INCREASING-CHORD GRAPHS ON POINT SETS

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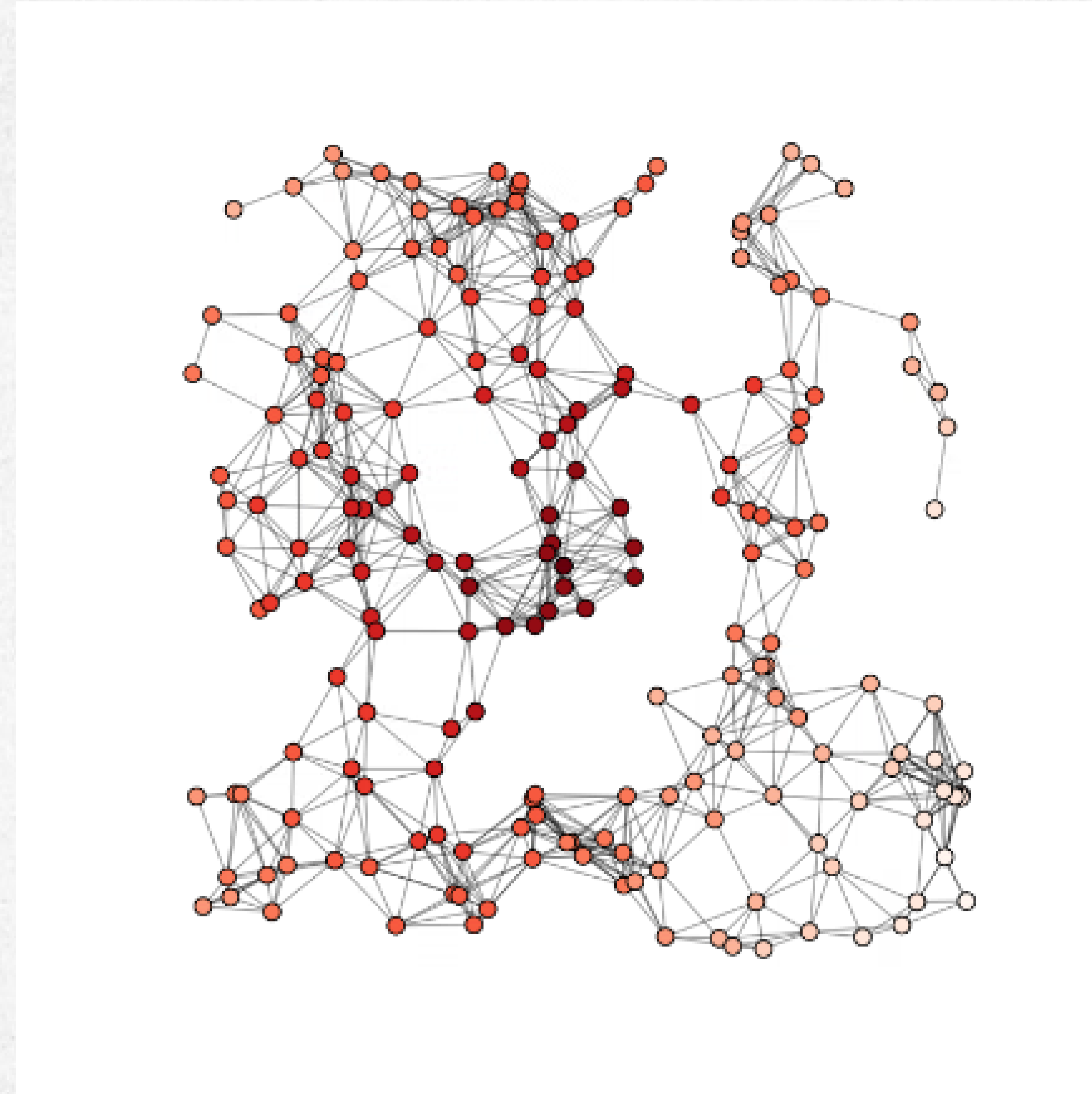
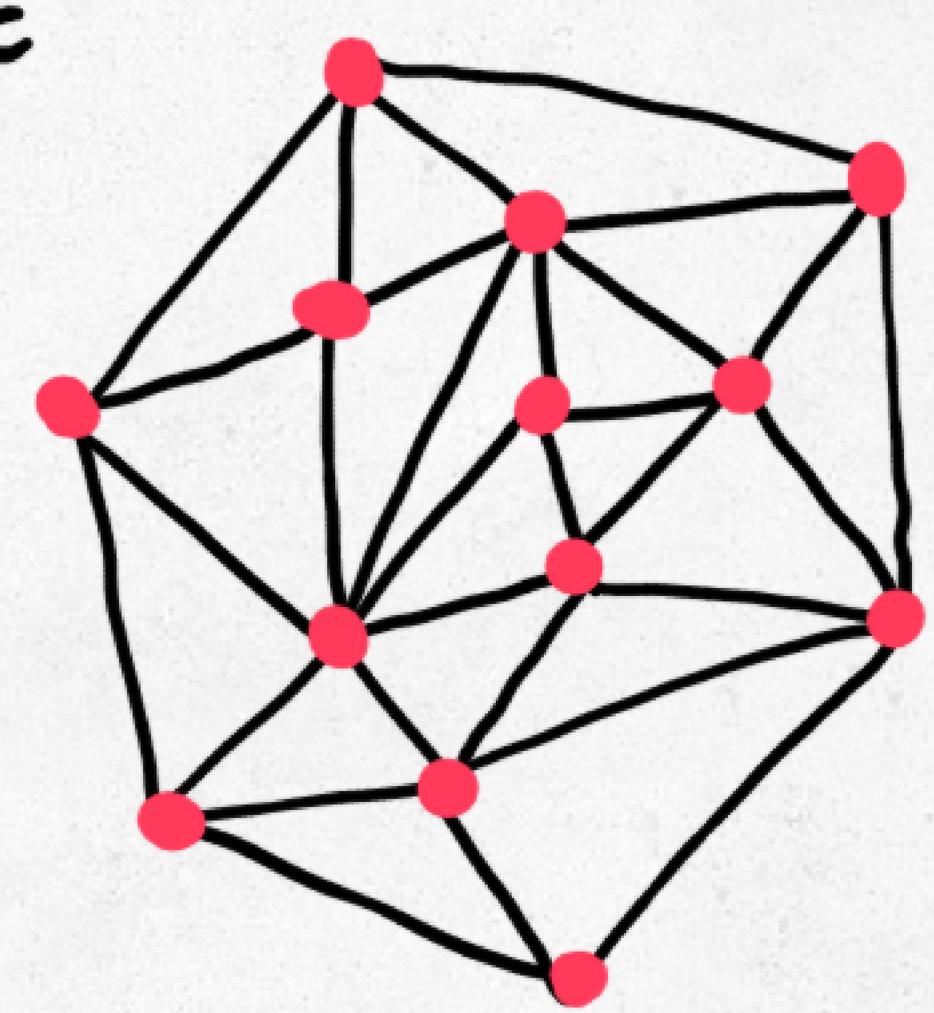
GEOMETRIC GRAPHS



VERTICES ARE POINTS IN THE PLANE

EDGES ARE STRAIGHT-LINE SEGMENTS

TRIANGULATION: PLANAR, INTERNAL FACES ARE DELIMITED BY TRIANGLES, THE OUTER FACE IS DELIMITED BY A CONVEX POLYGON



PROXIMITY GRAPHS....

A PROXIMITY GRAPH IS A GEOMETRIC GRAPH THAT CAN BE CONSTRUCTED FROM A POINT SET P BY CONNECTING POINTS THAT ARE "CLOSE" TO EACH OTHER [LIOTTA '13]

- GABRIEL GRAPHS
- DELAUNAY TRIANGULATIONS
- RELATIVE NEIGHBORHOOD GRAPHS
- RECTANGLE OF INFLUENCE GRAPHS
- NEAREST NEIGHBOR GRAPHS
- β-DRAWINGS
- EUCLIDEAN MSTs
- MINIMUM WEIGHT TRIANGULATIONS

... AND MORE

WE ARE INTERESTED IN GEOMETRIC GRAPHS SATISFYING SOME (LOCAL OR GLOBAL) GEOMETRIC PROPERTY

- MONOTONE GRAPHS
- GREEDY GRAPHS
- SELF-APPROACHING GRAPHS
- INCREASING-CHORD GRAPHS

COMPUTATIONAL GEOMETRY PERSPECTIVE

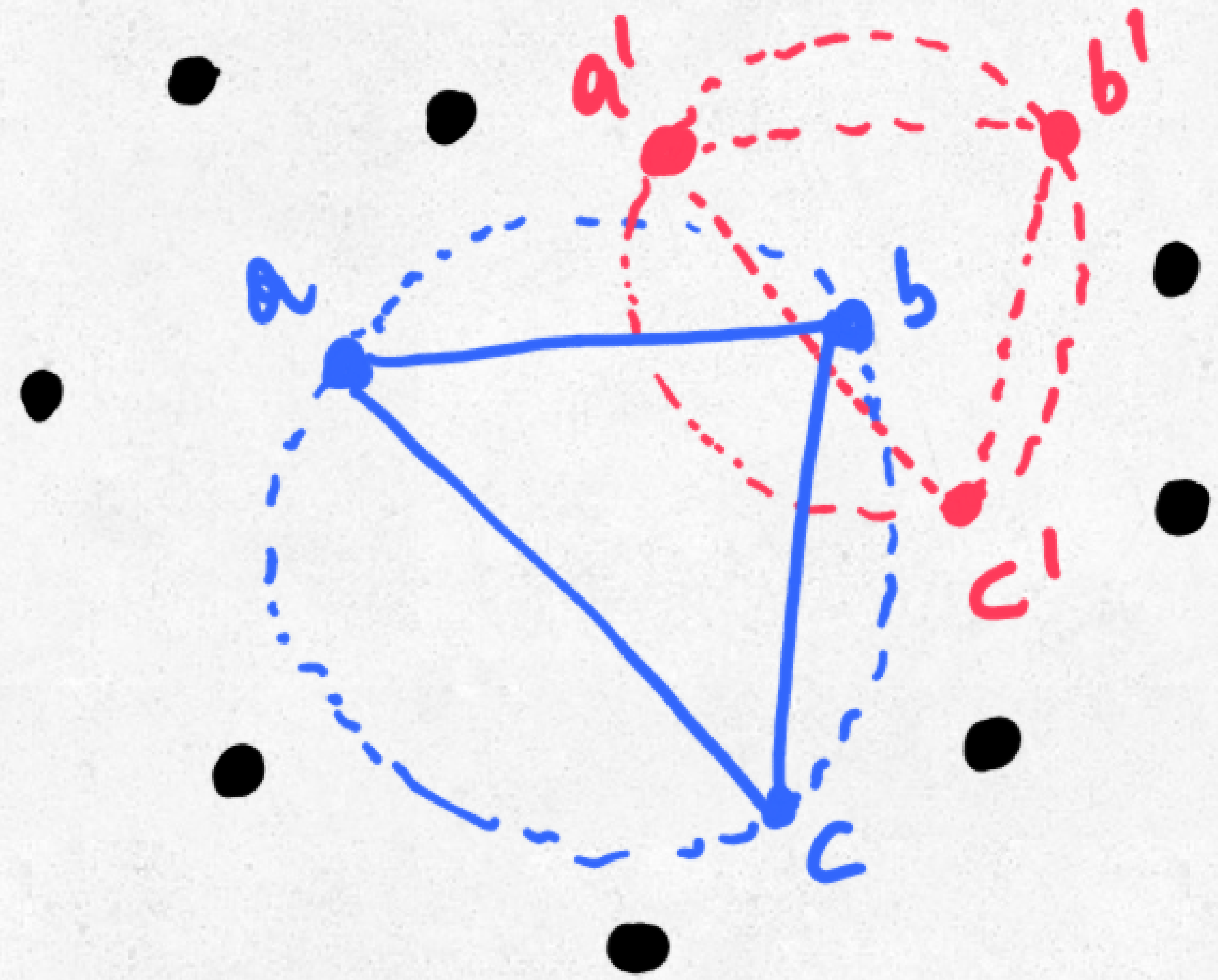
GIVEN A POINT SET, DOES A ( ) GRAPH ALWAYS EXIST?
COMPUTATIONAL COMPLEXITY?

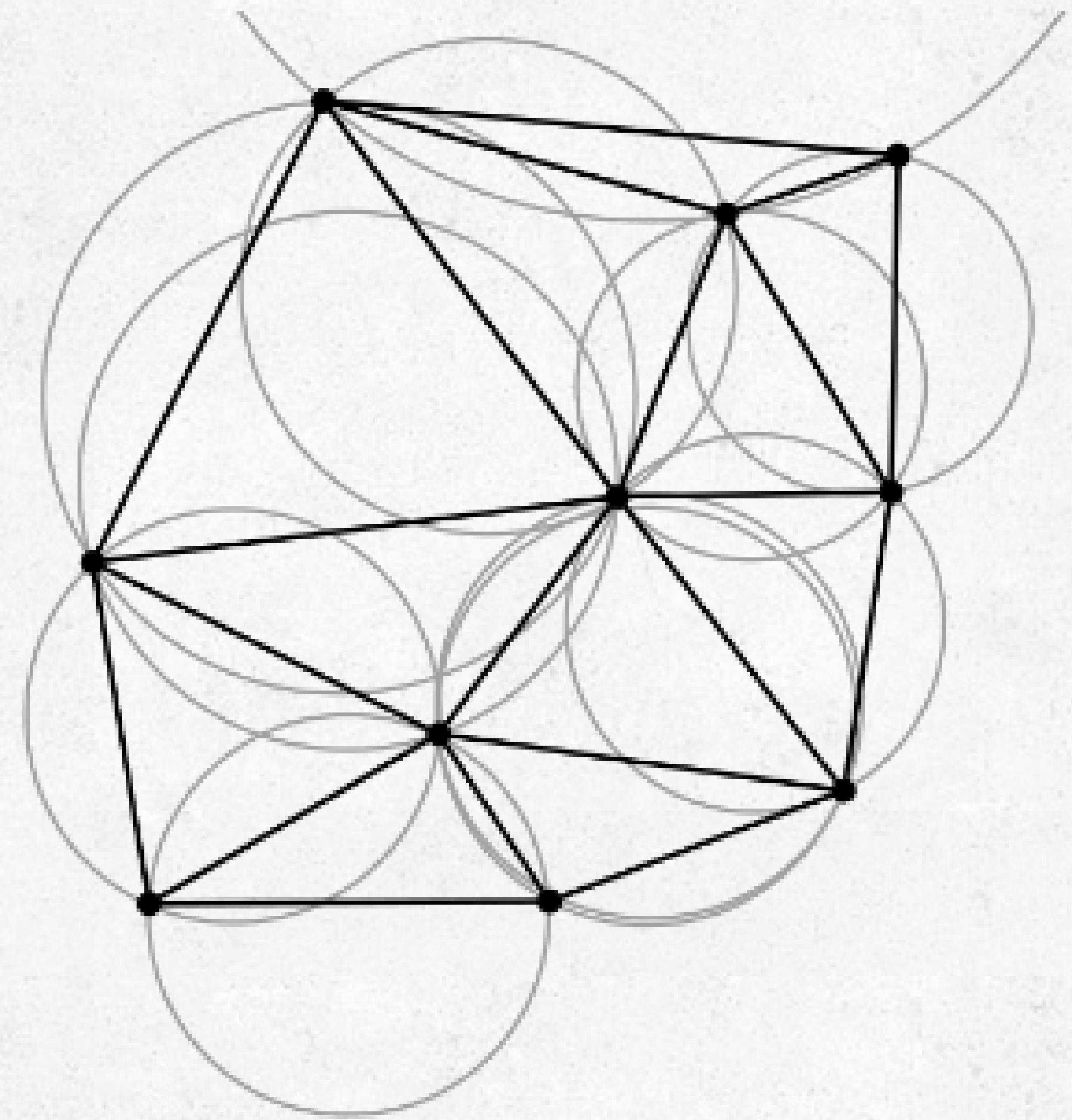
GRAPH DRAWING PERSPECTIVE

WHICH GRAPHS ADMIT A GEOMETRIC REPRESENTATION AS ( ) GRAPHS?
COMPUTATIONAL COMPLEXITY?

DELAUNAY TRIANGULATIONS

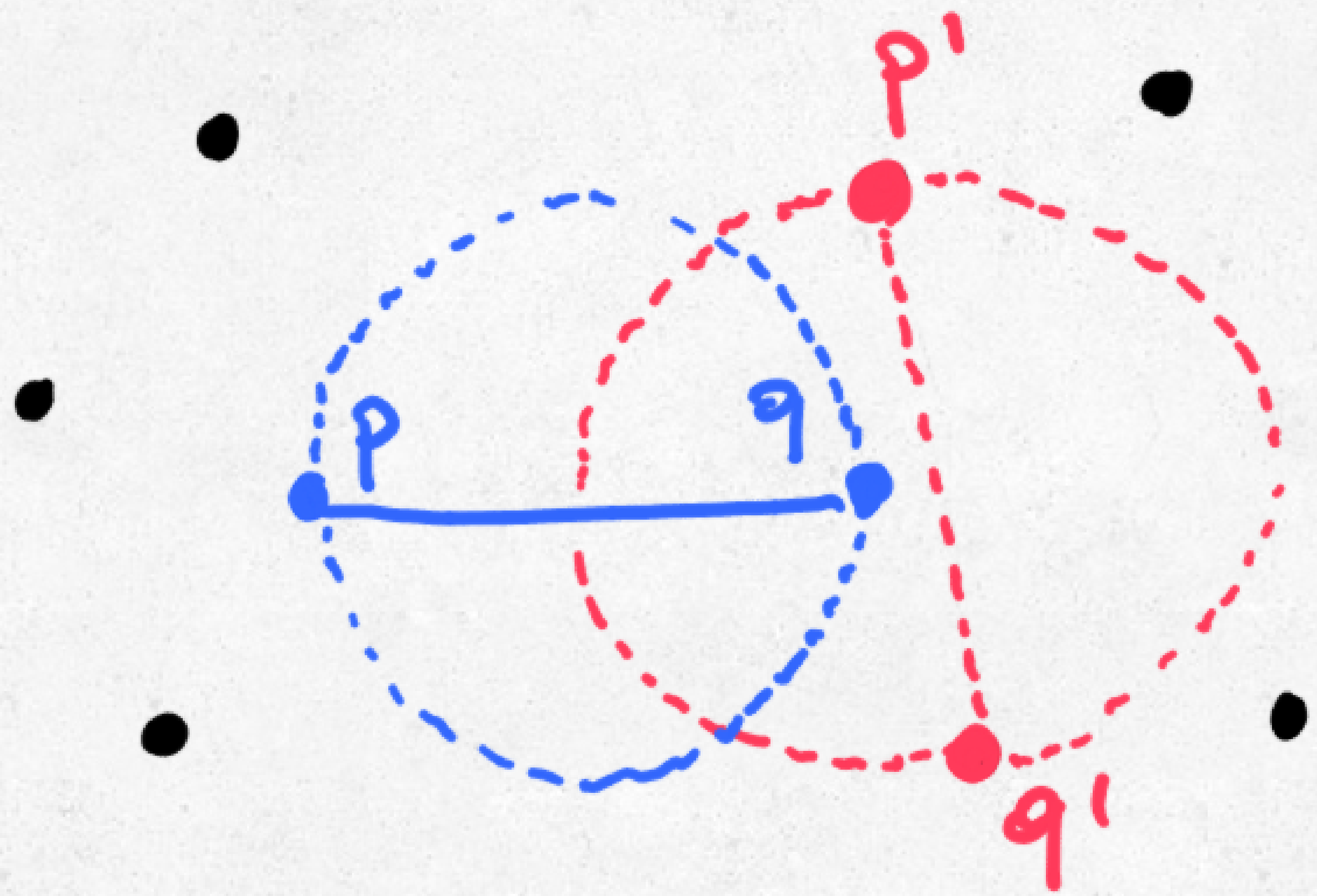
GIVEN A POINT SET P, THE DELAUNAY TRIANGULATION D OF P IS A TRIANGULATION SUCH THAT NO POINT IS INSIDE THE CIRCUMCIRCLE OF ANY TRIANGLE

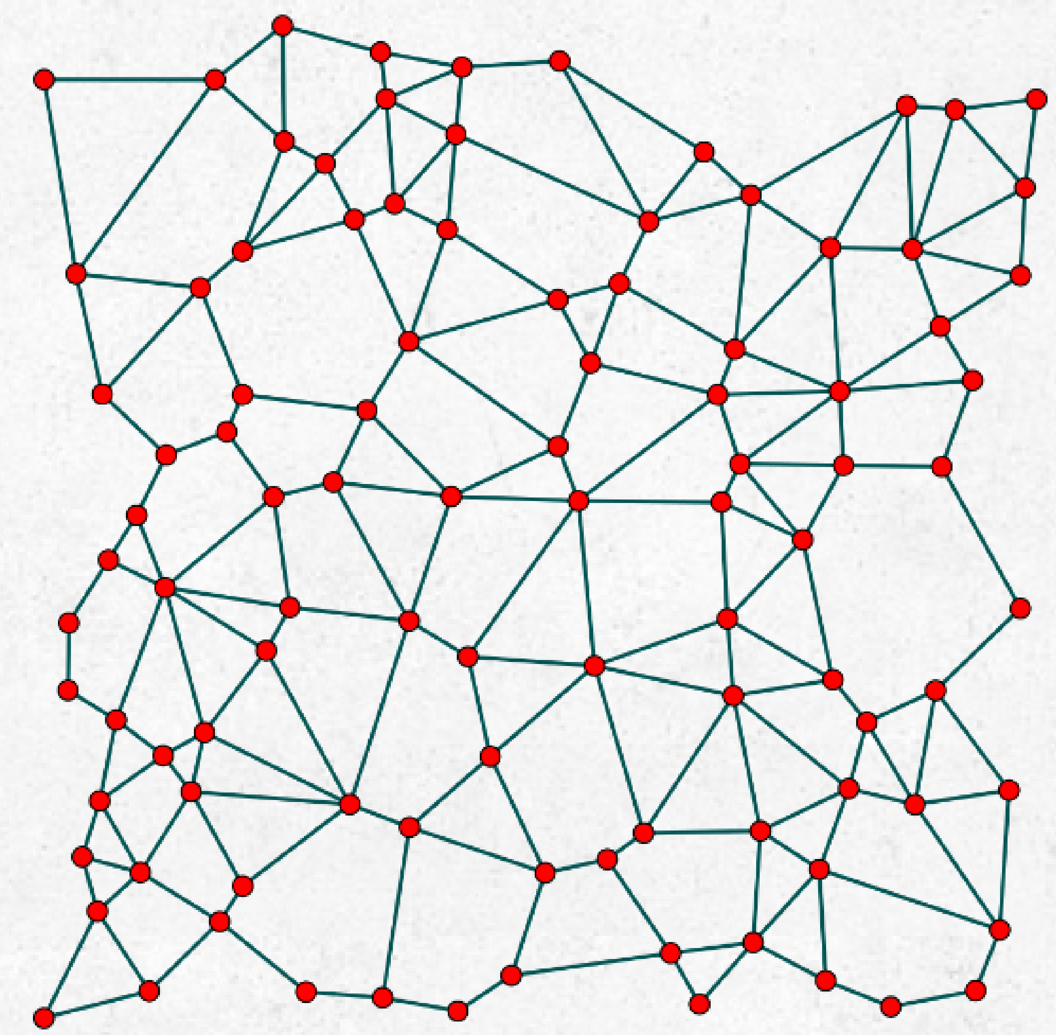




GABRIEL GRAPHS

GIVEN A POINT SET P, THE GABRIEL GRAPH G ON P HAS AN EDGE (p, q) IF AND ONLY IF THE CIRCLE WITH DIAMETER $\overline{pq}$ HAS NO POINT OF $P / \{p, q\}$ IN ITS INTERIOR OR ON ITS BOUNDARY.





THE GABRIEL GRAPH OF A POINT SET P IS A SUBGRAPH OF THE DELAUNAY TRIANGULATION D OF P AND IT CAN BE COMPUTED IN O(|P|) TIME FROM D [MATULA-SOKAL '80]

THE GABRIEL GRAPH OF ANY POINT SET IS PLANAR.

THE GABRIEL GRAPH OF A POINT SET P IS NOT ALWAYS A TRIANGULATION. IF IT IS, THEN IT IS CALLED GABRIEL TRIANGULATION AND WE SAY THAT P ADMITS A GABRIEL TRIANGULATION.

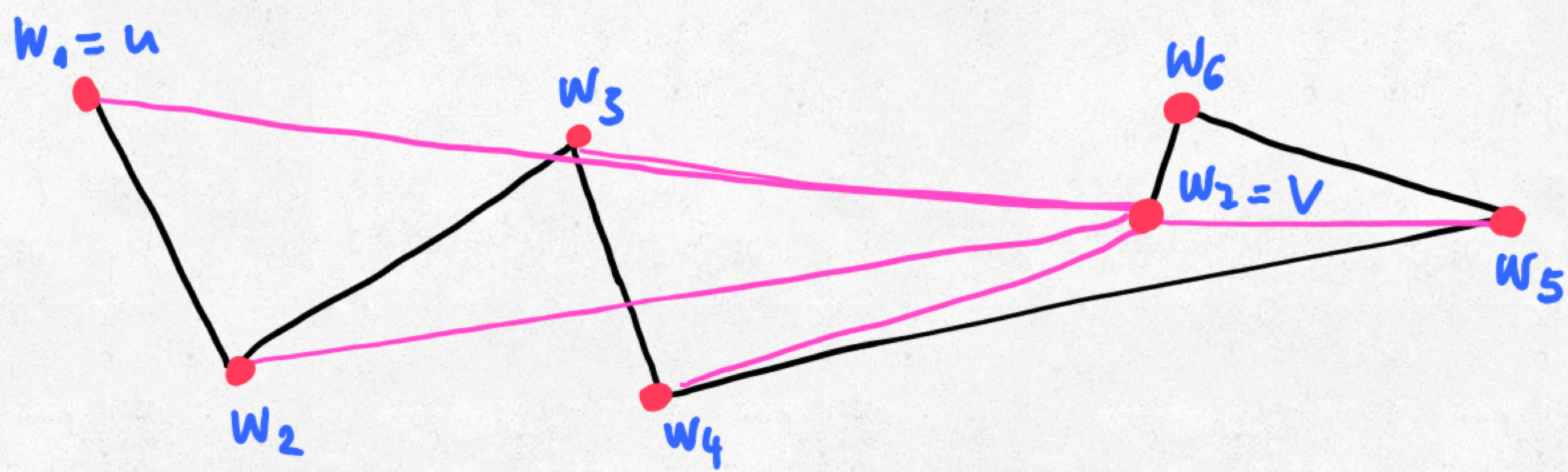
A TRIANGULATION IS A GABRIEL TRIANGULATION IF AND ONLY IF EVERY INTERIOR ANGLE IS ACUTE.

GREEDY GRAPHS $G(V,E)$

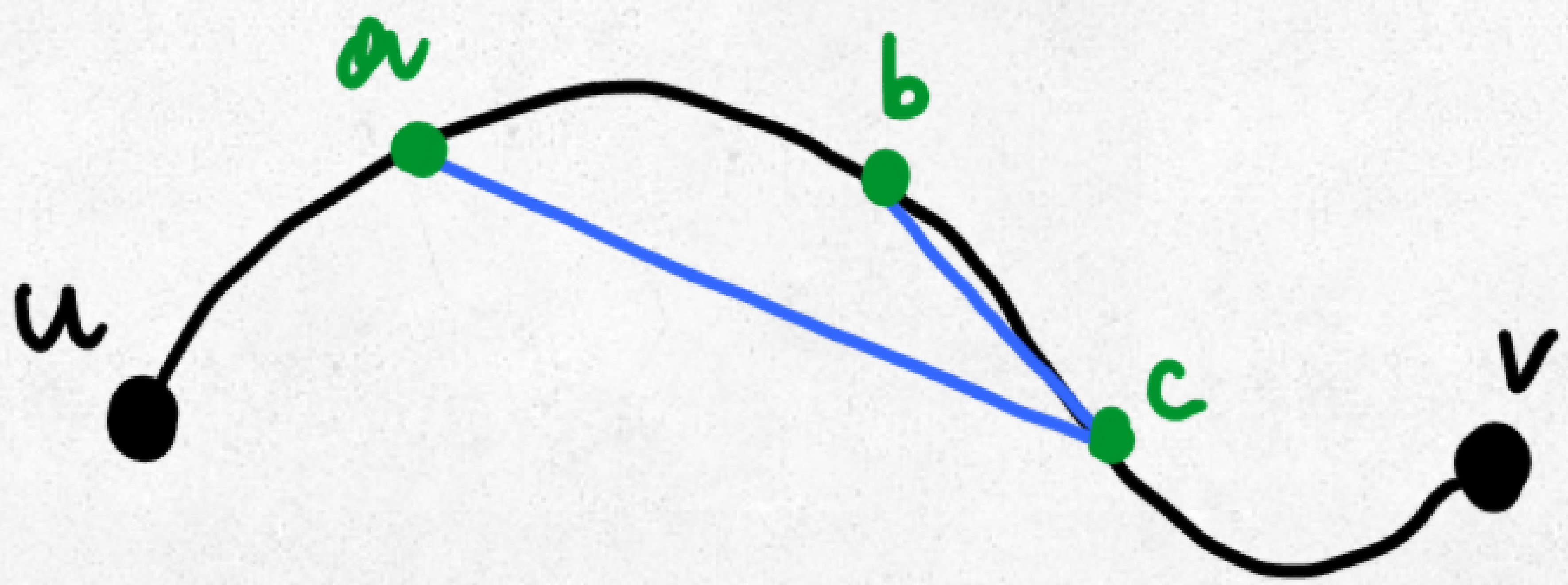
FOR EVERY PAIR OF VERTICES $u, v \in V$, $\exists$ PATH $(w_1 = u, w_2, \ldots, w_k = v)$ SUCH THAT

$$\text{DISTANCE}(w_i, w_k) \geq \text{DISTANCE}(w_{i+1}, w_k)$$

FOR EVERY $1 \leq i \leq k-2$



SELF-APPROACHING CURVE C FROM u TO v



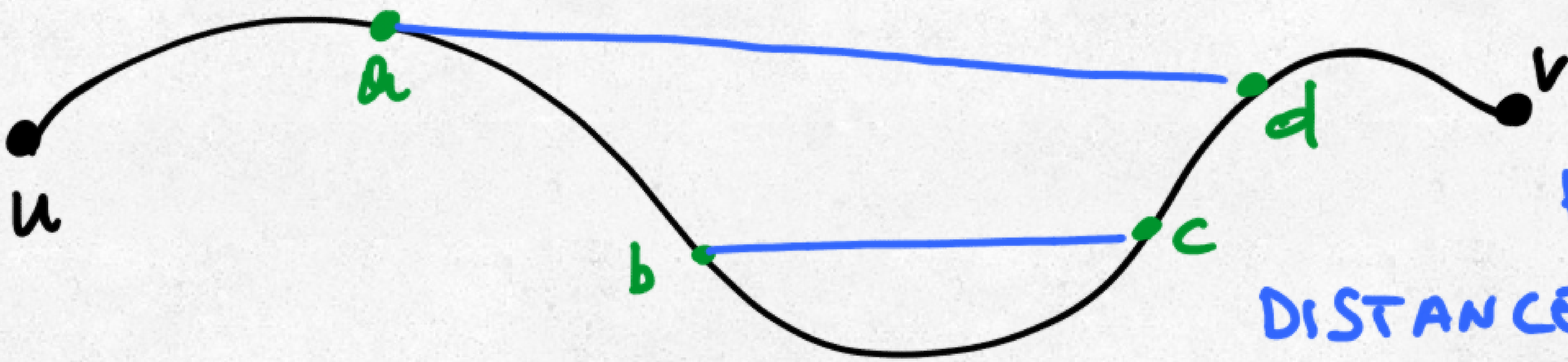
$\forall$ a, b, c ON C:

DISTANCE $(a, c) \geq$ DISTANCE (b, c)

INCREASING-CHORD CURVE BETWEEN u AND v

SELF-APPROACHING IN BOTH DIRECTIONS!

EQUIVALENTLY



$\forall$ a, b, c, d ON C:

DISTANCE $(a, d) \geq$ DISTANCE (b, c)

SELF-APPROACHING GRAPH $G(V,E)$

$\forall u,v \in V$: $\exists$ PATH IN G FROM u TO v THAT IS SELF-APPROACHING FROM u TO v.

INCREASING-CHORD GRAPH $G(V,E)$

$\forall u,v \in V$: $\exists$ PATH IN G BETWEEN u AND v THAT IS INCREASING-CHORD BETWEEN u AND v

WHY

SMALL GEOMETRIC DILATION ~5.3 SELF-APPROACHING GRAPHS
~2.1 INCREASING-CHORD GRAPHS

STRONGLY RELATED TO GREEDY GRAPHS

INCREASING-CHORD $\Rightarrow$ SELF-APPROACHING $\Rightarrow$ GREEDY

THEY SEEM TO BE INTERESTING THEORETICAL OBJECTS.

WHAT ARE THE QUESTIONS

IS IT TRUE THAT, FOR EVERY POINT SET P, THERE EXISTS AN INCREASING-CHORD (OR SELF-APPROACHING) PLANAR GRAPH G ON P?

RELAXATIONS: ALLOW STEINER POINTS;
ALLOW CROSSINGS AND GUARANTEE THAT G HAS FEW EDGES.

WHAT'S THE COMPLEXITY OF RECOGNIZING INCREASING-CHORD OR SELF-APPROACHING GRAPHS?

CHARACTERIZE (CLASSES OF) INCREASING-CHORD AND SELF-APPROACHING GRAPHS.

WHAT IS KNOWN (1)

INTRODUCED BY ALAMDARI, CHAN, GRANT, LUBIW, PATHAK [GD '12]

LINEAR-TIME ALGORITHM TO RECOGNIZE SELF-APPROACHING PATHS IN $\mathbb{R}^2$

ALMOST-LINEAR-TIME ALGORITHM TO RECOGNIZE SELF-APPROACHING PATHS IN $\mathbb{R}^3$

CHARACTERIZATION OF THE TREES THAT CAN BE REPRESENTED AS SELF-APPROACHING GRAPHS

MANY MORE GRAPH DRAWING RESULTS IN THE NEXT TALK!

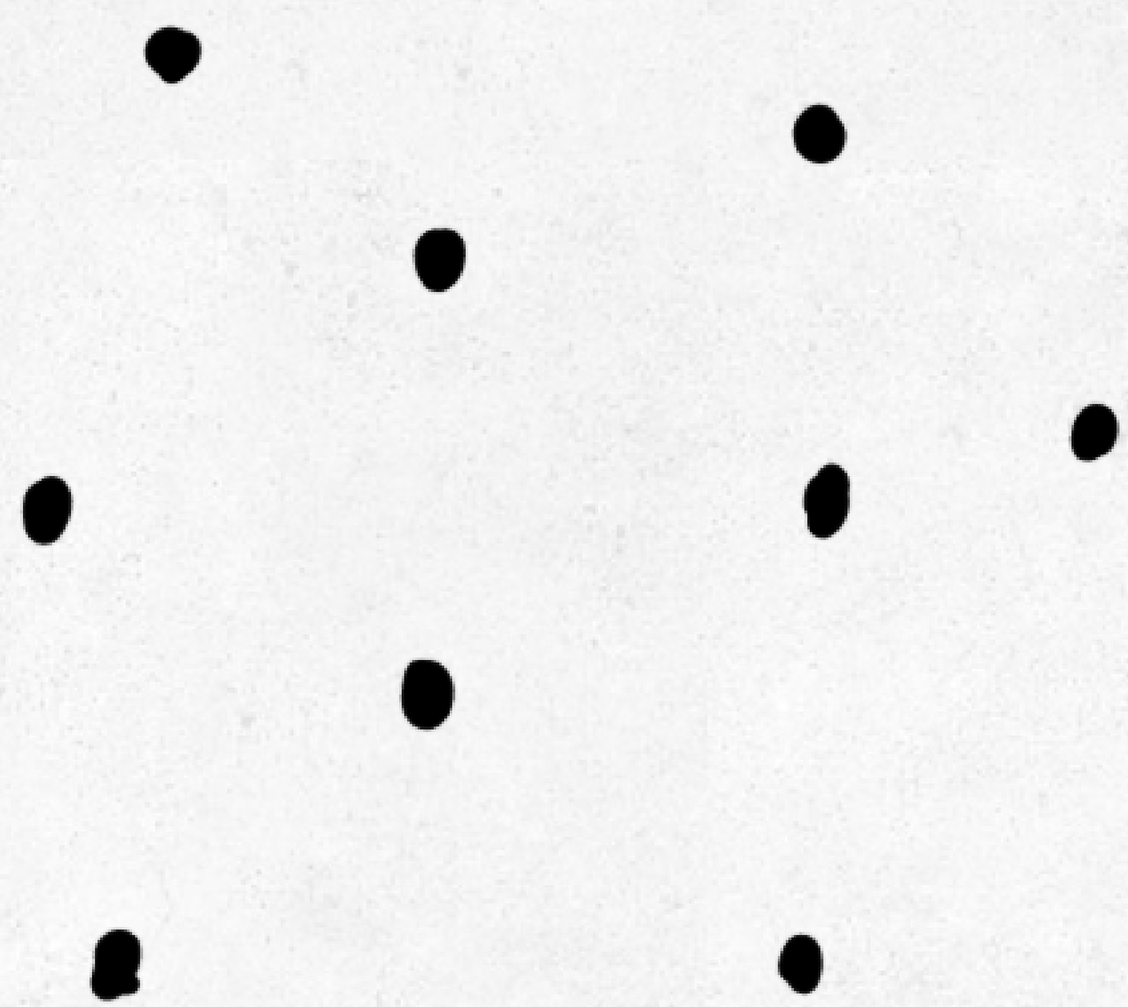
WHAT IS KNOWN (2)

IS IT TRUE THAT, FOR EVERY POINT SET P, THERE EXISTS AN INCREASING-CHORD (OR SELF-APPROACHING) PLANAR GRAPH G ON P?

THERE EXIST POINT SETS P SUCH THAT THE DELAUNAY TRIANGULATION OF P IS NOT A SELF-APPROACHING GRAPH.

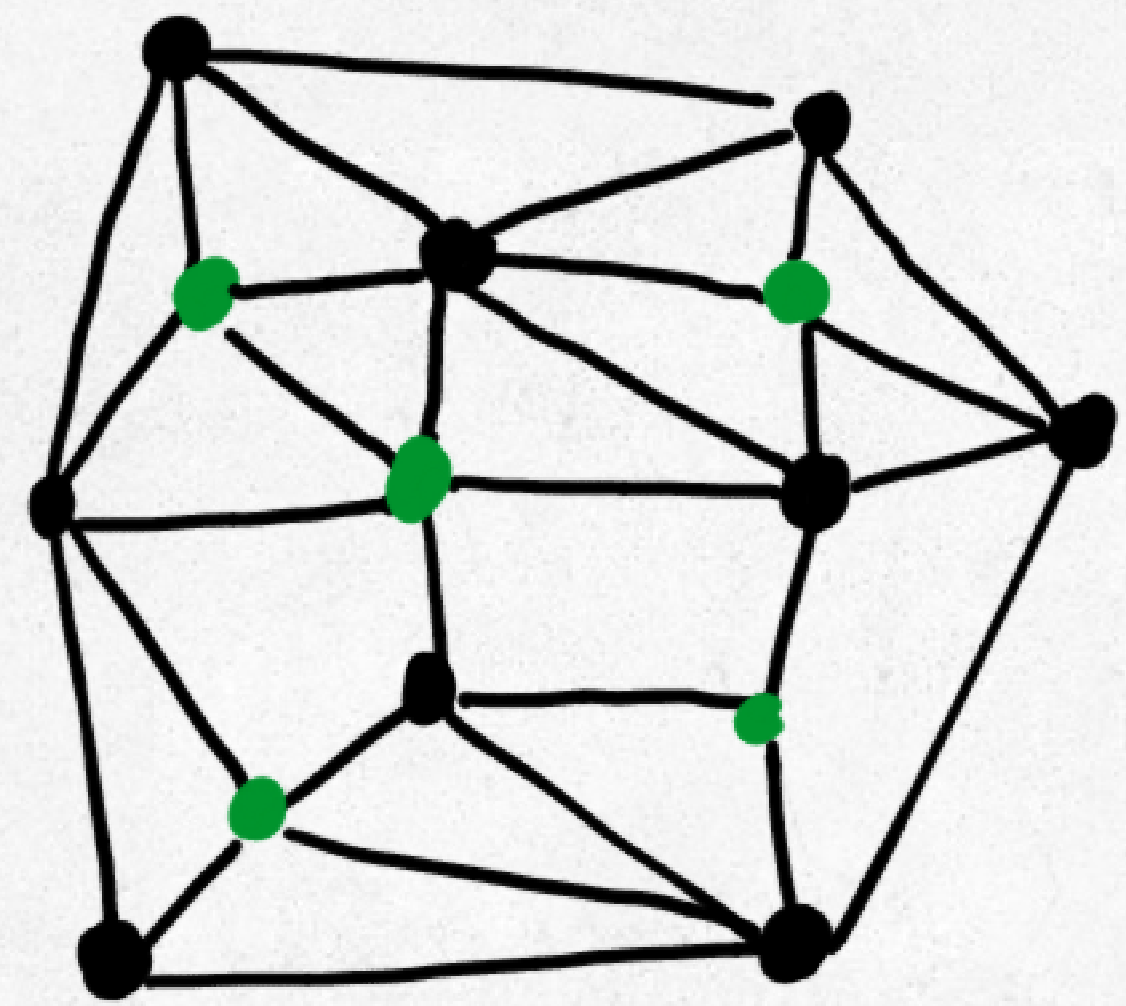
FOR EVERY POINT SET P, THERE EXISTS A GEOMETRIC GRAPH $G(P', E)$ SUCH THAT $P \subseteq P'$, $|P'| \in O(|P|)$, AND G CONTAINS AN INCREASING-CHORD PATH BETWEEN EVERY TWO POINTS IN P.

THEOREM 1 FOR EVERY POINT SET P, THERE EXISTS AN INCREASING-CHORD PLANAR GRAPH G SPANNING P WITH $O(|P|)$ VERTICES



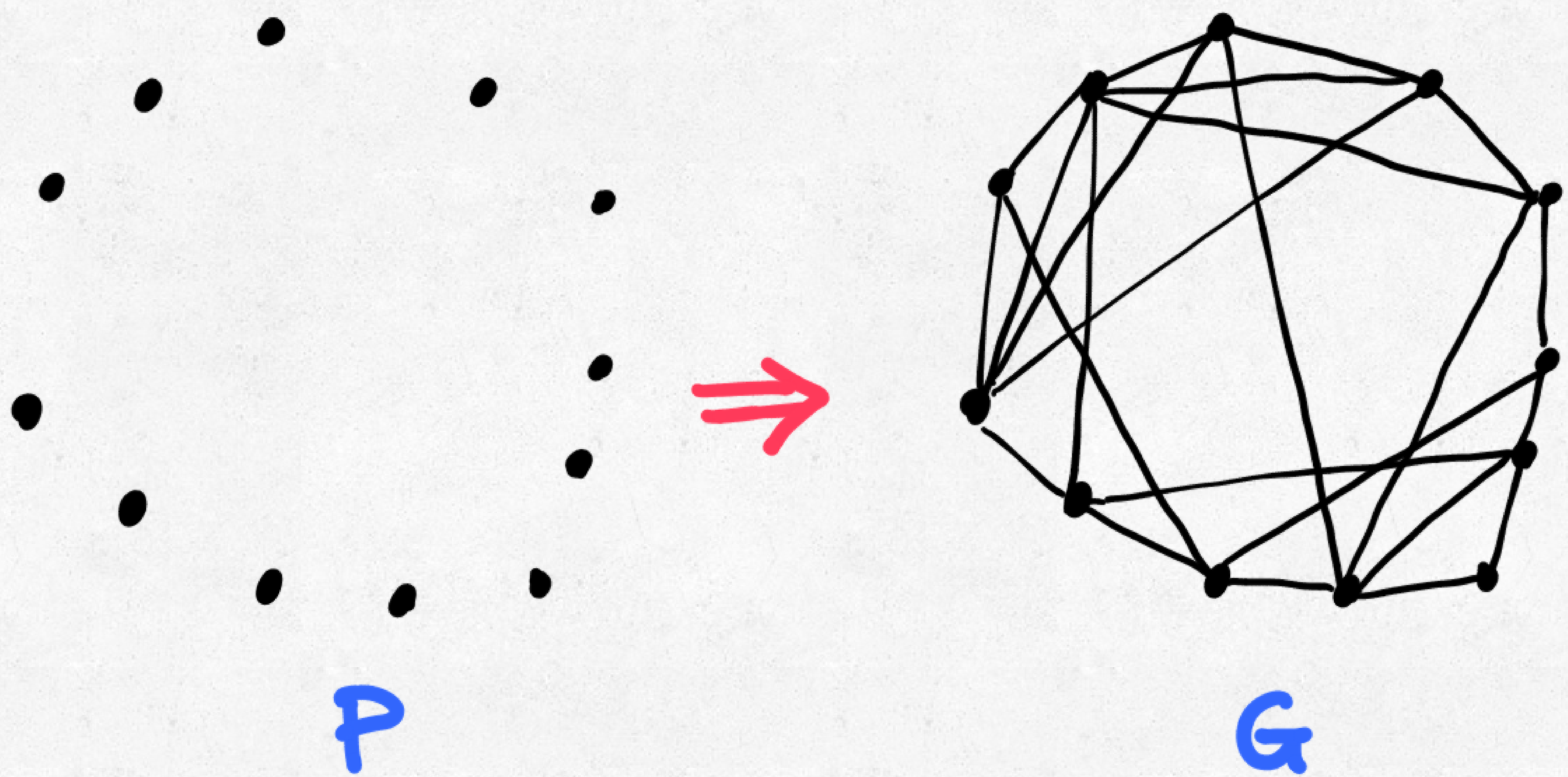
P

$\Rightarrow$



G

THEOREM 2 FOR EVERY CONVEX POINT SET P, THERE EXISTS AN INCREASING-CHORD GRAPH G SPANNING P WITH $O(|P| \log |P|)$ EDGES (AND NO STEINER POINTS)



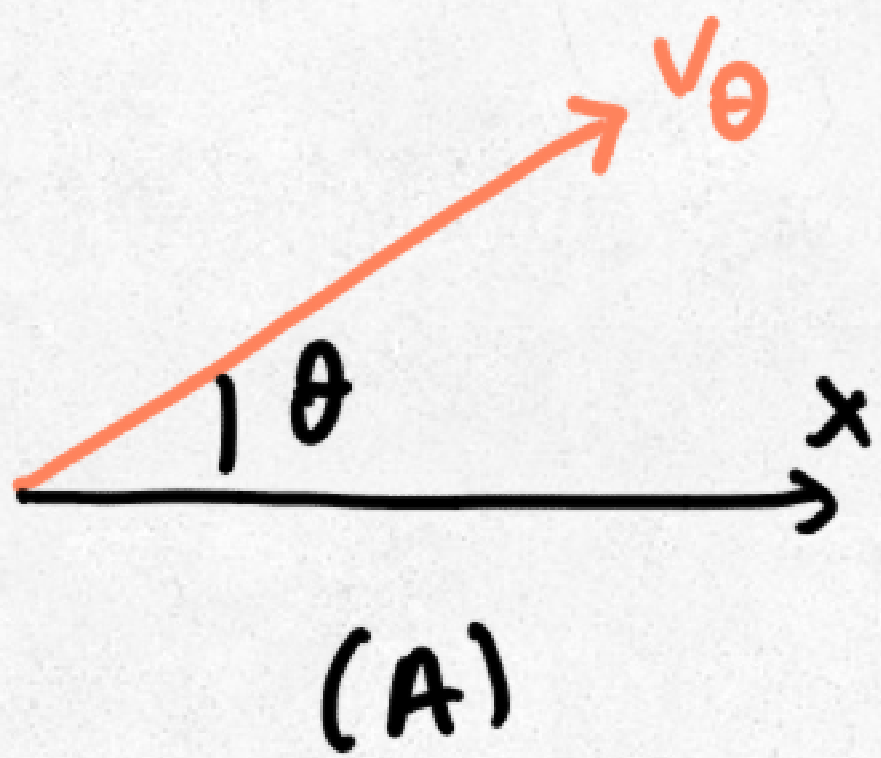
THEOREM 1 FOR EVERY POINT SET P THERE EXISTS AN INCREASING-CHORD PLANAR GRAPH G SPANNING P WITH $O(|P|)$ VERTICES

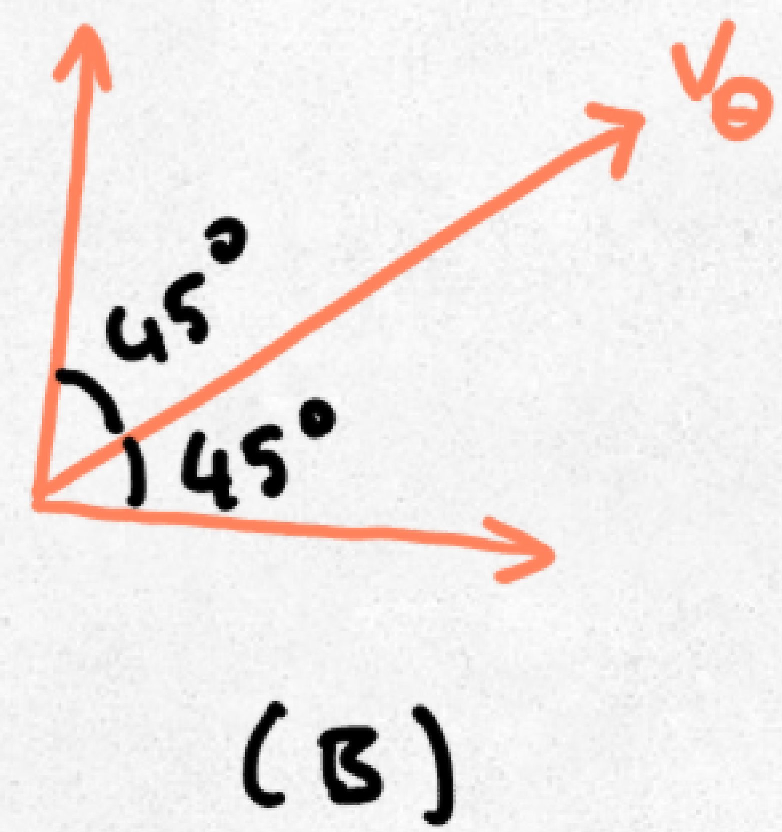
LEMMA 1 GABRIEL TRIANGULATIONS ARE INCREASING-CHORD PLANAR GRAPHS

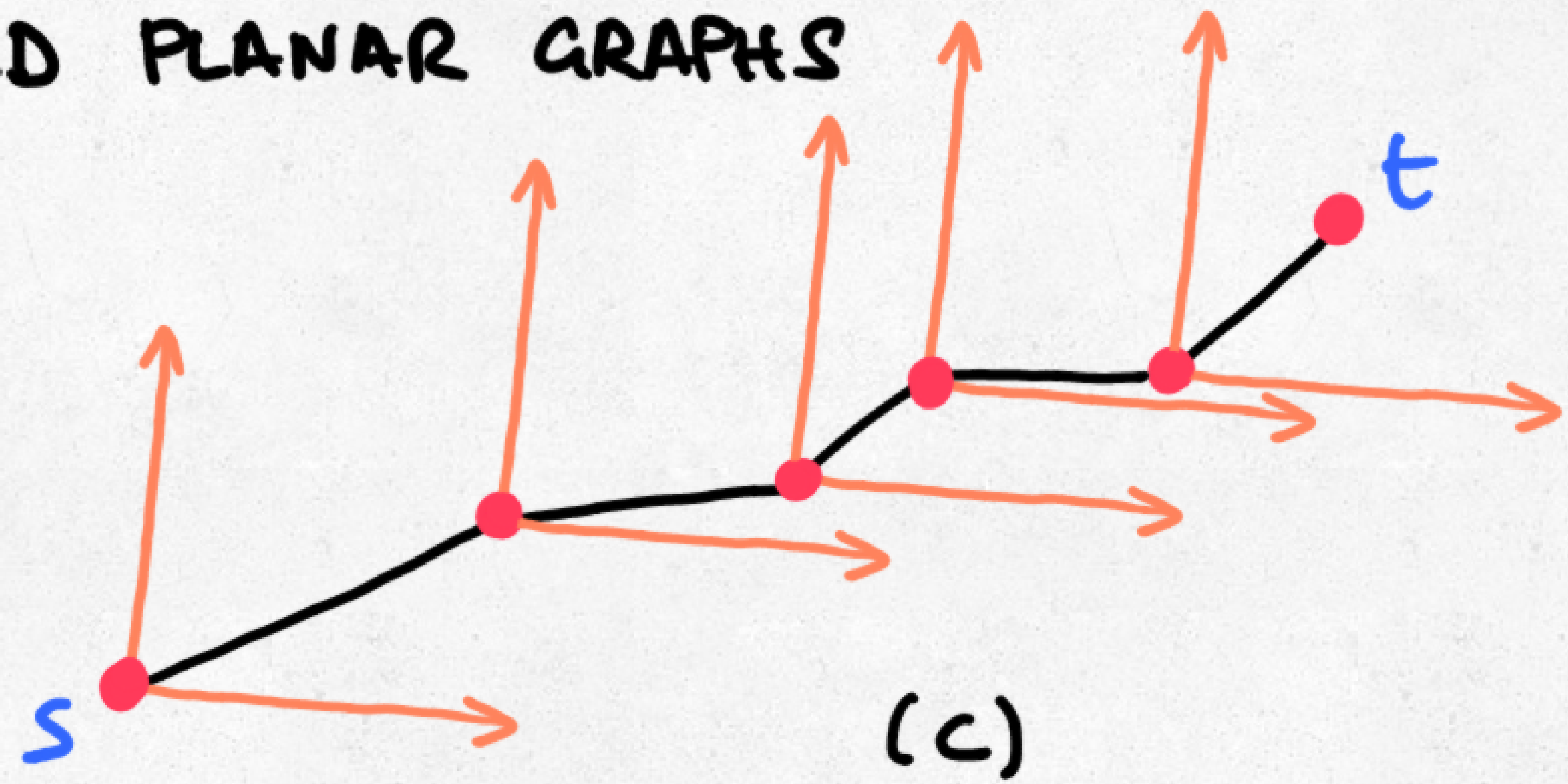
LEMMA 2 [BERN, EPPSTEIN, GILBERT '94]
FOR EVERY POINT SET P, THERE EXISTS A POINT SET P', WITH $|P'| \in O(|P|)$, SUCH THAT P' ADMITS A GABRIEL TRIANGULATION.

LEMMA 1 GABRIEL TRIANGULATIONS ARE INCREASING-CHORD PLANAR GRAPHS

θ-PATH





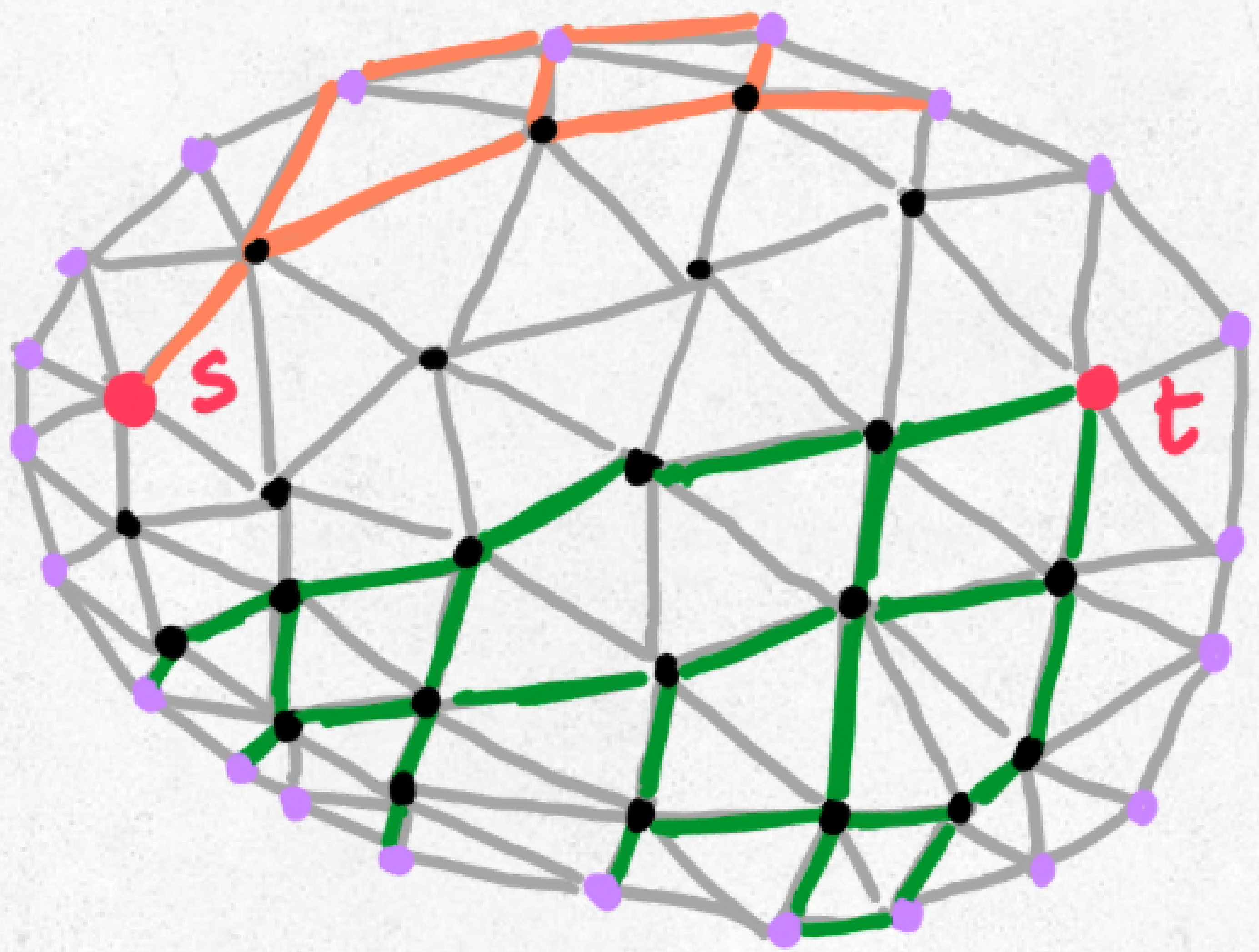


LEMMA 3 [ICKING, KLEIN, LANGETEPE '99]
θ-PATHS ARE INCREASING-CHORD PATHS.

LEMMA 4 LET $G(P,E)$ BE A GABRIEL TRIANGULATION. FOR EVERY TWO POINTS $s,t \in P$, $\exists\, \theta$ SUCH THAT G CONTAINS A θ-PATH FROM s TO t.

LEMMA 4 LET $G(P,E)$ BE A GABRIEL TRIANGULATION.
$\forall s,t \in P$, $\exists\, \theta$ SUCH THAT G CONTAINS A θ-PATH FROM s TO t.

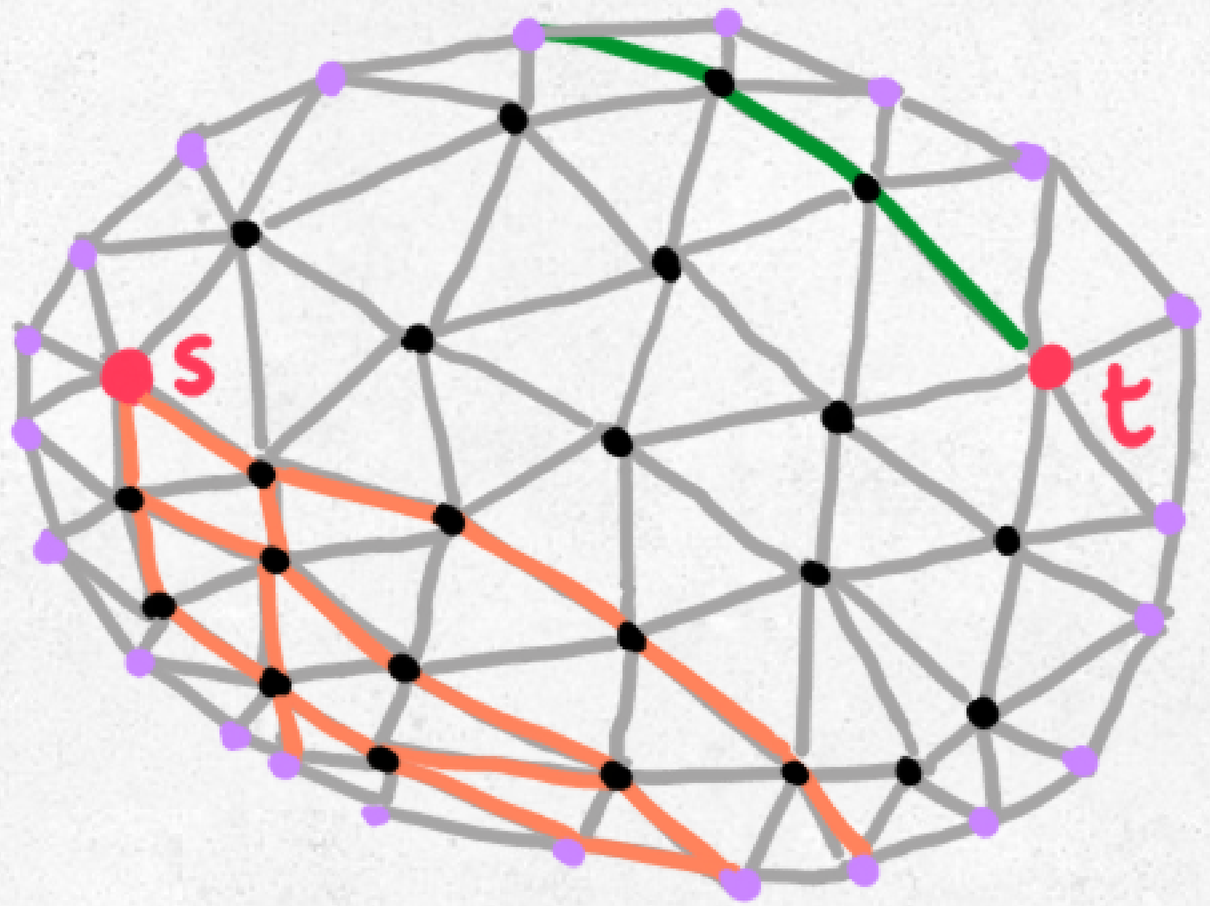
$\theta = 45°$



ALL θ-PATHS FROM s ARE "HIGH"

ALL $(\theta+180°)$- PATHS FROM t ARE "LOW"

$\theta = -45^\circ$

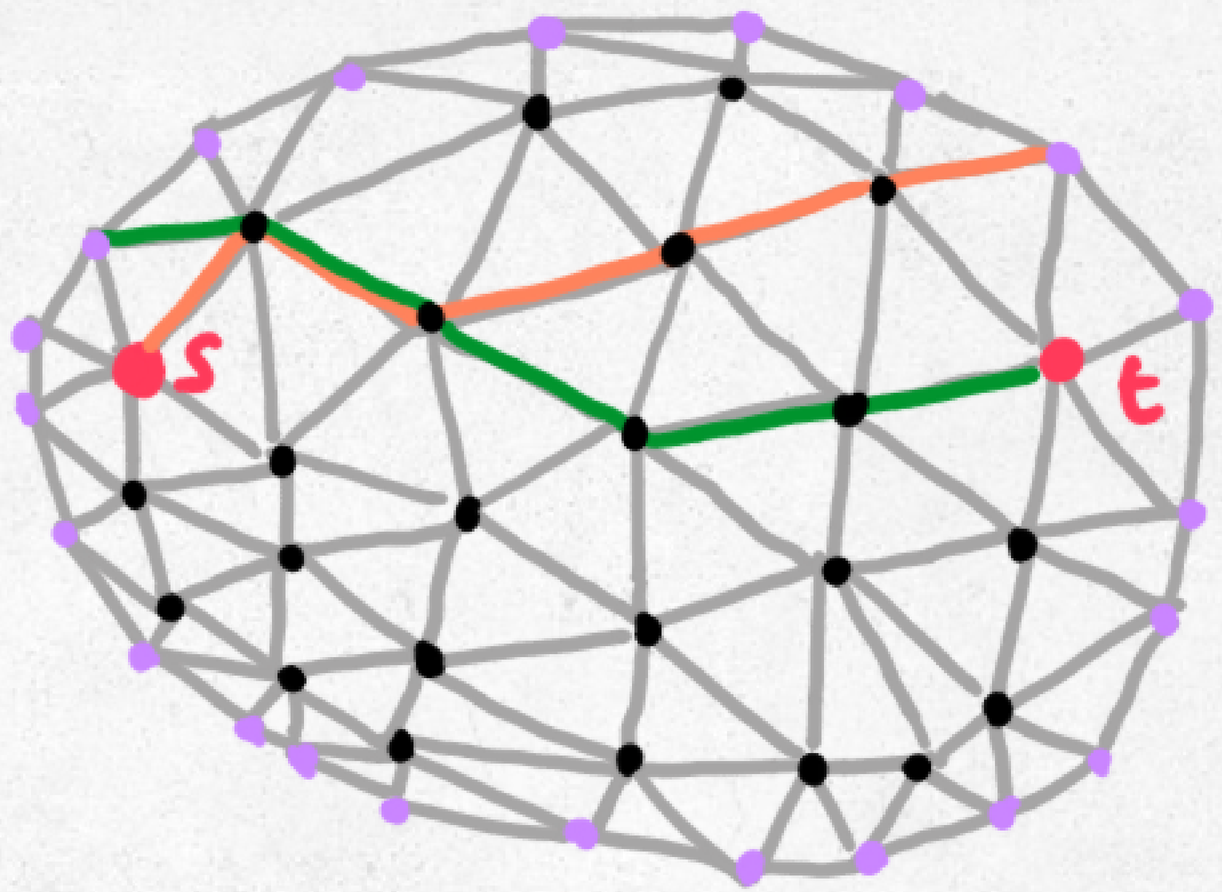


ALL θ-PATHS FROM s ARE "LOW"

ALL $(\theta + 180^\circ)$-PATHS FROM t ARE "HIGH".

SINCE, AS YOU DECREASE θ FROM $+45°$ TO $-45°$, THE θ-PATHS FROM s TURN FROM BEING ALL HIGH TO BEING ALL LOW, AND

SINCE, AS YOU DECREASE θ FROM $+45°$ TO $-45°$, THE $(\theta+180°)$-PATHS FROM t TURN FROM BEING ALL LOW TO BEING ALL HIGH



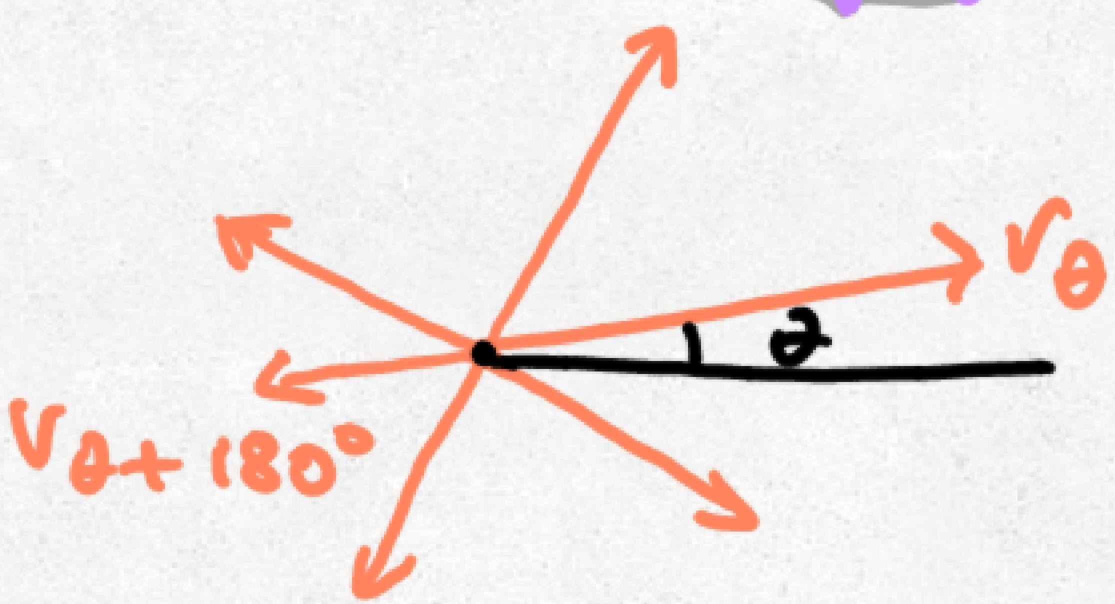
BY CONTINUITY AND SINCE ALL ANGLES ARE ACUTE THERE EXISTS A VALUE OF θ SUCH THAT:

THERE EXIST A HIGH θ-PATH FROM s AND A HIGH $(\theta+180°)$-PATH FROM t OR

THERE EXIST A LOW θ-PATH FROM s AND A LOW θ-PATH FROM t

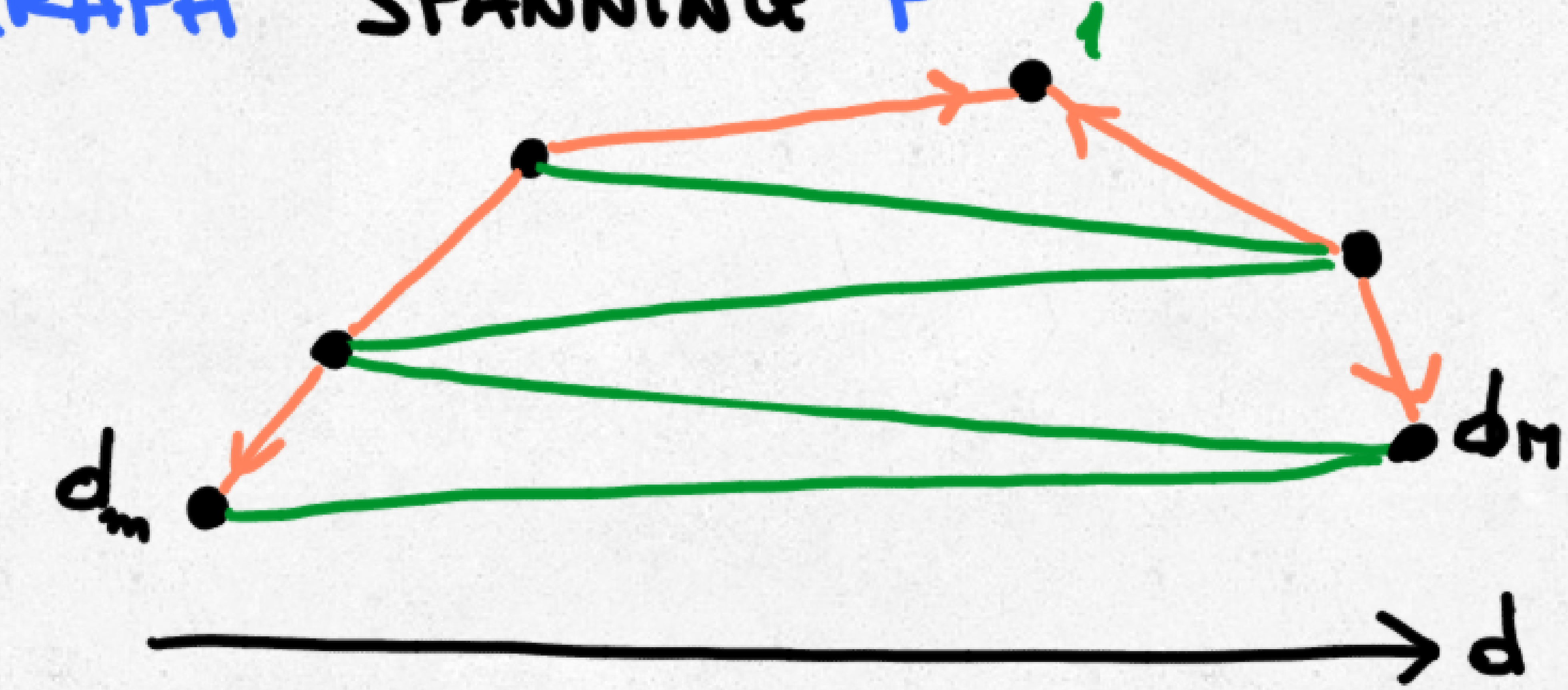
$\Downarrow$

THESE TWO PATHS TOGETHER PROVIDE THE DESIRED θ-PATH FROM s TO t.



THEOREM 2 FOR EVERY CONVEX POINT SET P, THERE EXISTS AN INCREASING-CHORD GRAPH G SPANNING P WITH $O(|P| \log |P|)$ EDGES (AND NO STEINER POINTS)

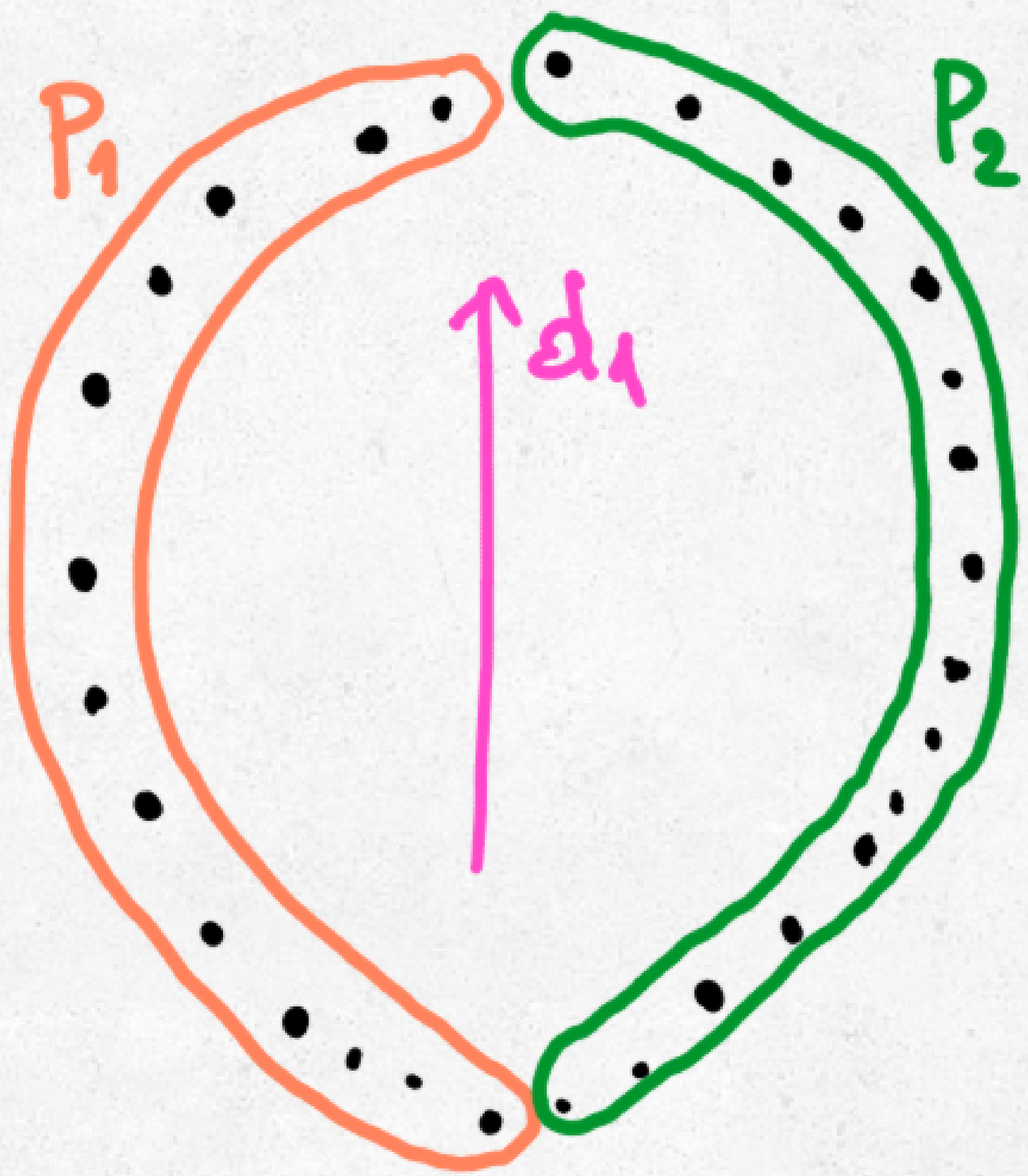
LEMMA 5 LET P BE A ONE-SIDED CONVEX POINT SET. THERE EXISTS AN INCREASING-CHORD OUTERPLANAR GRAPH SPANNING P



PROOF USES: (X,Y)-MONOTONE PATHS ARE INCREASING-CHORD [ALAMDARI et al.]

LEMMA 6 FOR EVERY CONVEX POINT SET P, THERE EXIST ONE-SIDED CONVEX POINT SETS $P_1, \ldots, P_k$ SUCH THAT

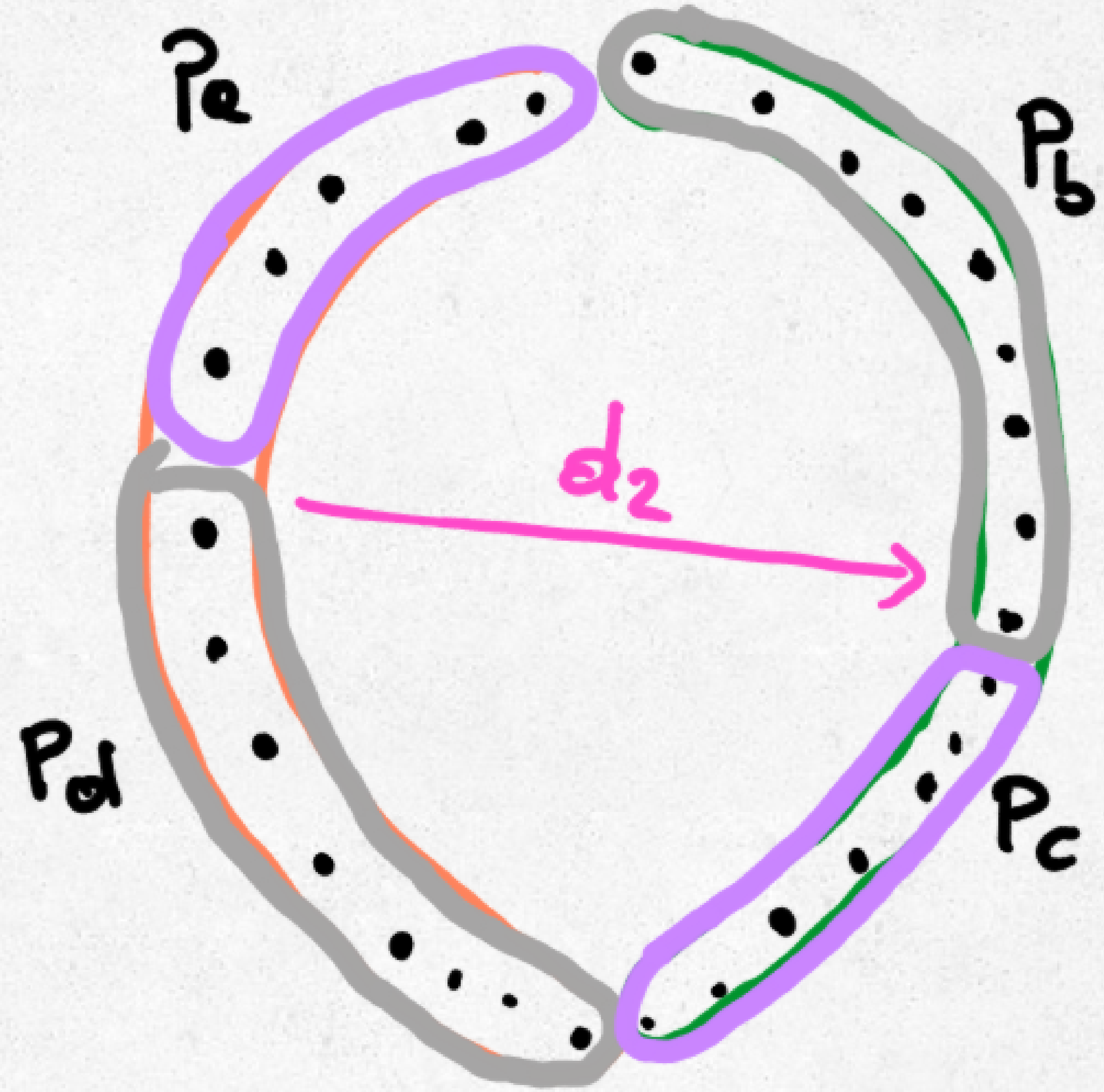
- $P_i \subseteq P$
- $\sum |P_i| \in O(|P| \log |P|)$
- $\forall\, p, q \in P,\ \exists\, P_i : p, q \in P_i$



$P_3 = P_a \cup P_b$

$P_4 = P_c \cup P_d$

P_3 AND P_4 ARE ONE-SIDED



$|P_a| + |P_c| \leq \lceil \frac{|P|}{2} \rceil + 1$

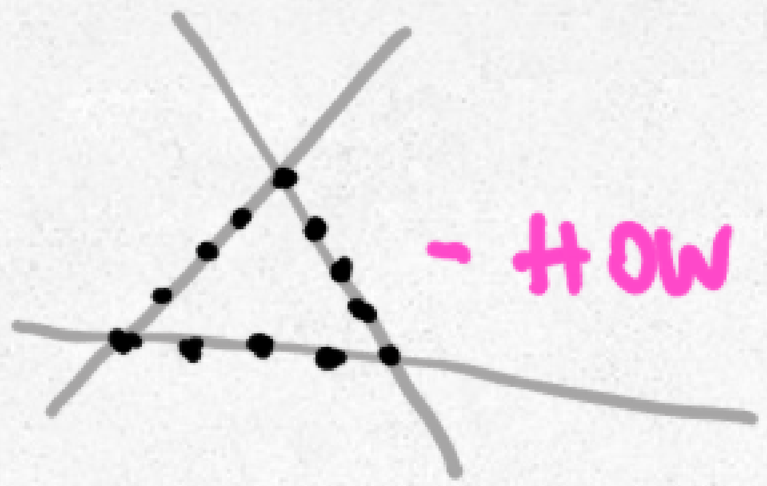
$|P_b| + |P_d| \leq \lceil \frac{|P|}{2} \rceil + 1$

CONTINUE ON $P_a \cup P_c$ AND ON $P_b \cup P_d$ (INDEPENDENTLY)

$$\sum |P_i| \simeq |P| + 2\frac{|P|}{2} + 4\frac{|P|}{4} + \dots \in O(|P| \log |P|)$$

OPEN PROBLEM 1 IS IT TRUE THAT, FOR EVERY (CONVEX) POINT SET P, THERE EXISTS AN INCREASING-CHORD (SELF-APPROACHING) PLANAR GRAPH SPANNING P?

- HOW ABOUT NON-PLANAR GRAPHS WITH $o(|P|^2)$ EDGES?



- HOW ABOUT POINTS ON THE BOUNDARY OF A TRIANGLE?

OPEN PROBLEM 2 IS IT TRUE THAT A GABRIEL TRIANGULATION IN $\mathbb{R}^d$ IS SELF-APPROACHING?

THANKS!