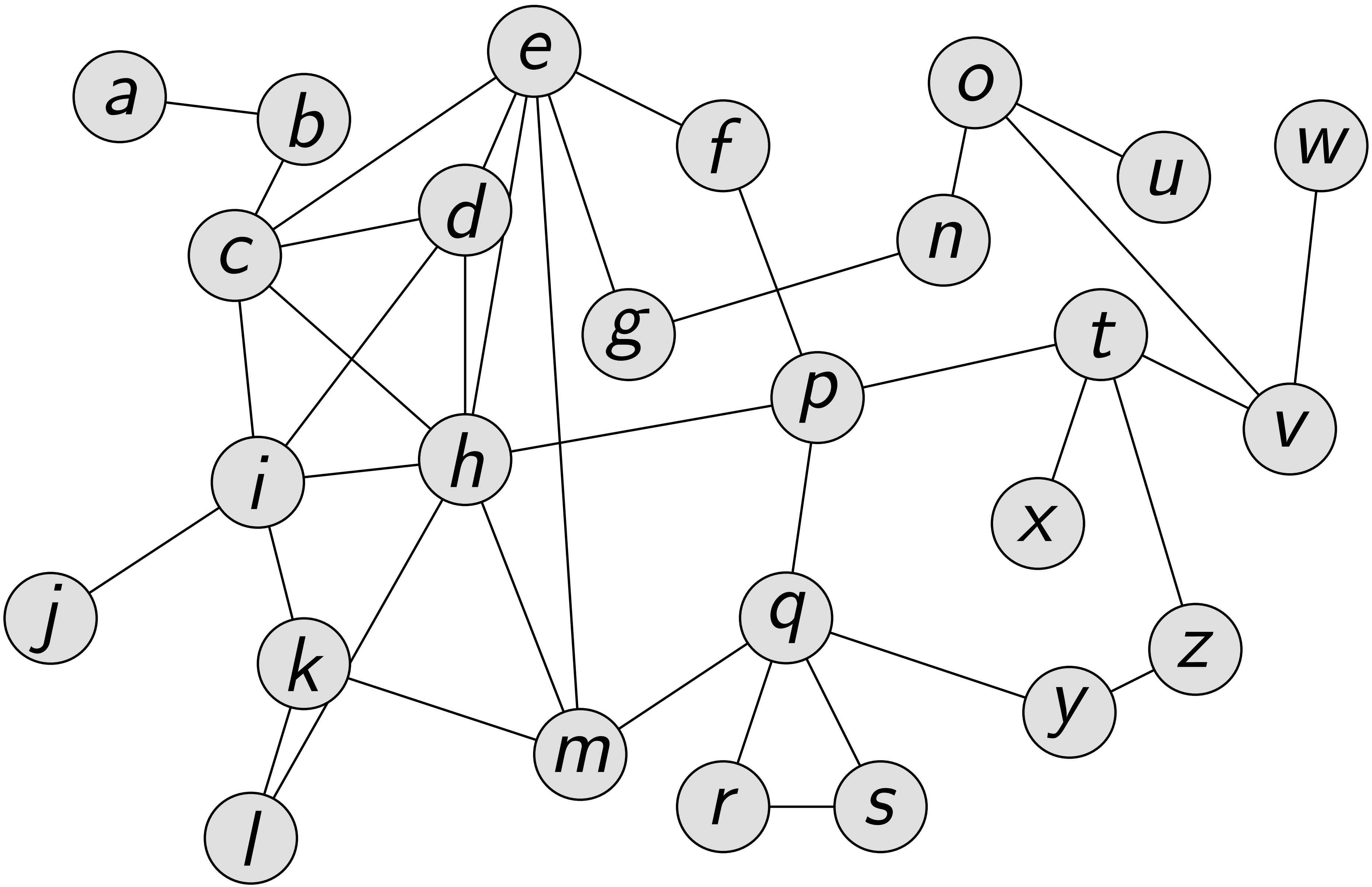
- especially with vertex/edge reinsertion: no significant influence of layer assignment algorithm

General Graphs



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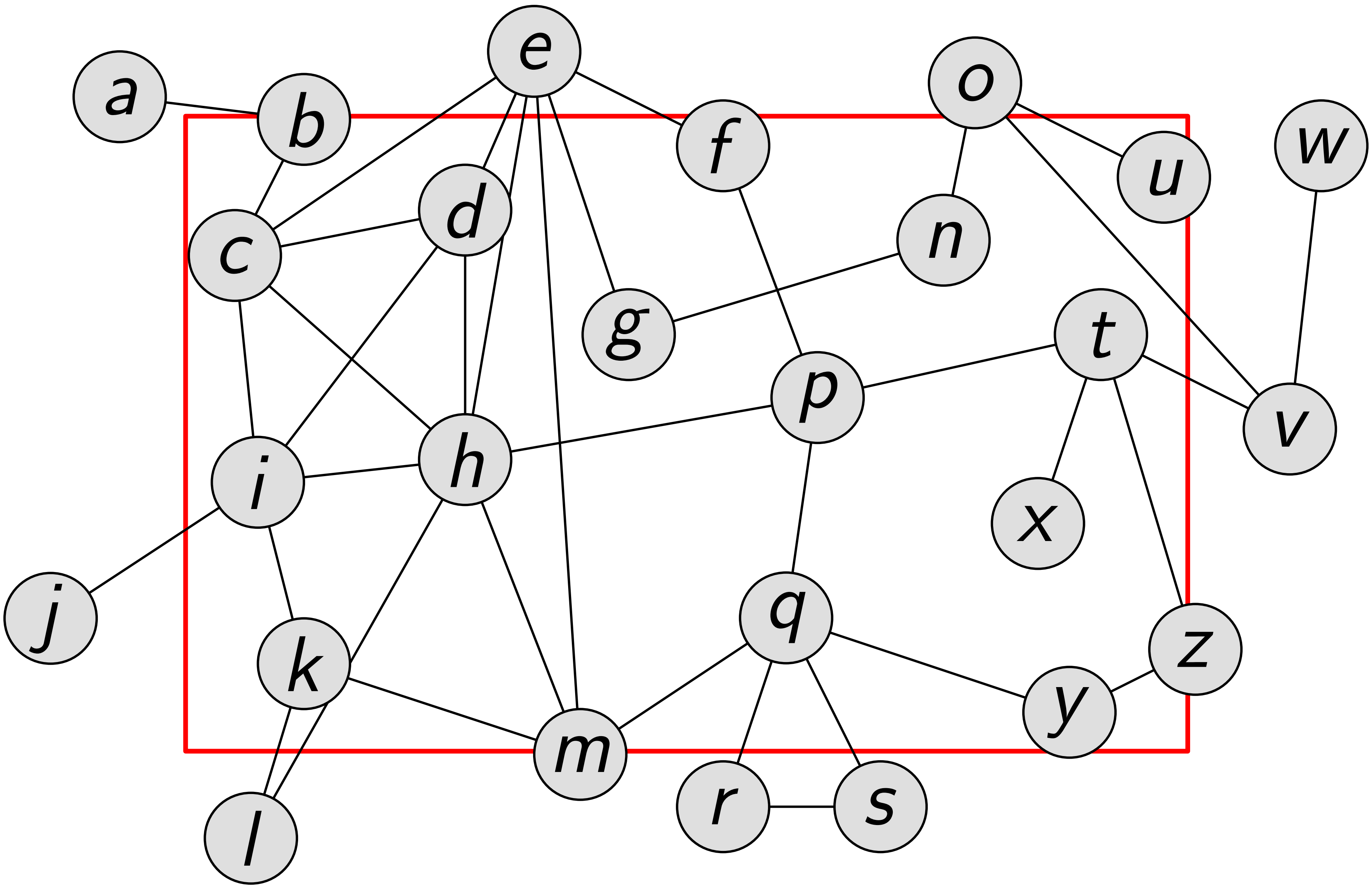

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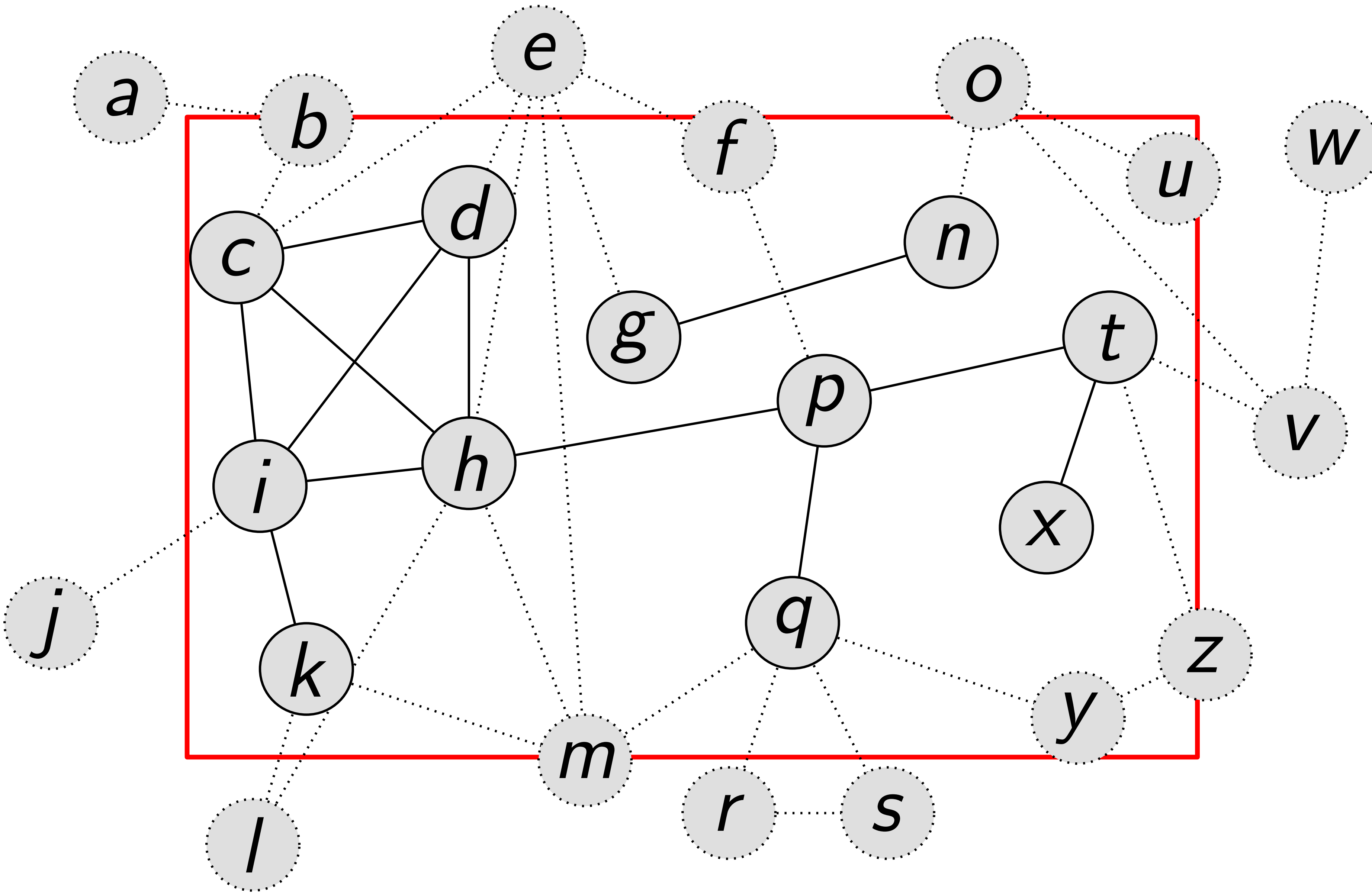

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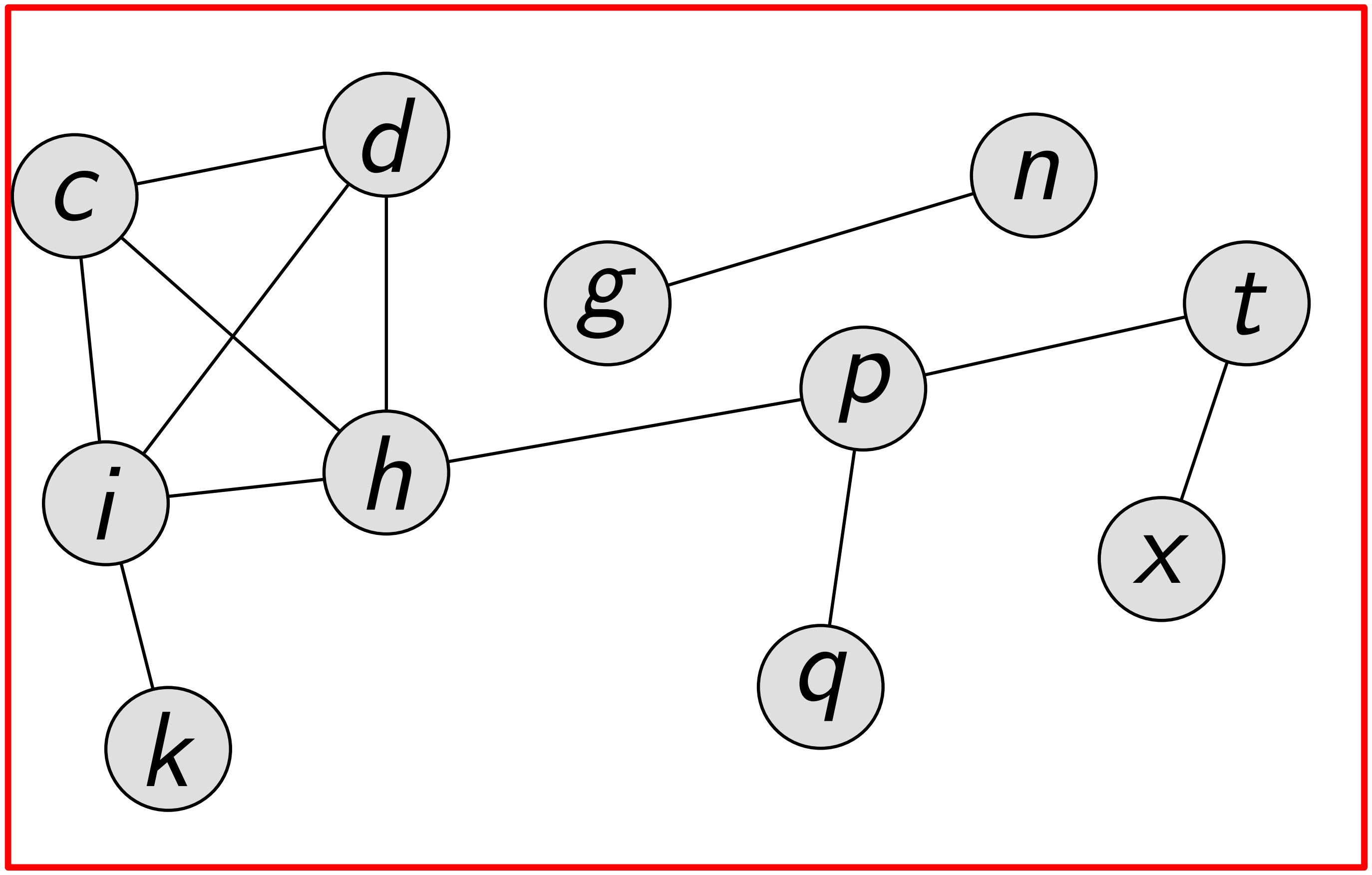

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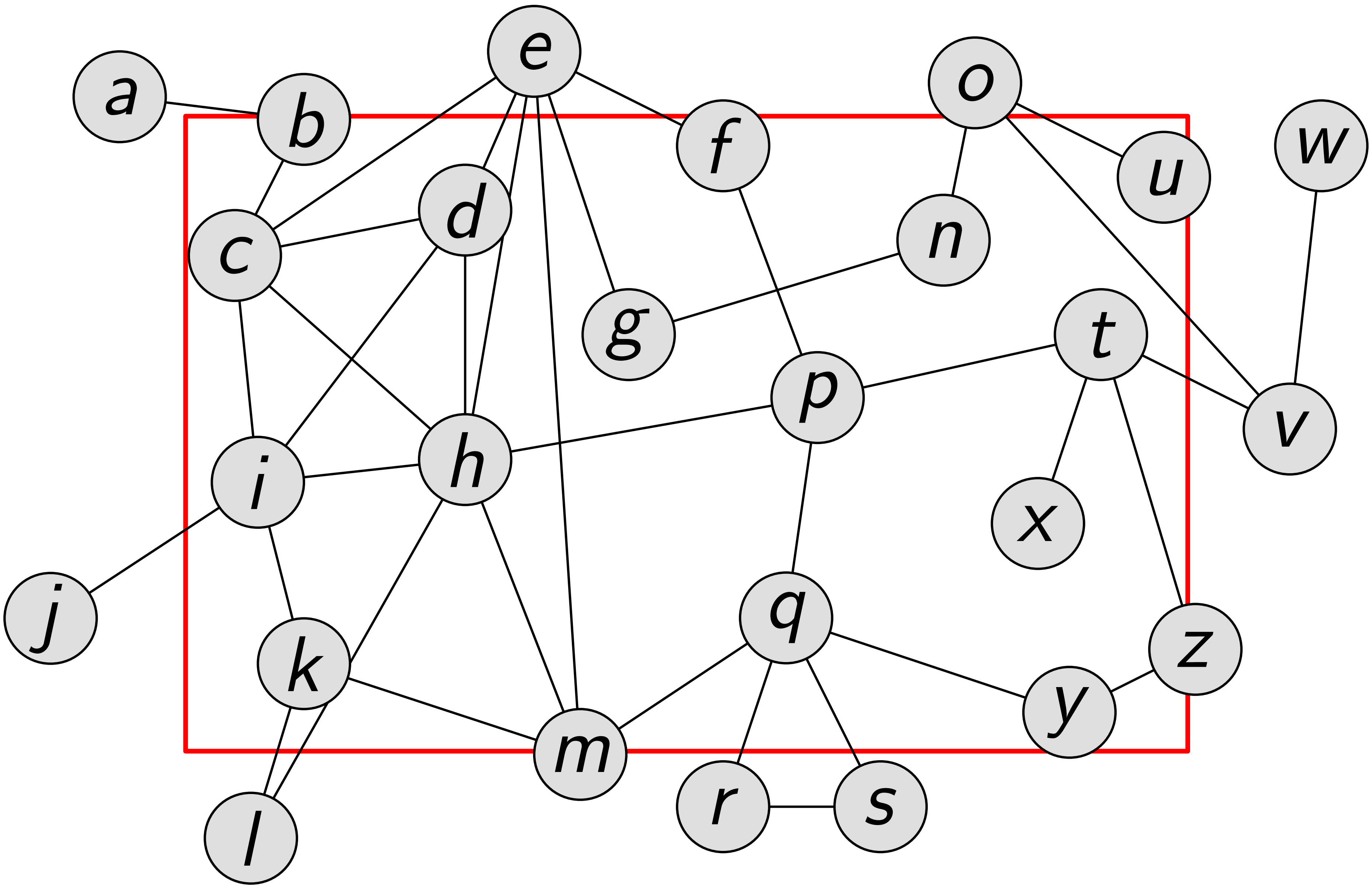

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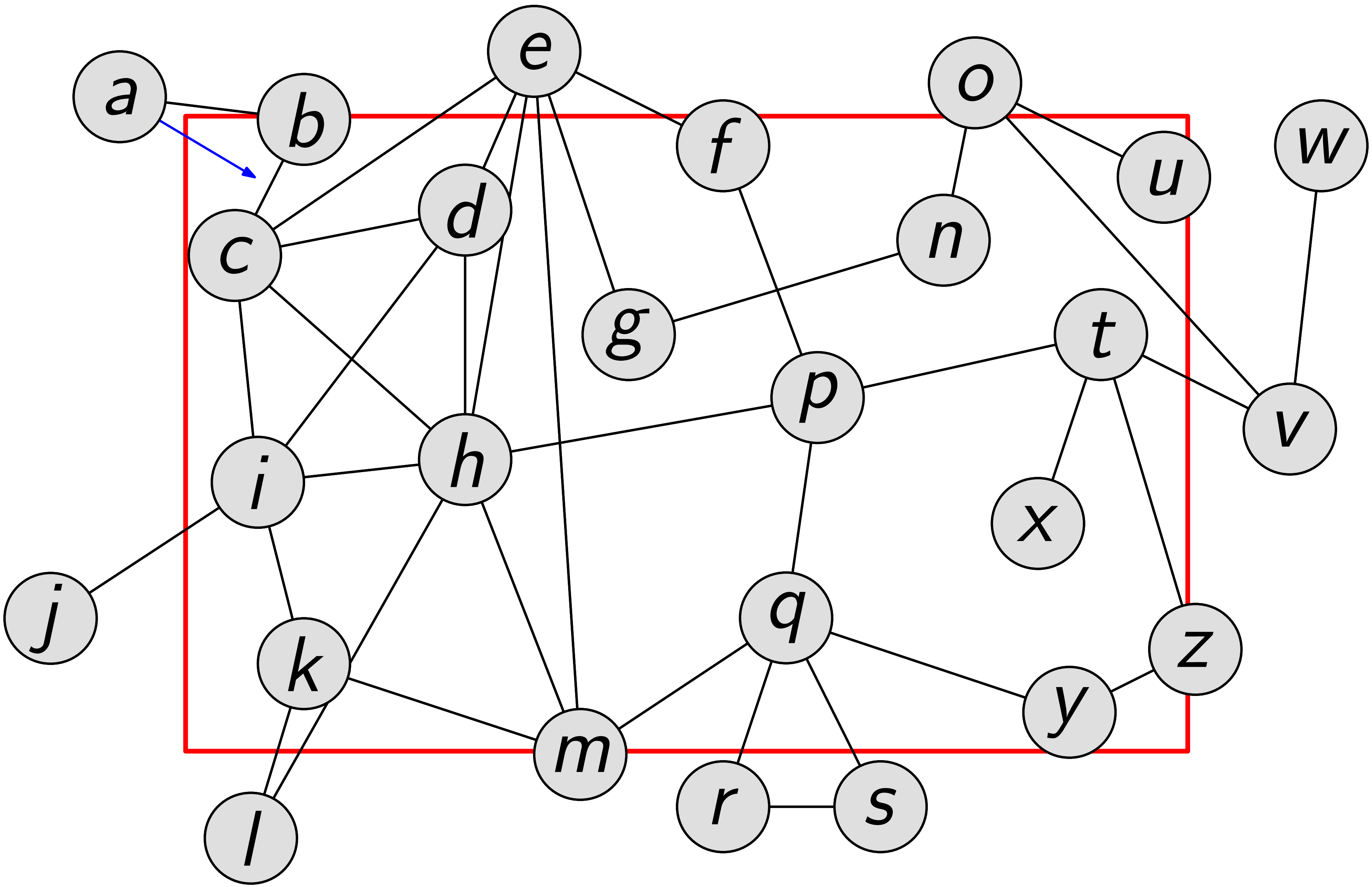

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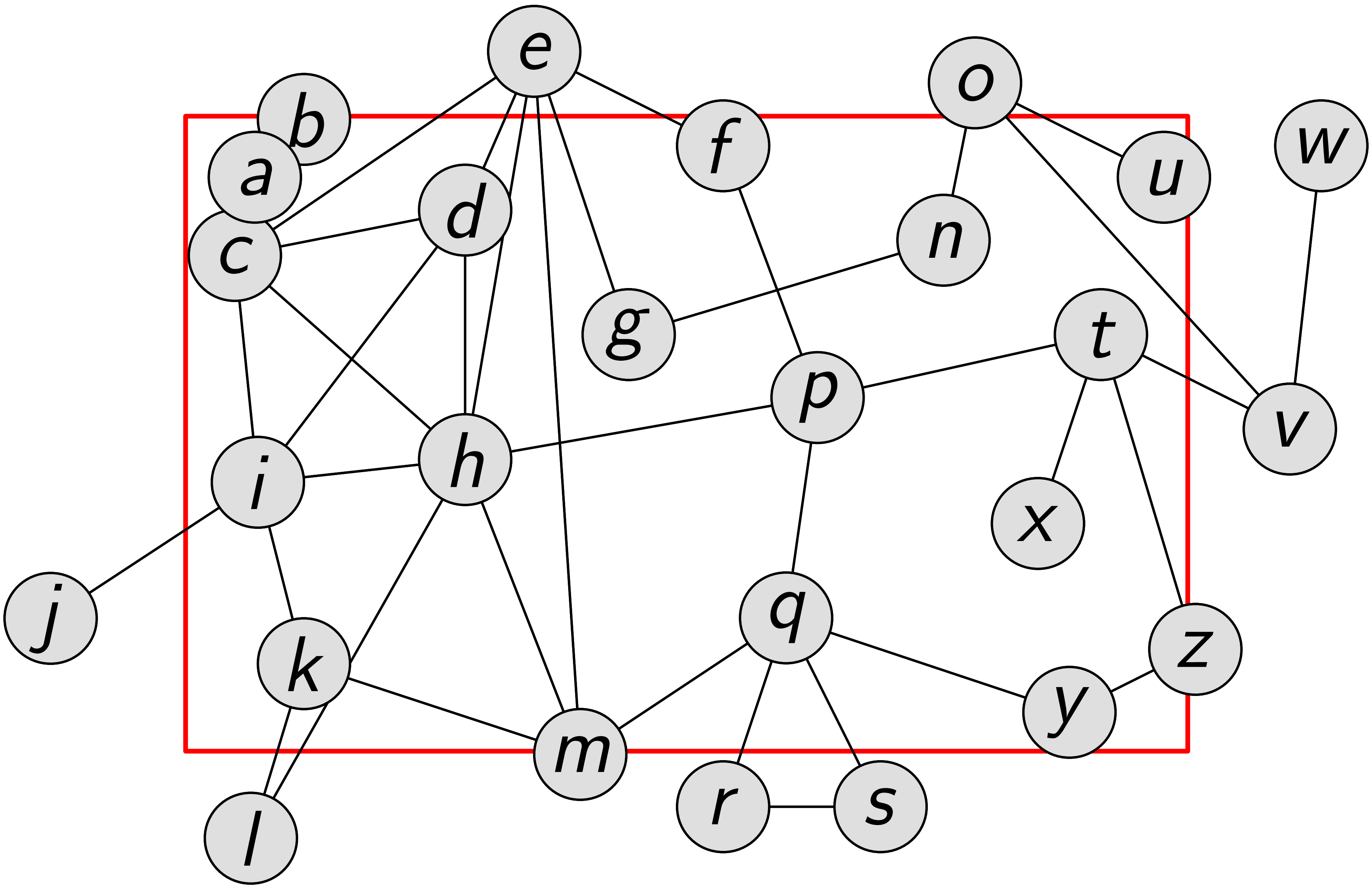

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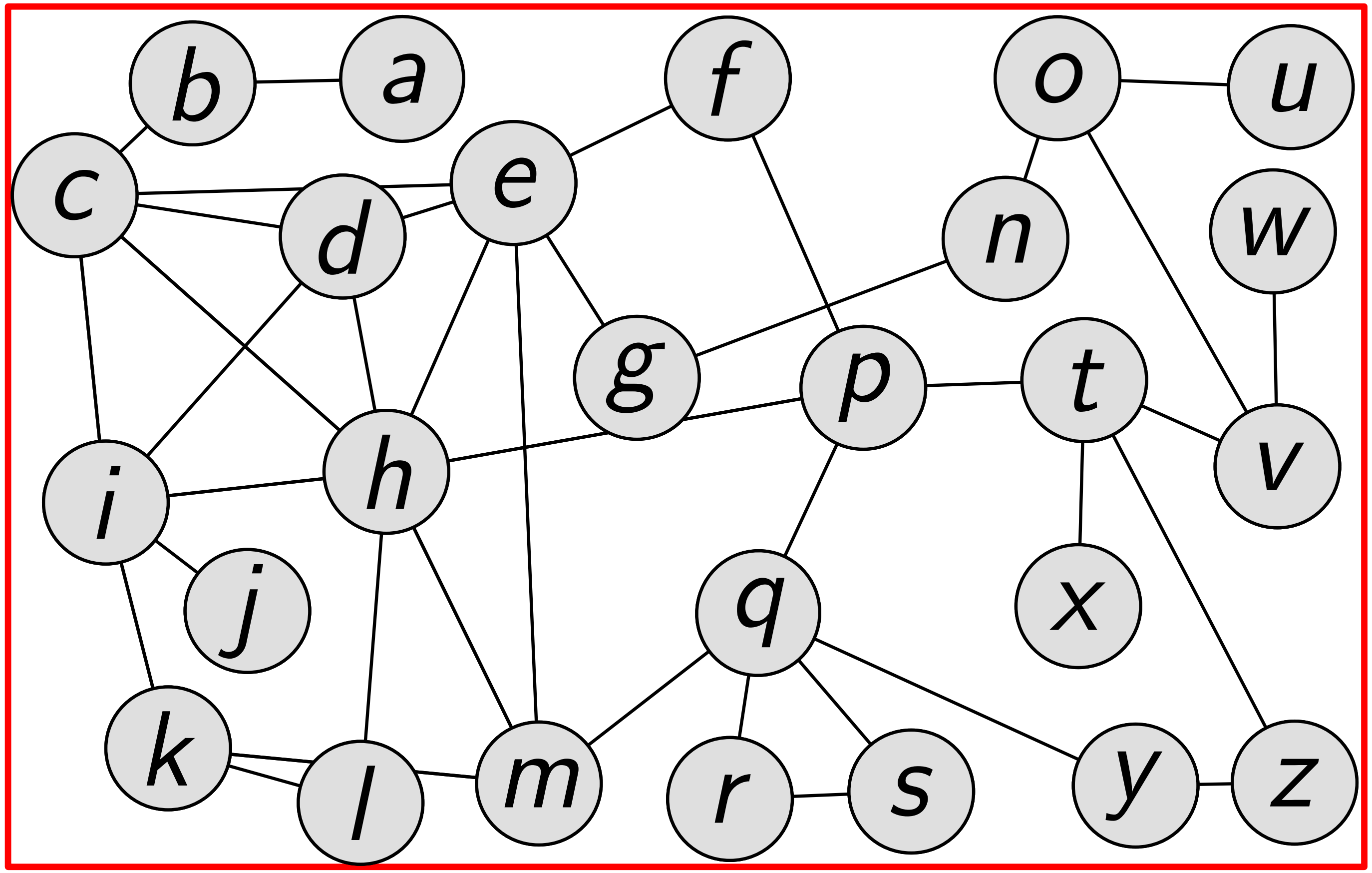
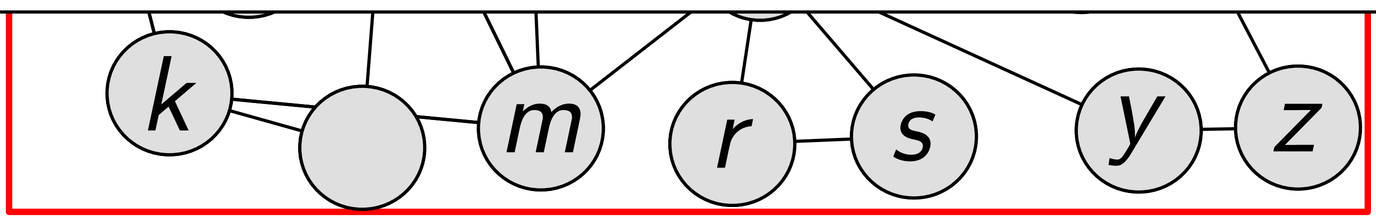


General Graphs – Basic Ideas

- use force-directed approach + standard forces



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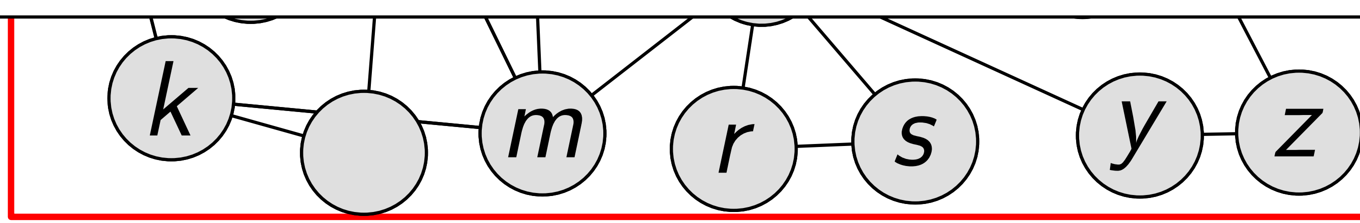
- use force-directed approach + standard forces
- keep drawing in prescribed *frame*



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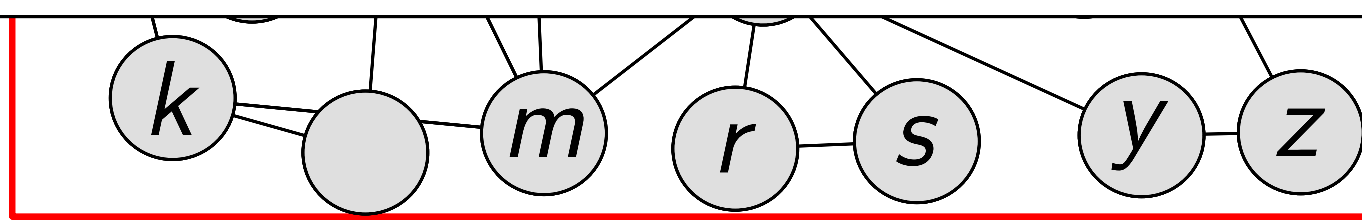
compare [Fruchterman, Reingold '91]



General Graphs – Basic Ideas

- use force-directed approach + standard forces
- ~~keep drawing in prescribed *frame*~~
- keep drawing in *frame*
- slowly shrink frame

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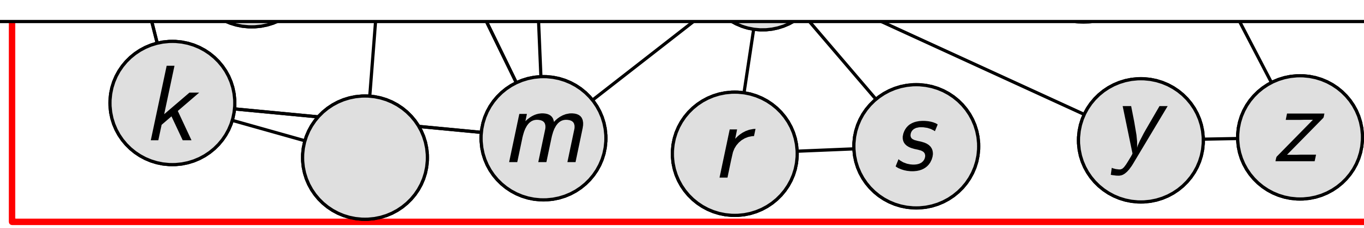

General Graphs – Basic Ideas

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- keep drawing in *frame*
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- remove vertices/edges if necessary after shrinking
- find new equilibrium

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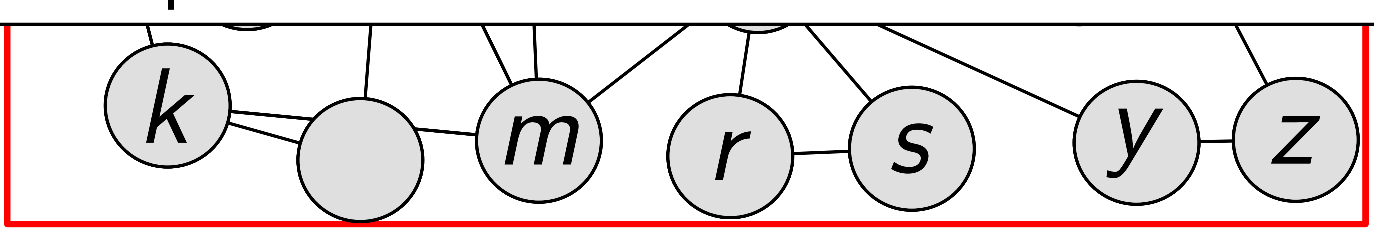

General Graphs – Outline

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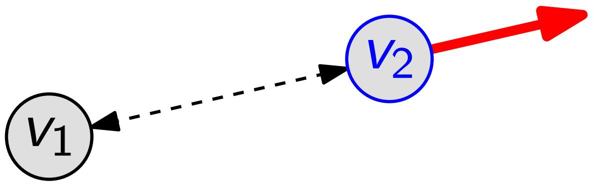
- postprocessing: transform straight-line edges to Bézier curves

Forces

- repulsion between vertices



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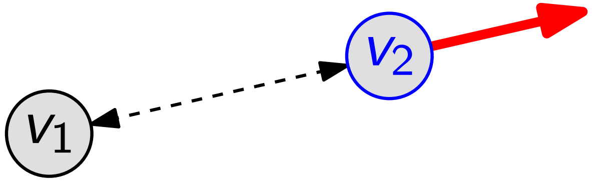
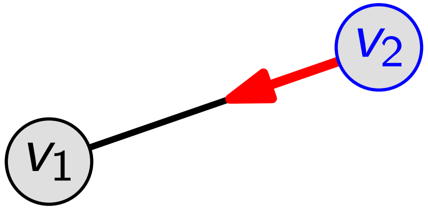


- attraction between *adjacent* vertices



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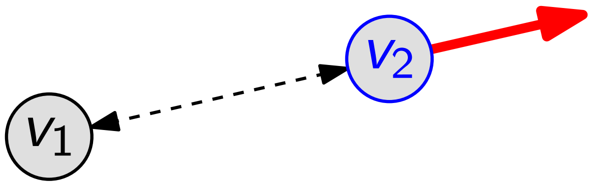

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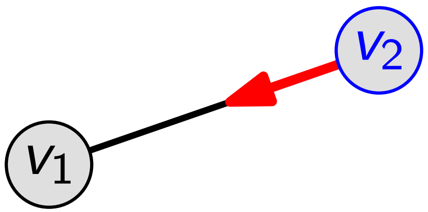

[Fruchterman, Reingold 1991]
parameterized by
desired edge length

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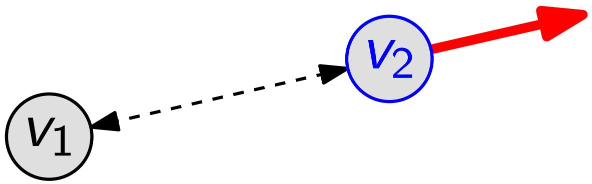

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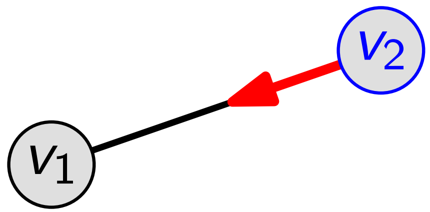
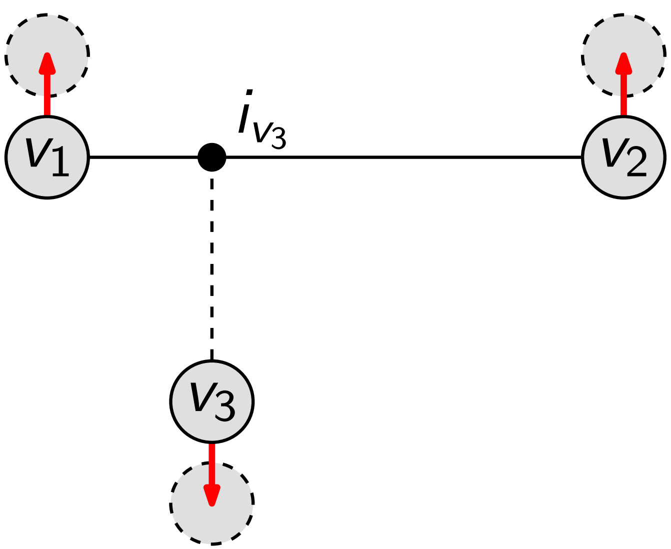


[Fruchterman, Reingold 1991] parameterized by desired edge length

- edge-vertex repulsion



[Bertault, 2000]

Forces

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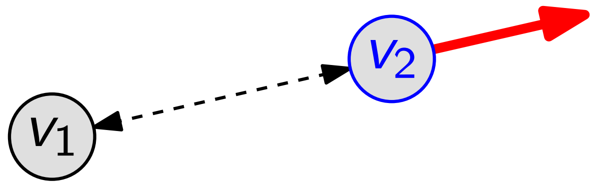

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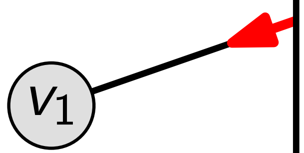

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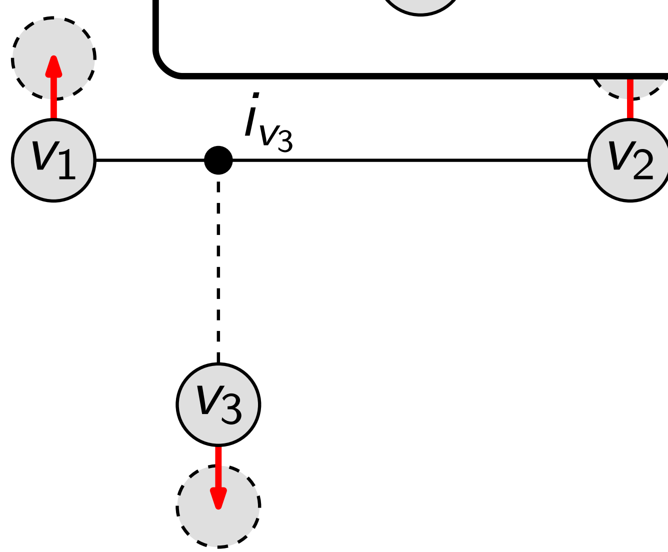
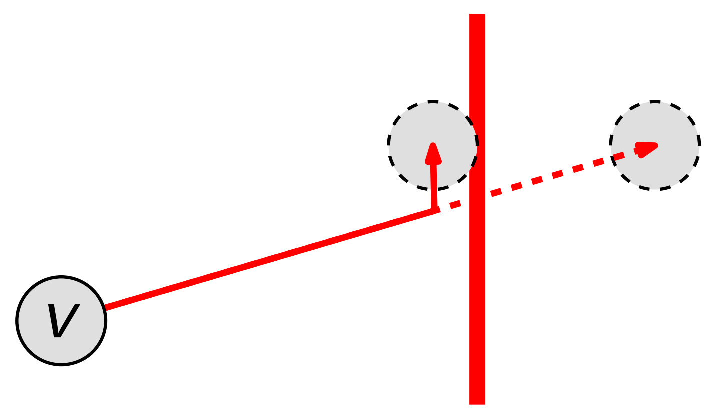


[Fruchterman, Reingold 1991]

parameterized by

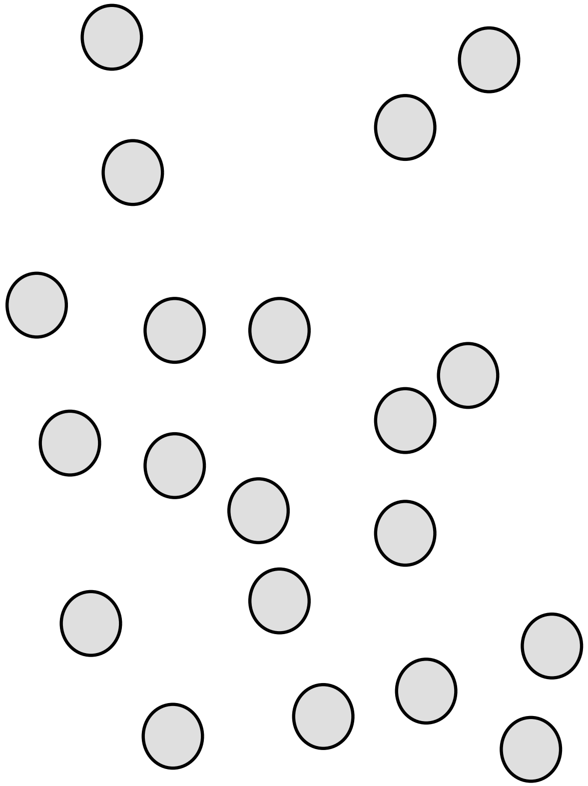
gth

frame boundary: cut off movement



[Bertault, 2000]

Handling the Frame



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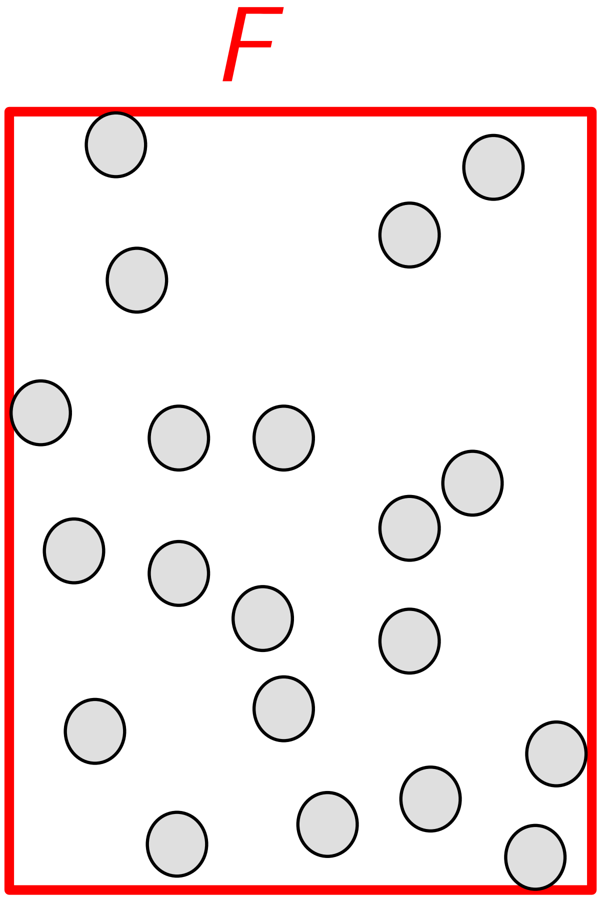

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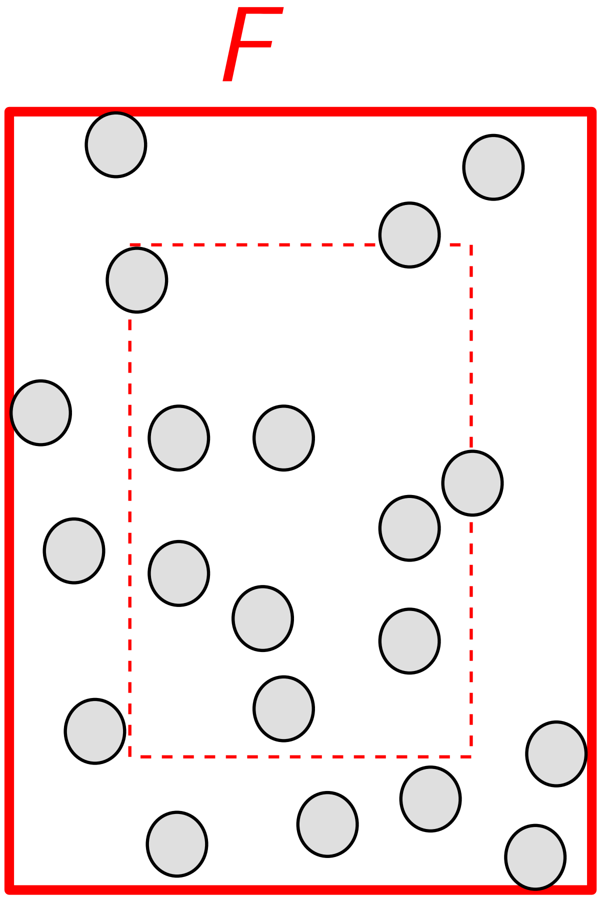
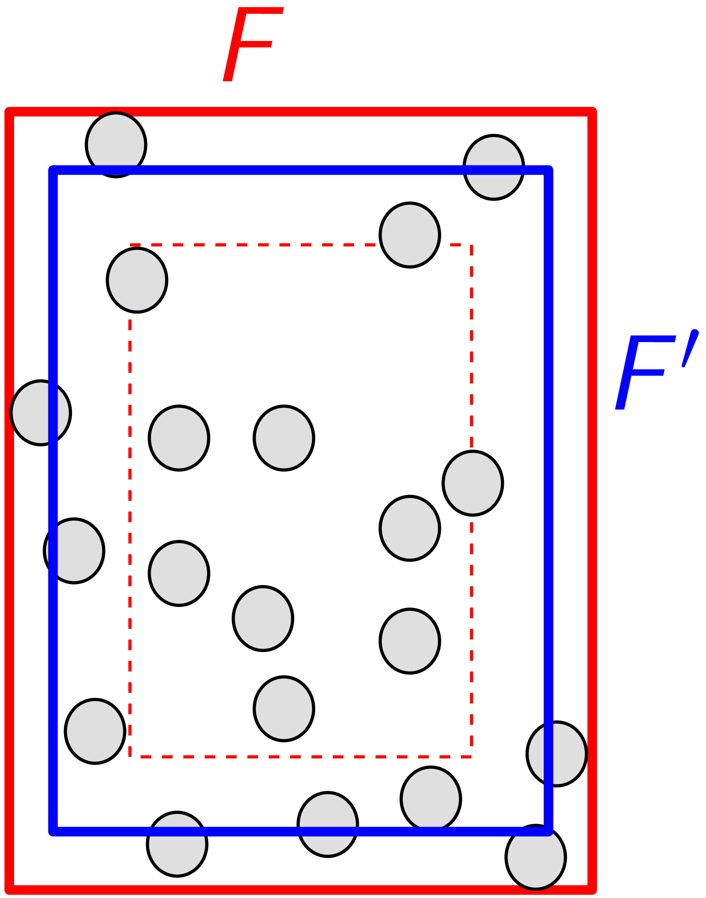


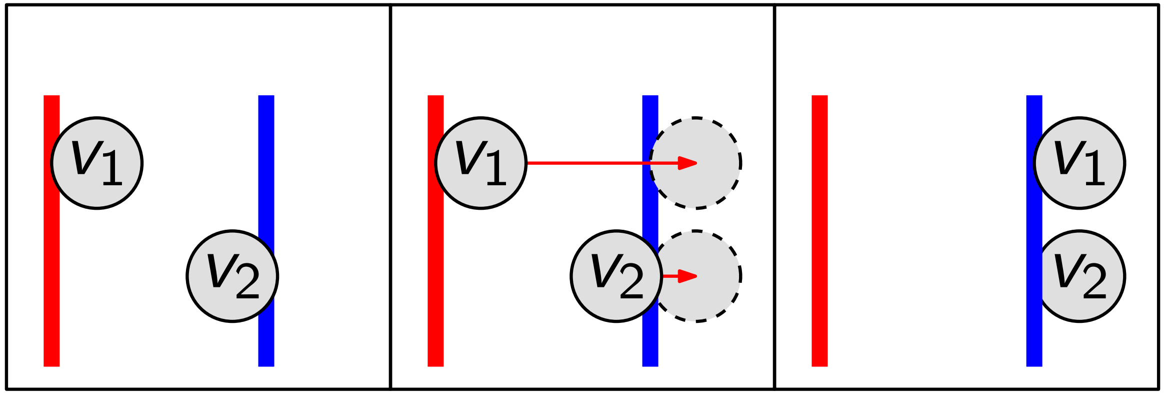
Handling the Frame

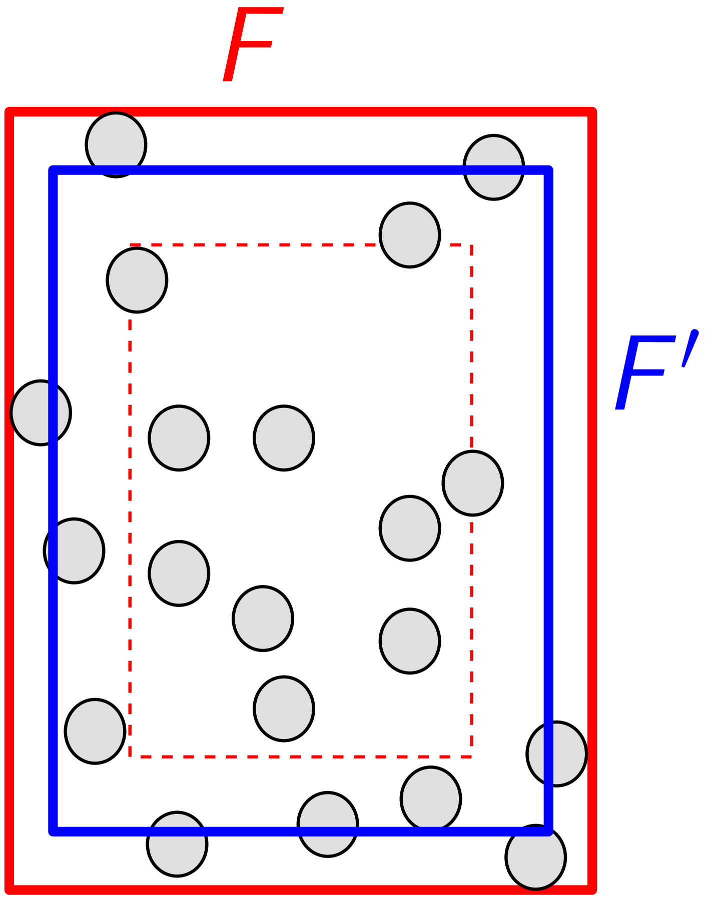
- iteratively shrink frame



Handling the Frame

- iteratively shrink frame
- push vertices into frame





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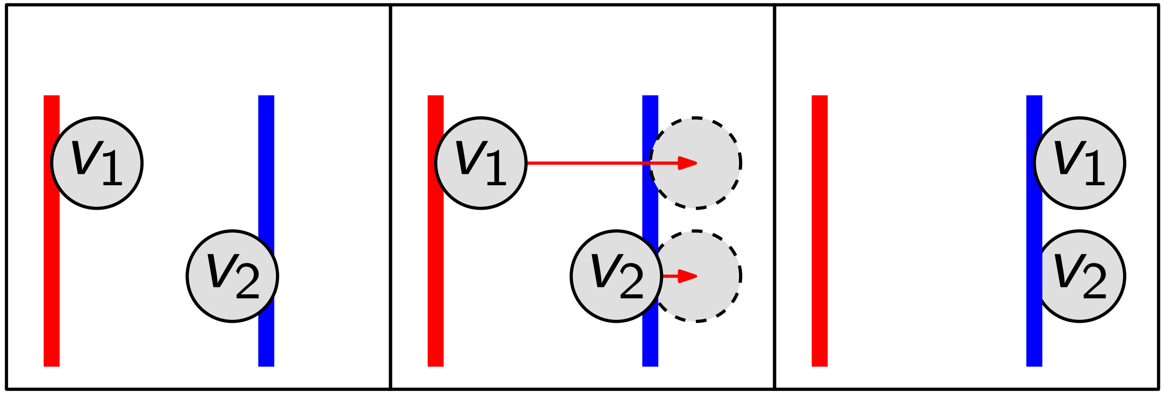

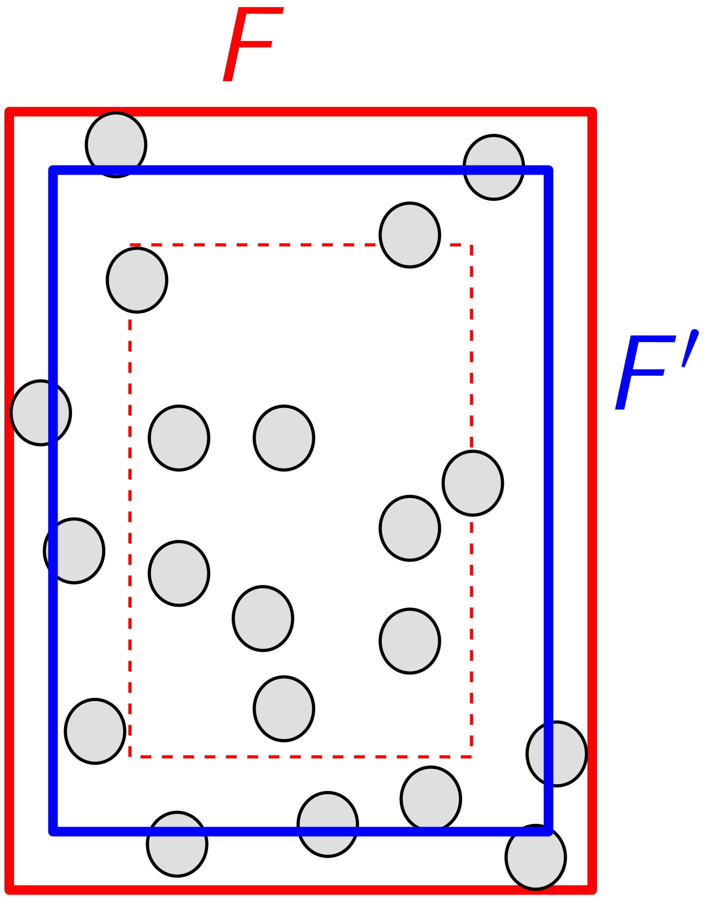

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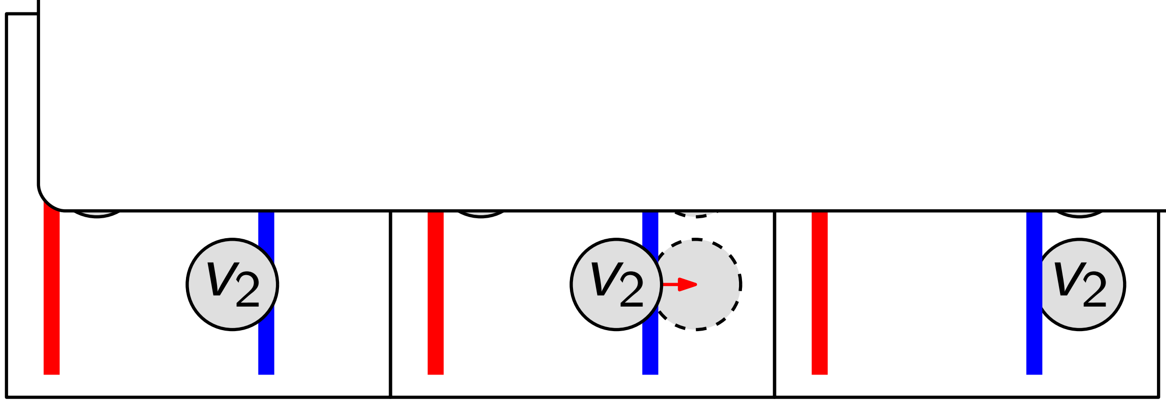
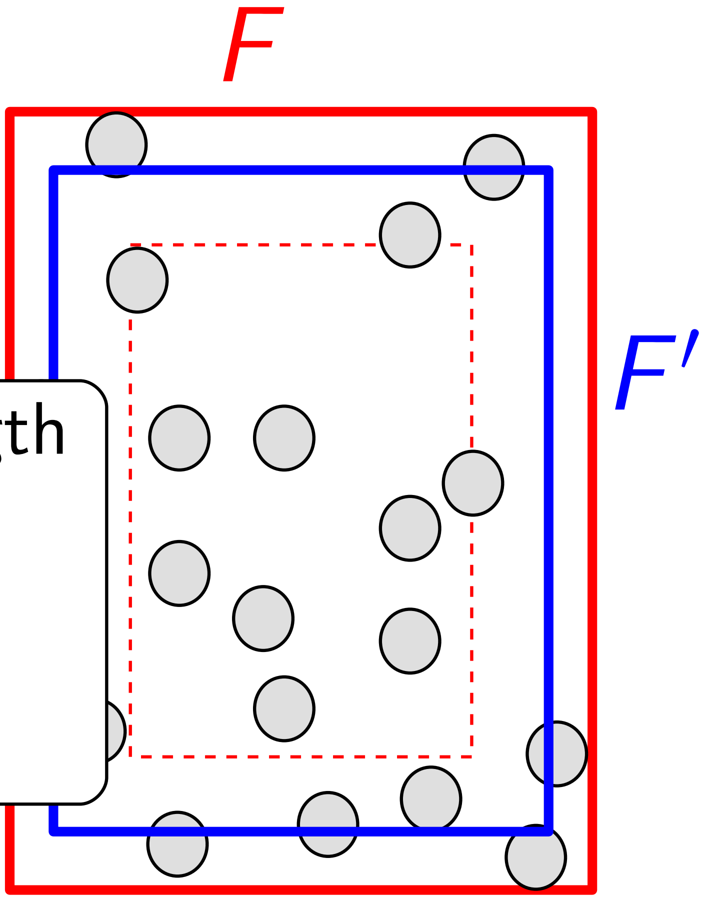
distances too short ↔ desired edge length





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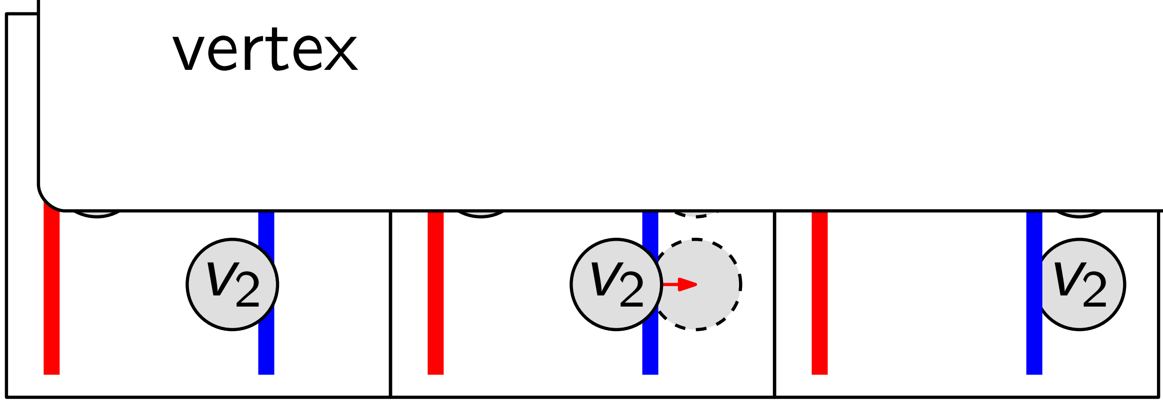

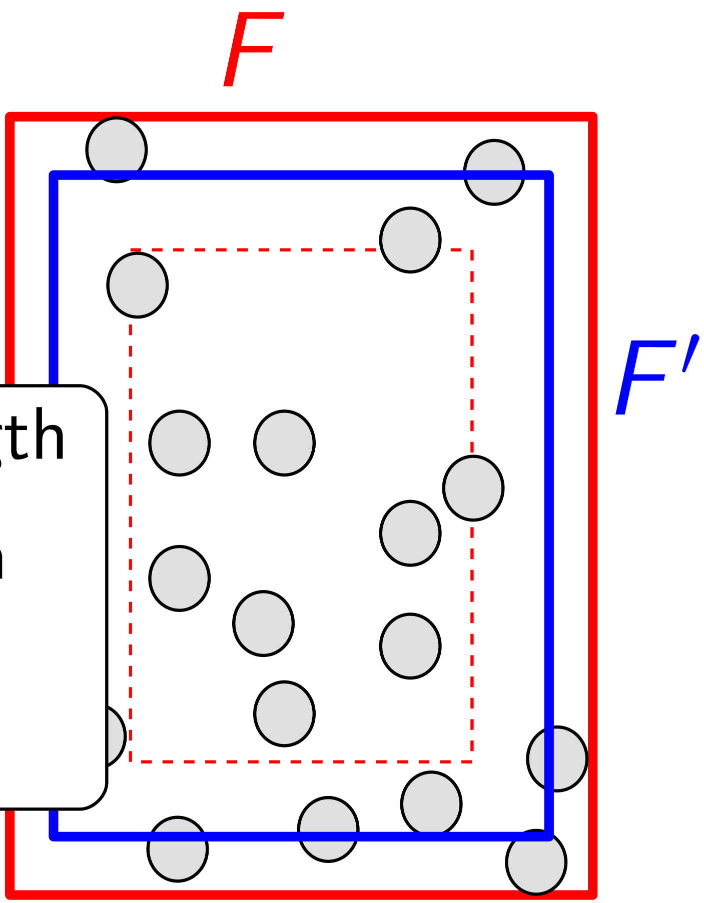

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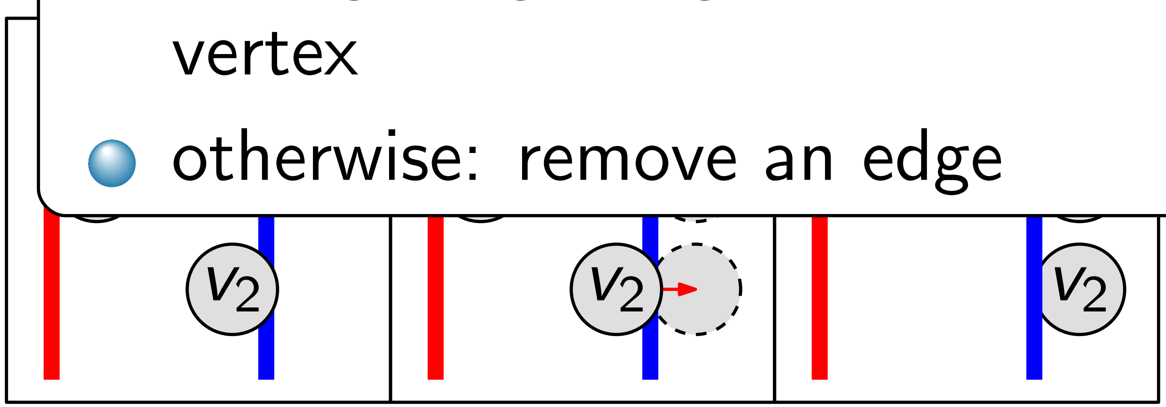
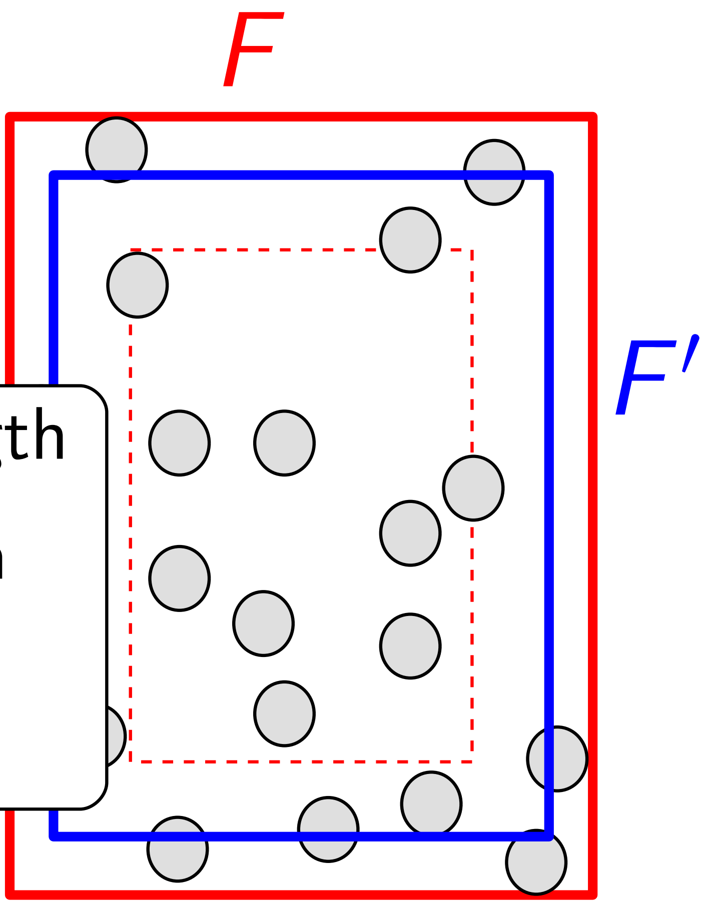
- otherwise: remove an edge





Removing vertices – Pressure and Stress

- remove lightest vertex
- remove vertex with worst weight-area ratio

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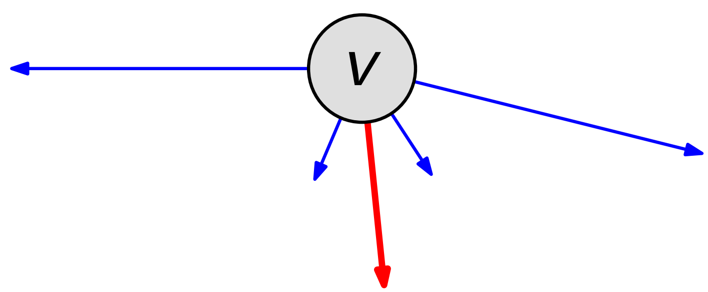
idea:



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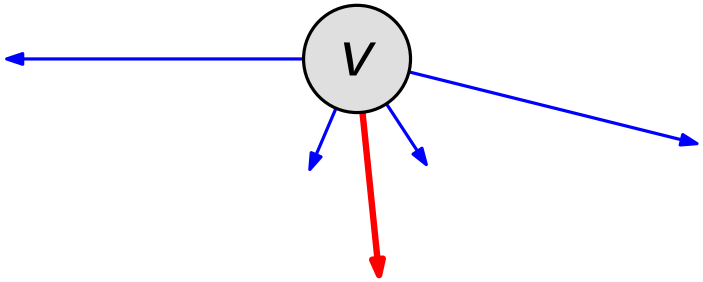
idea: forces in opposite directions



pressure $P(v)$

Removing vertices – Pressure and Stress

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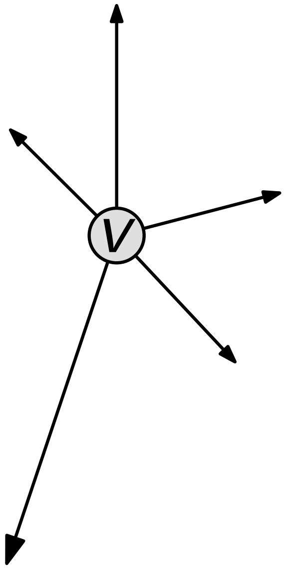

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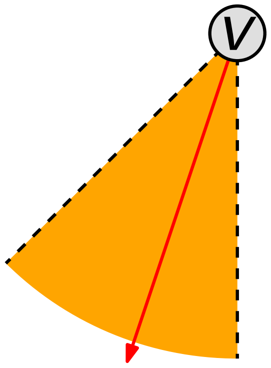

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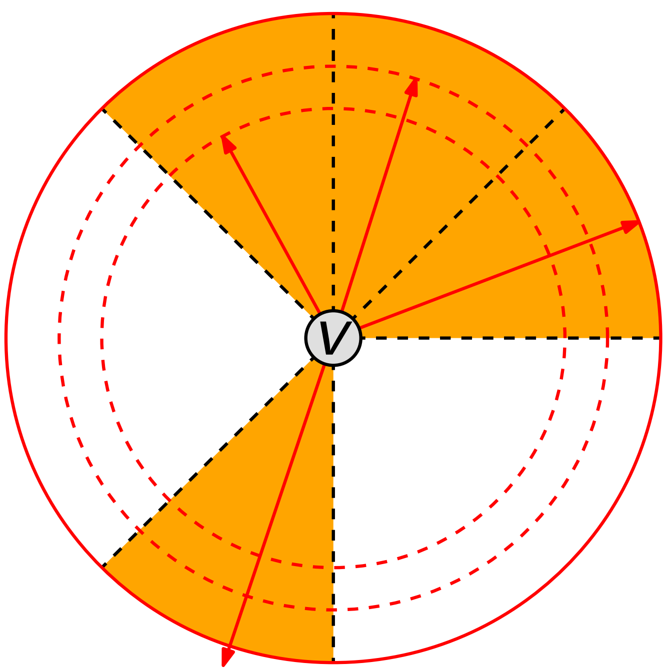

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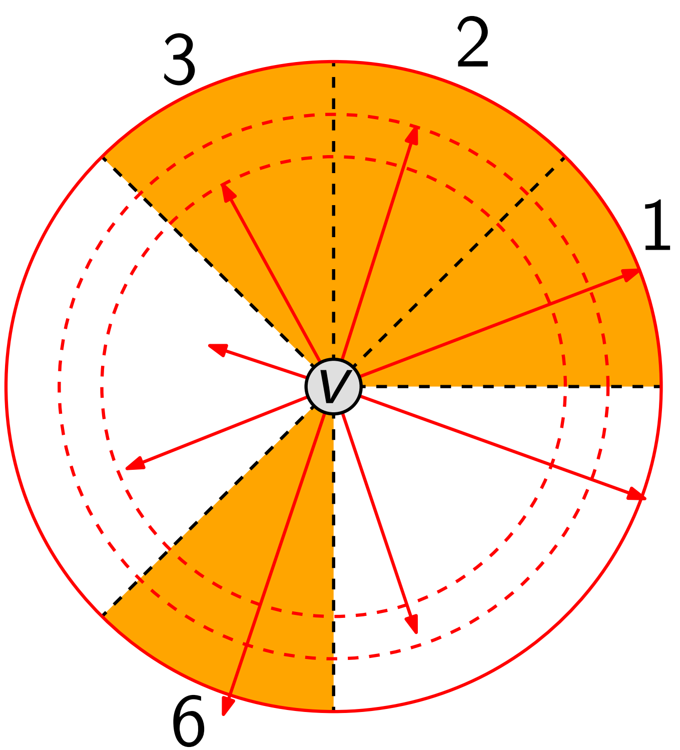

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– worst pair $\rightarrow$ pressure

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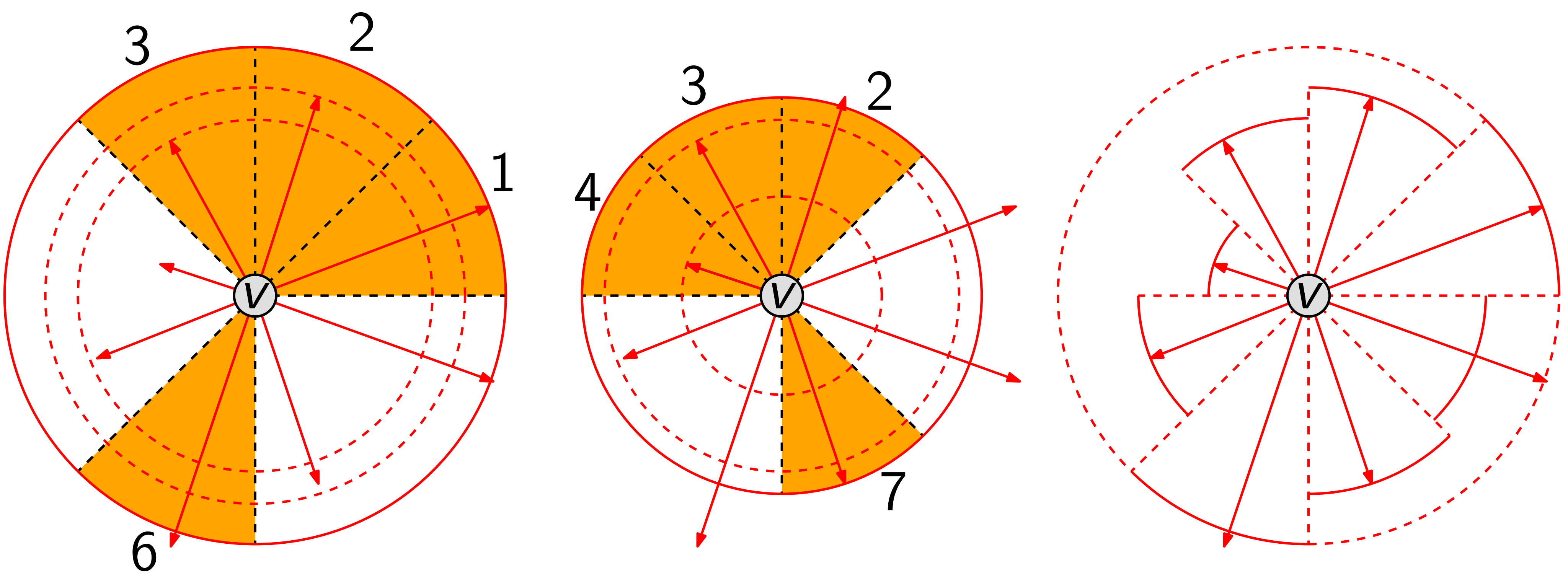

Removing vertices – Pressure and Stress

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- also take current drawing into account:
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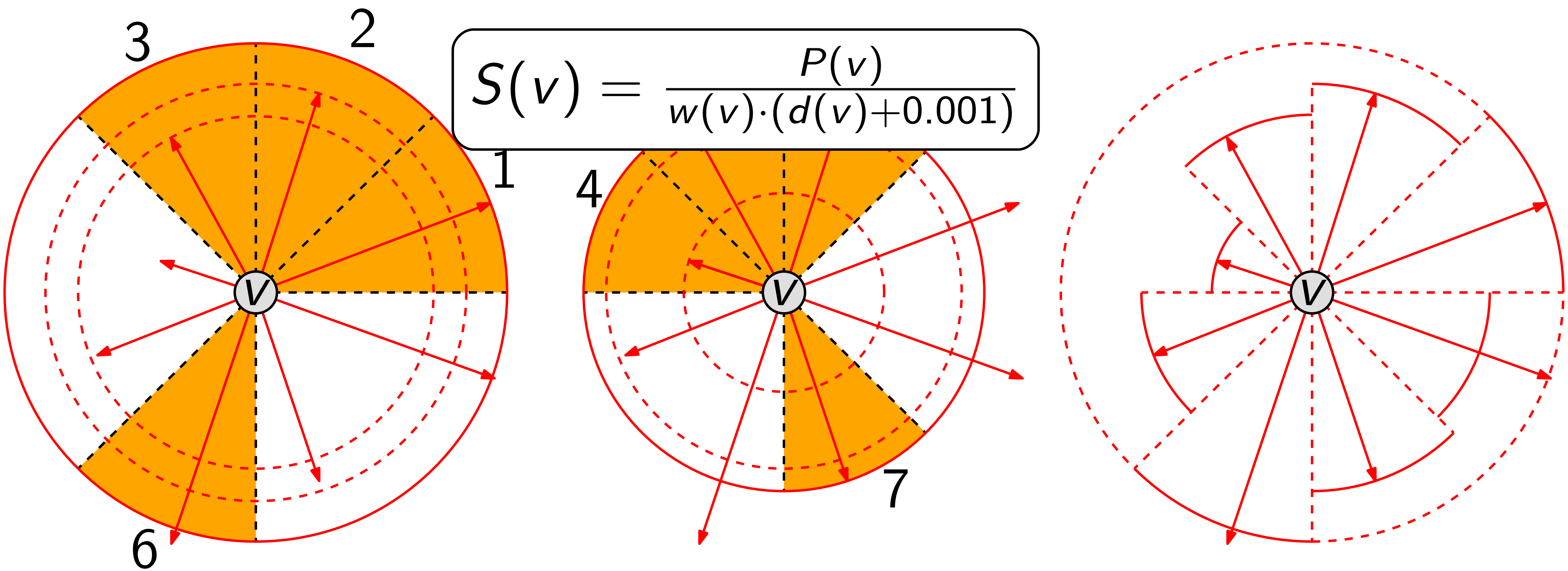
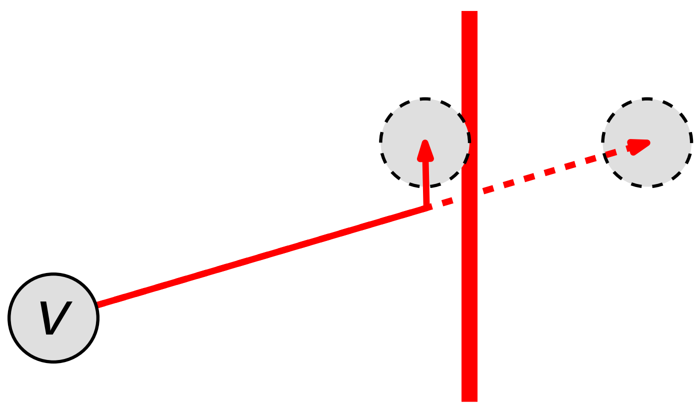
- remove vertex with highest stress value

$$S(v) = \frac{P(v)}{w(v)\cdot(d(v)+0.001)}$$



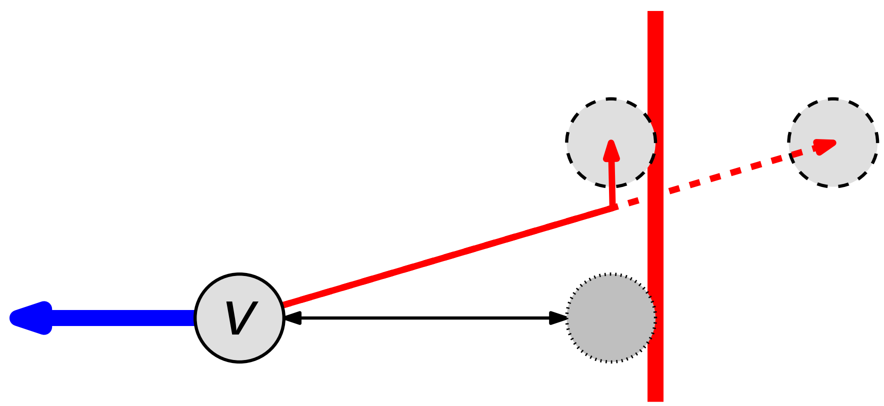
Remark: Boundary Vertices

- movement cut off



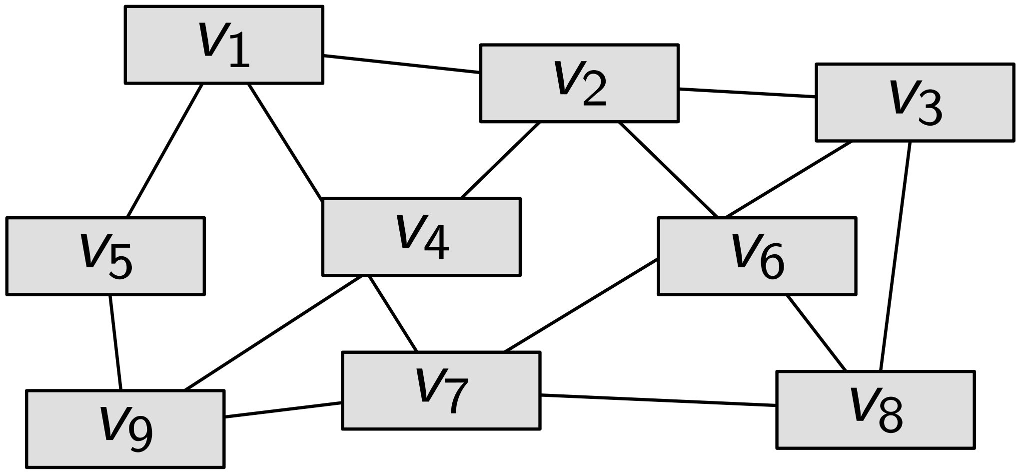
Remark: Boundary Vertices

- movement cut off
- model as force
- only considered for pressure computation



Postprocessing: Curved Edges

- avoid edge-vertex intersections



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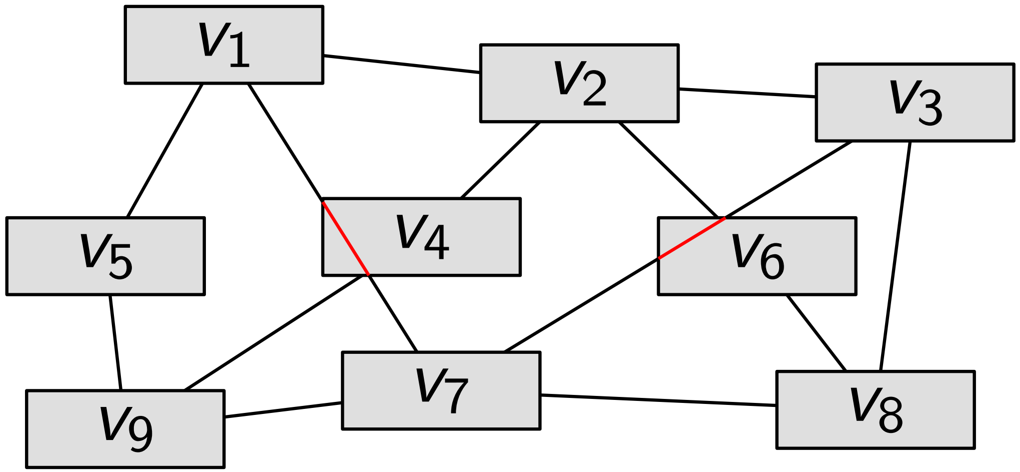

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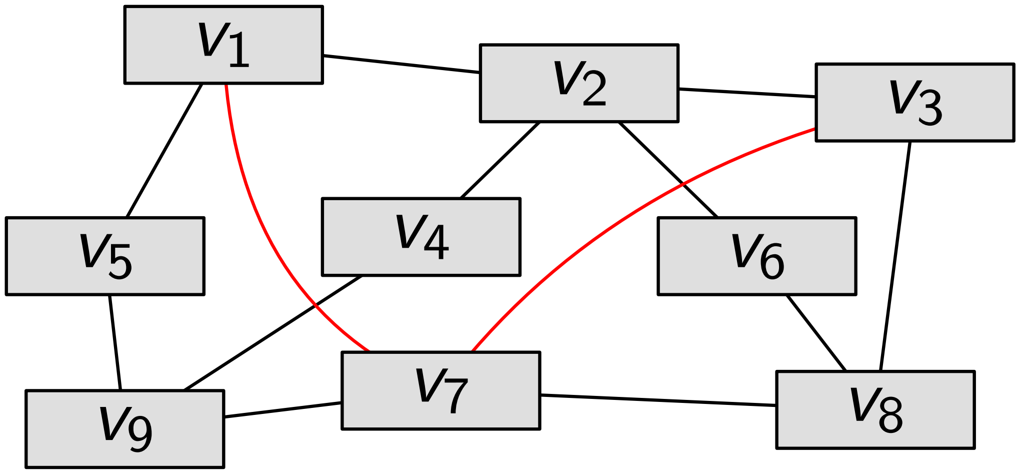

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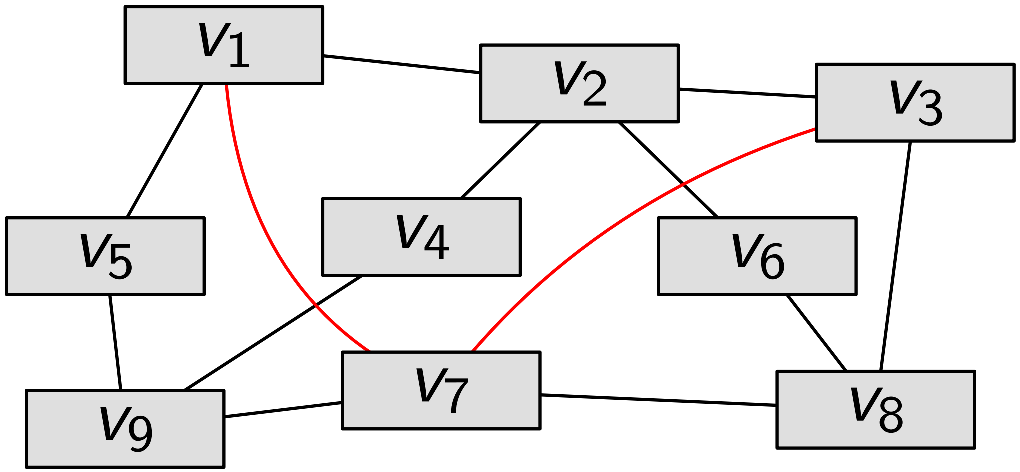
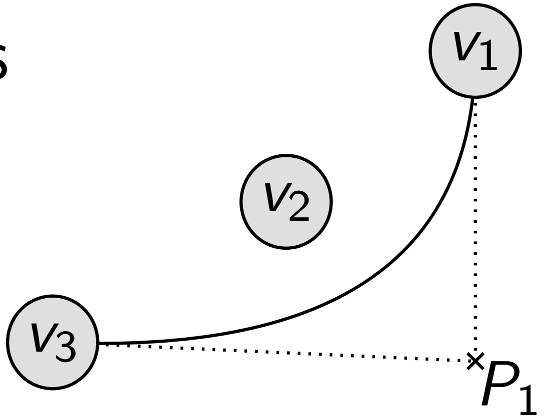
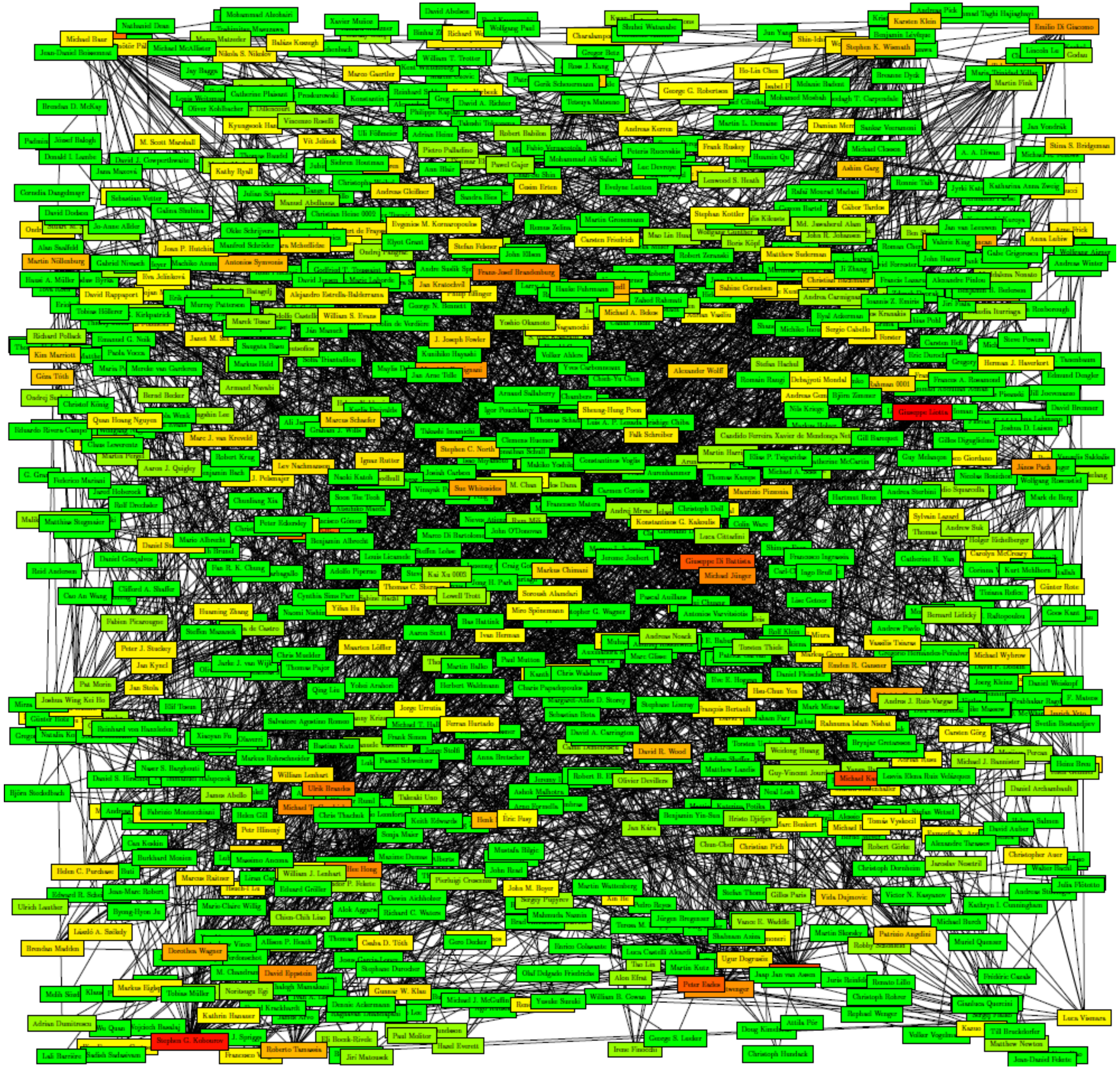


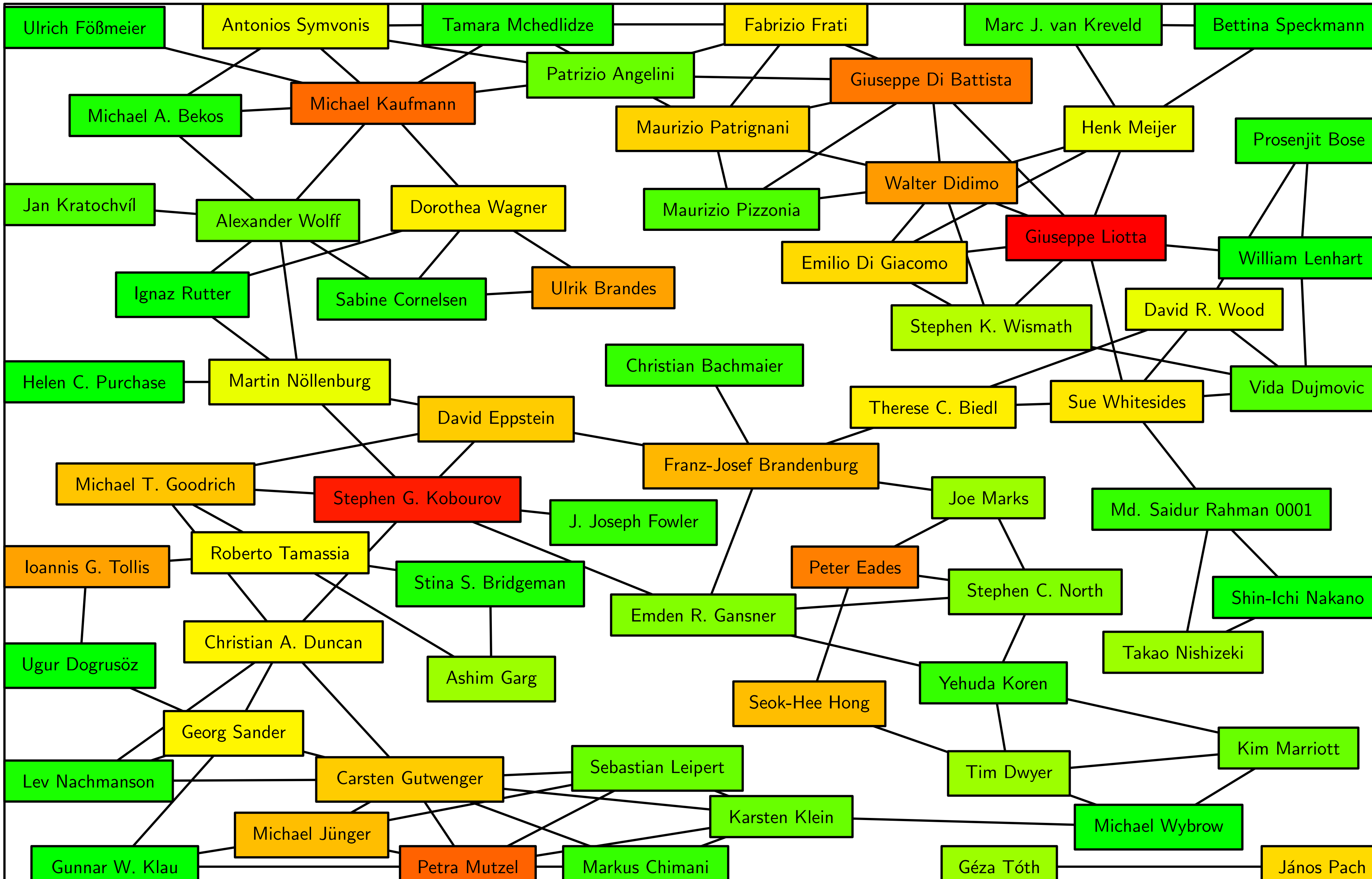
- draw some edges as quadratic Bézier curves
- force-directed computation of curve



Example – Input



Example – Output



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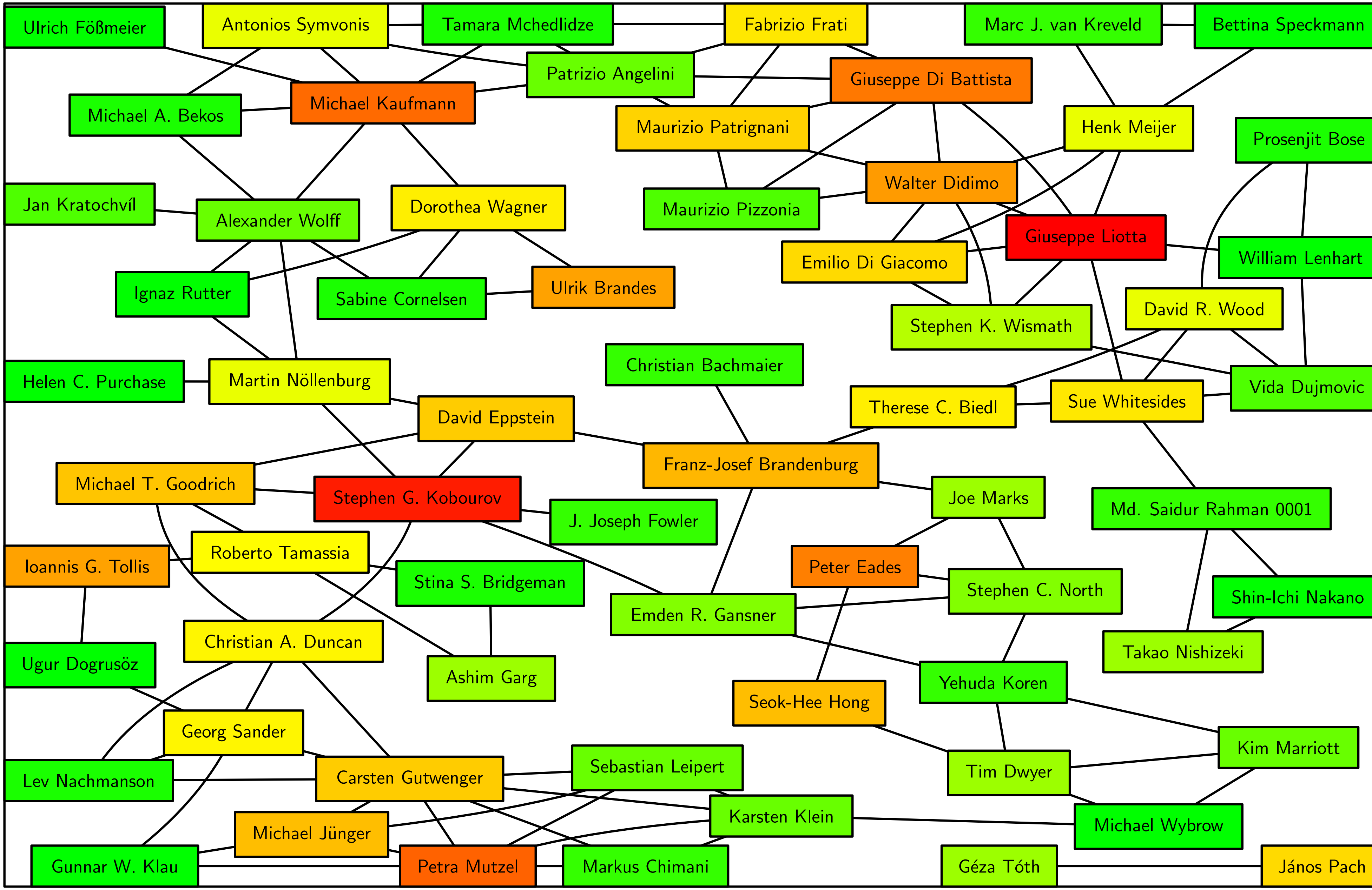
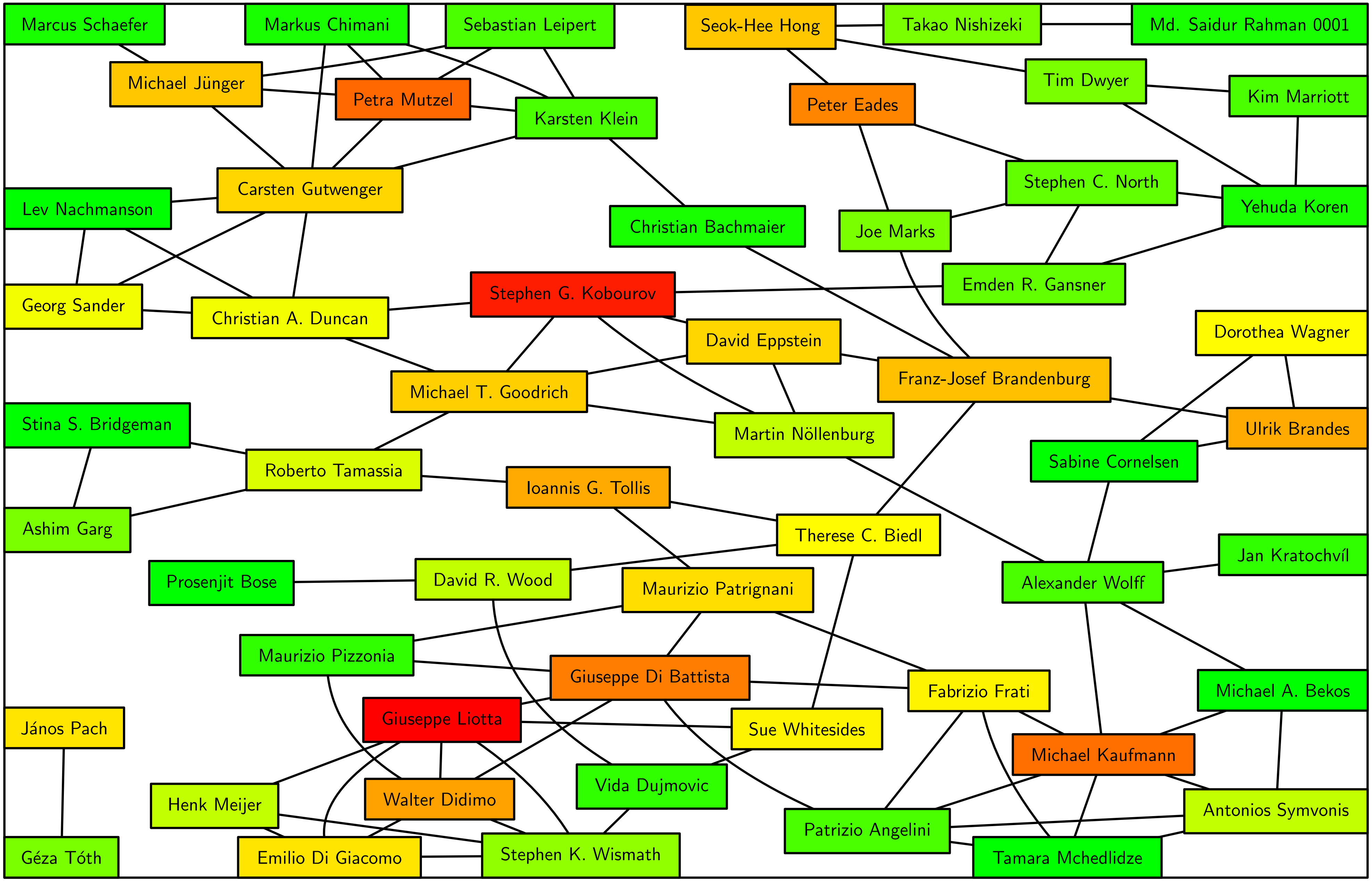


Example – Sparser Output



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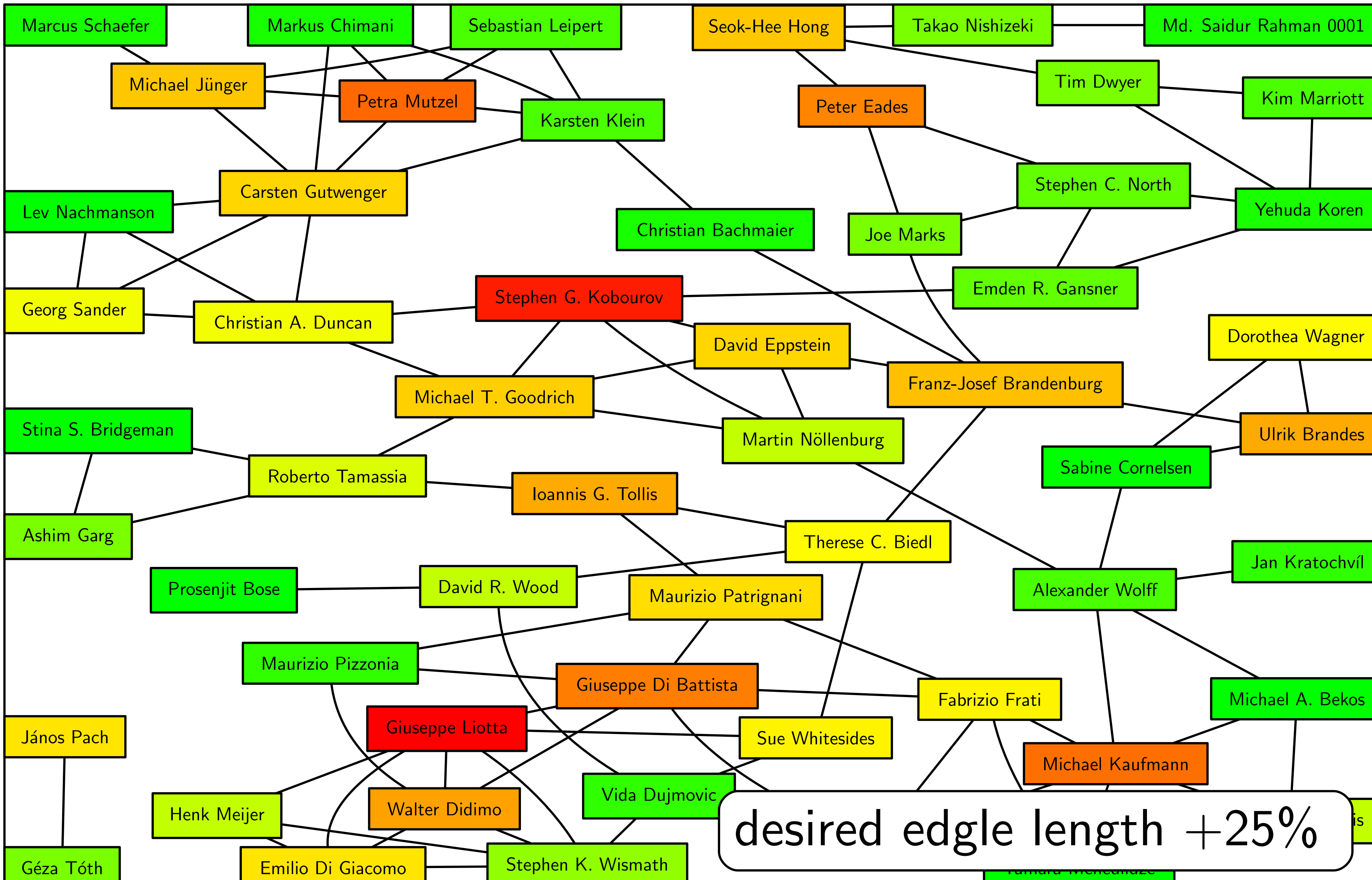

desired edgle length +25%

Remarks

- GD coauthor graph (1994 - 2012): 950, vertices 2559 edges $\rightarrow$ A4 paper, 10pt font:
  – runtime $\approx$ 2 minutes

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  – runtime $\approx$ 2 minutes
- preprocessing removing very light vertices yielded speedup $+$ heavier outputs
- activating edge-vertex repulsion only at the end of force-directed phase:
  weight of edges in output $+80\%$

Conclusion

- new problem: find + draw heavy subgraph within prescribed area

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- heuristic for calculation graphs
- force-directed approach for general graphs
- nice output drawings
- Problems:
  - – no alternative methods to compare with: real performance?
  - – speedup