On Monotone Drawings of Trees

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Joint work with
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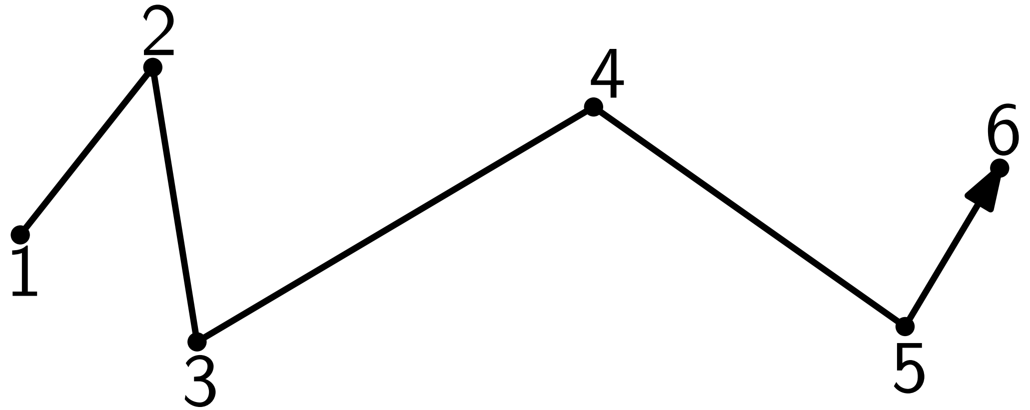

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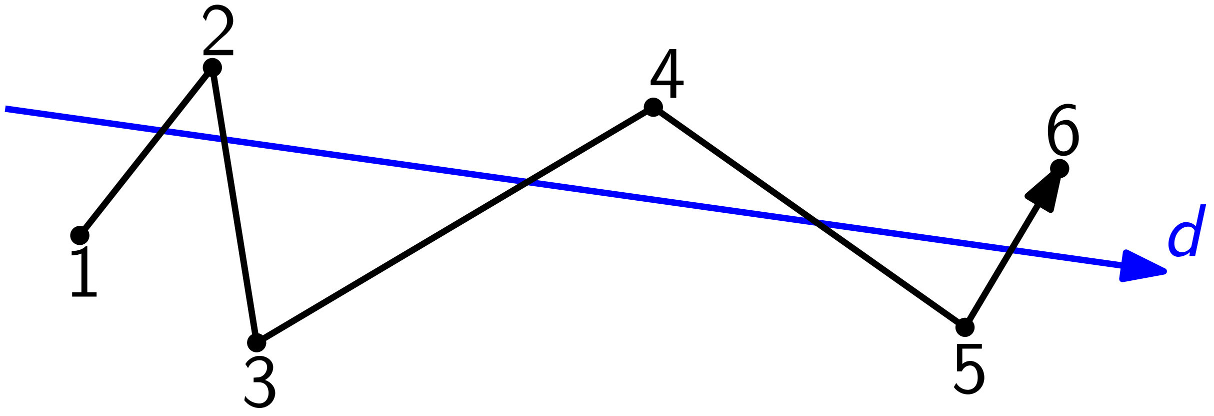

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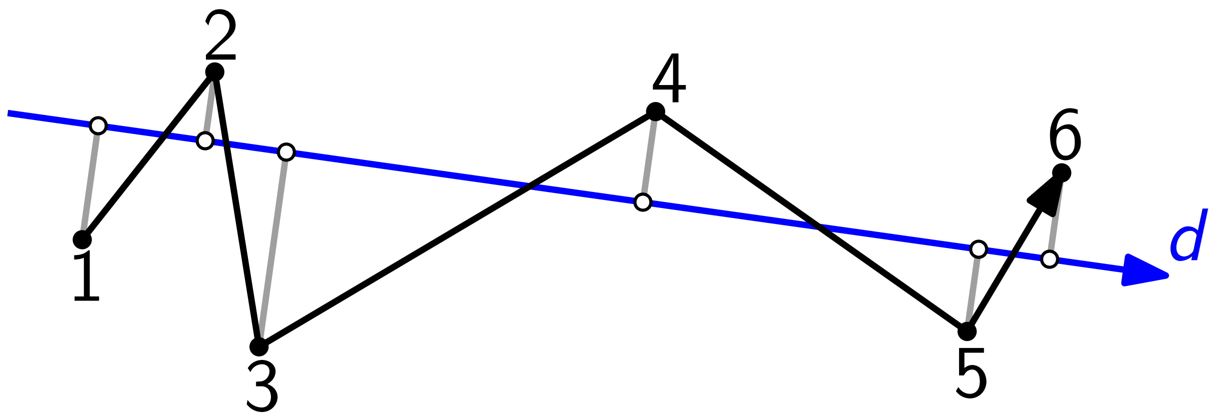

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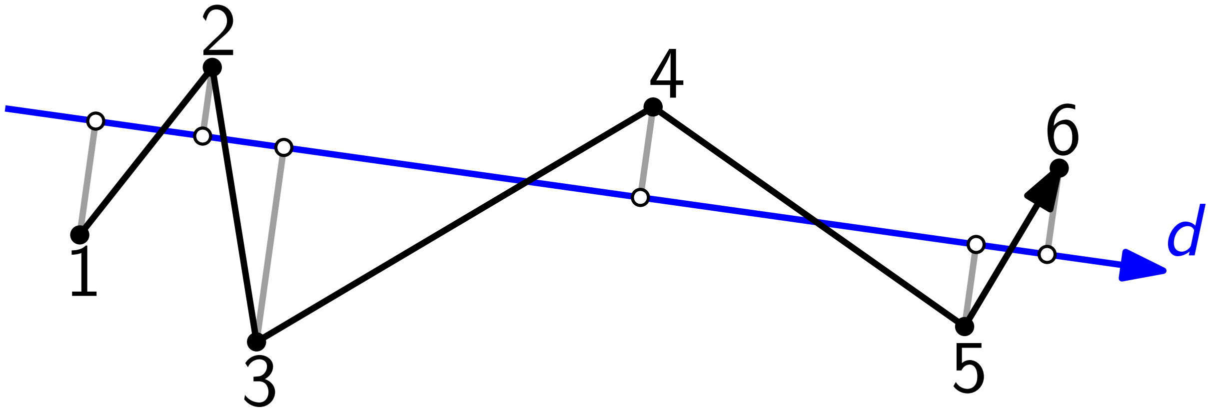

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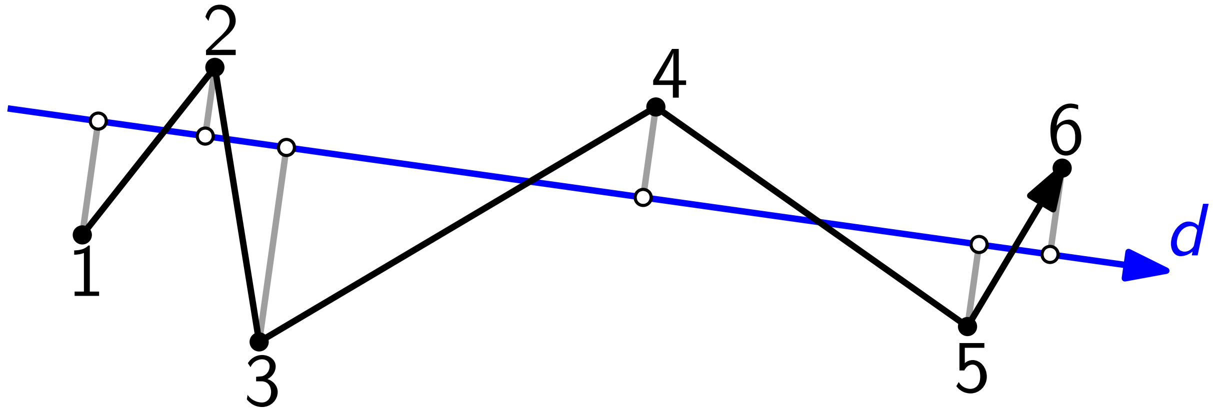

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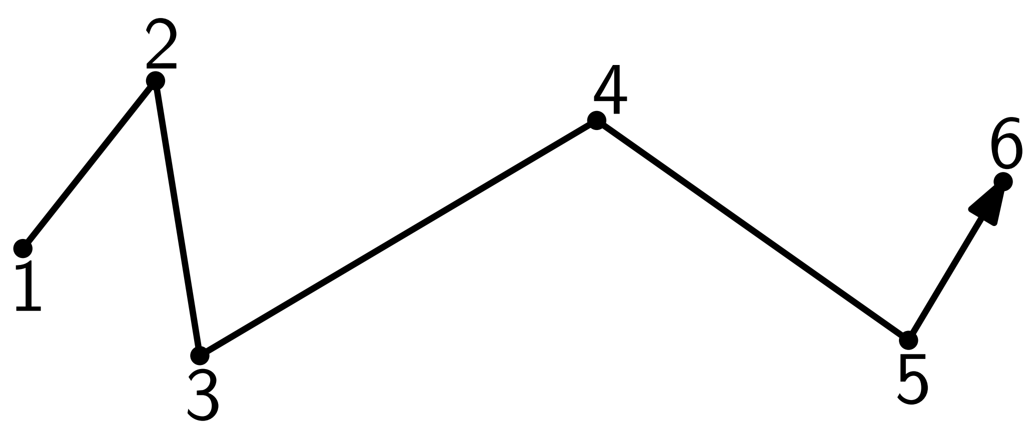

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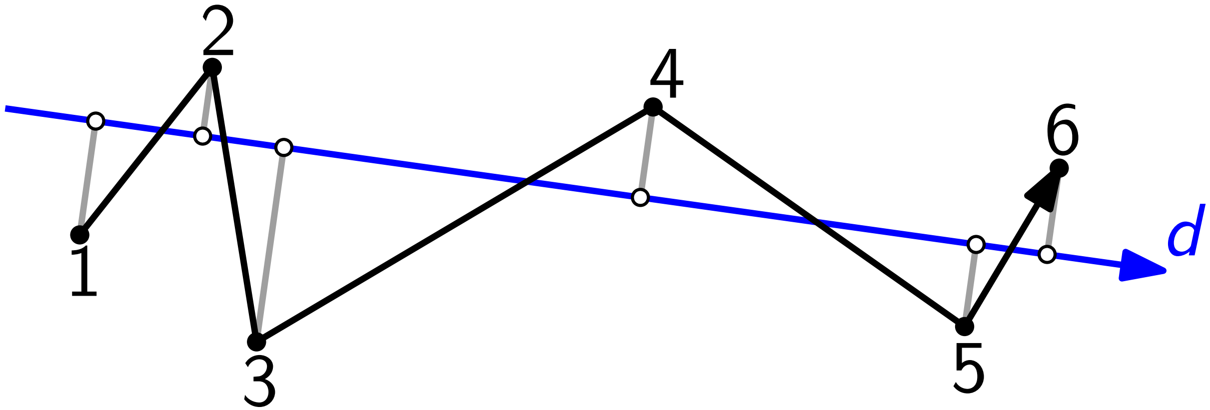

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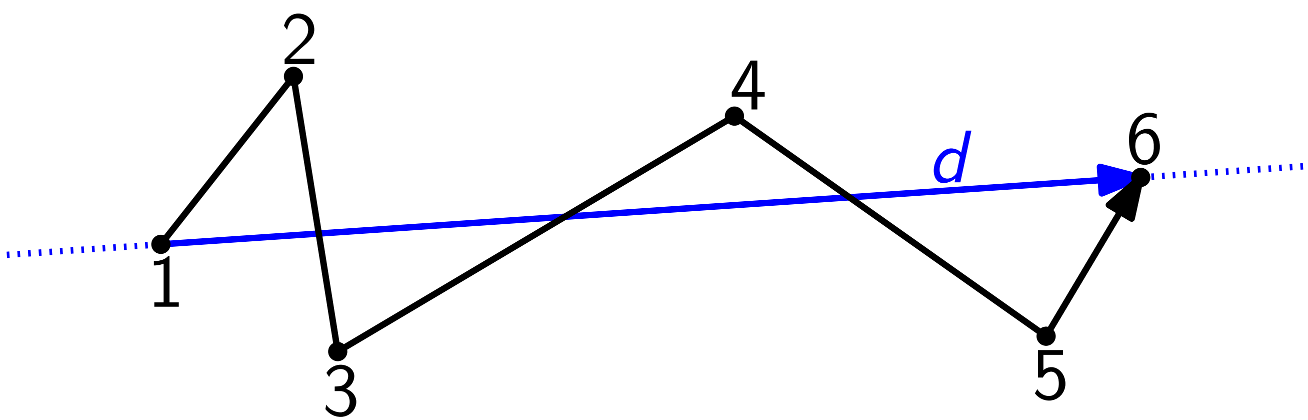

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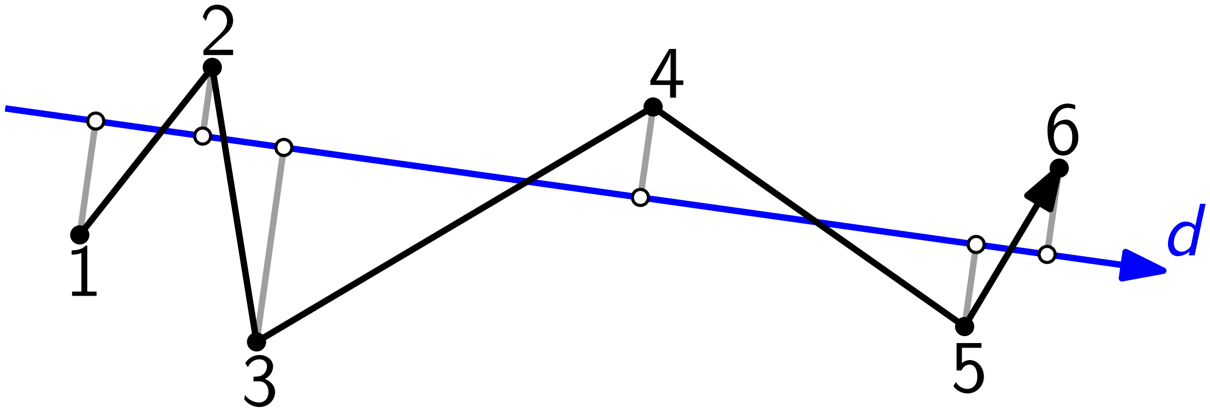

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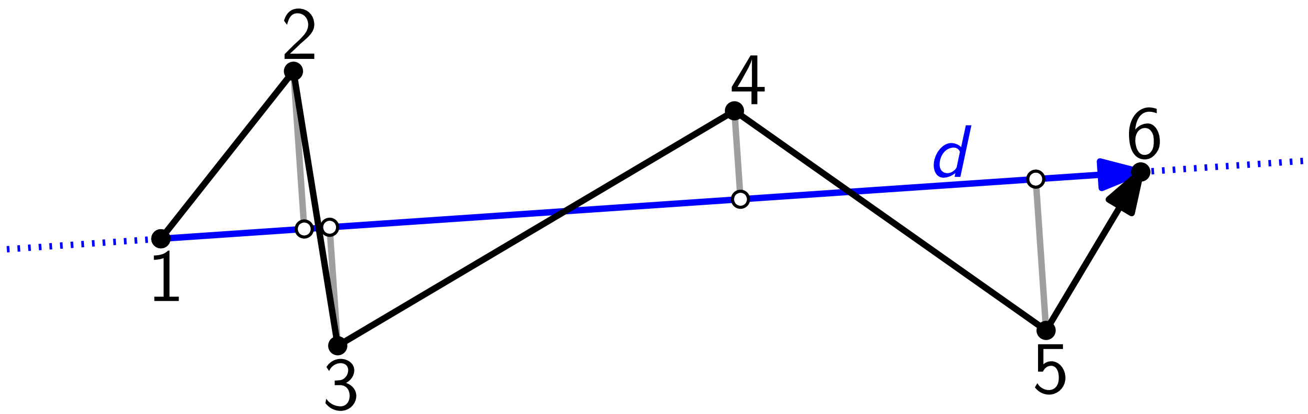

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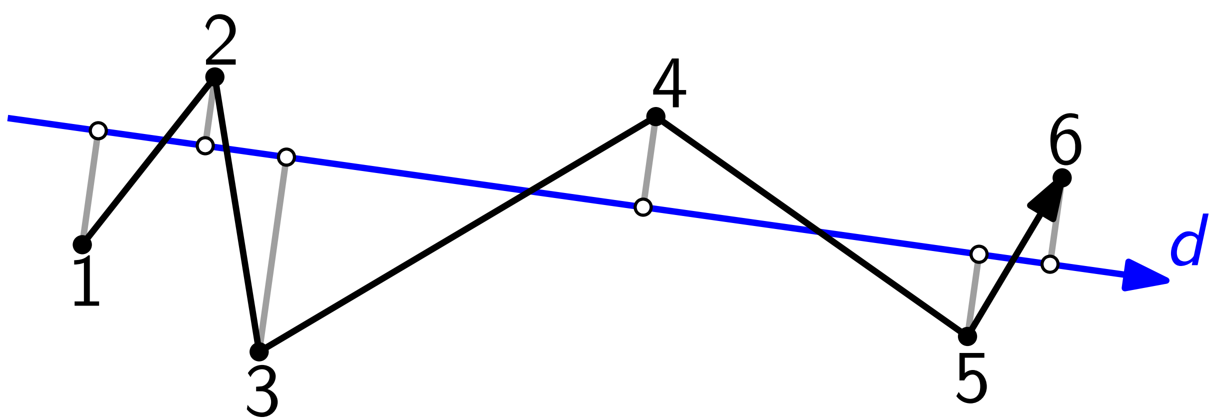

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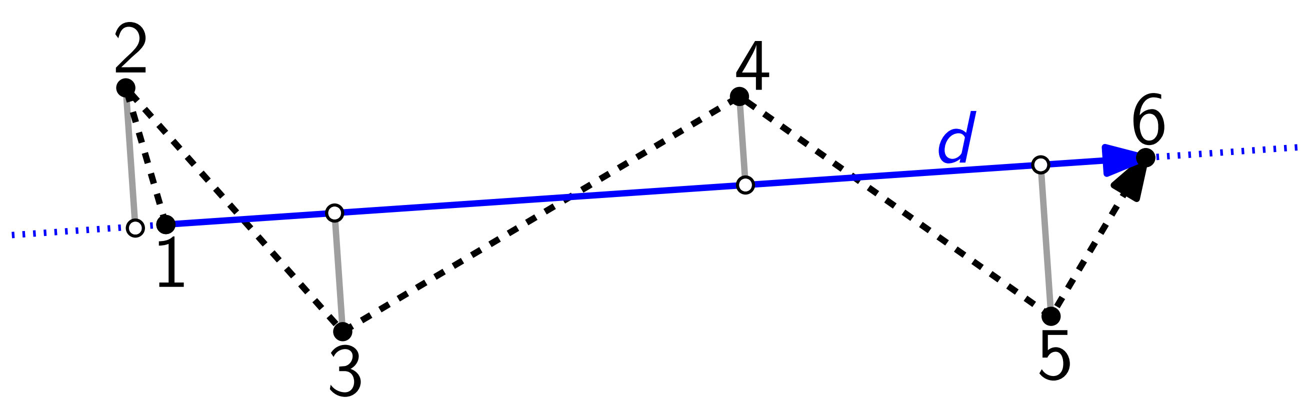

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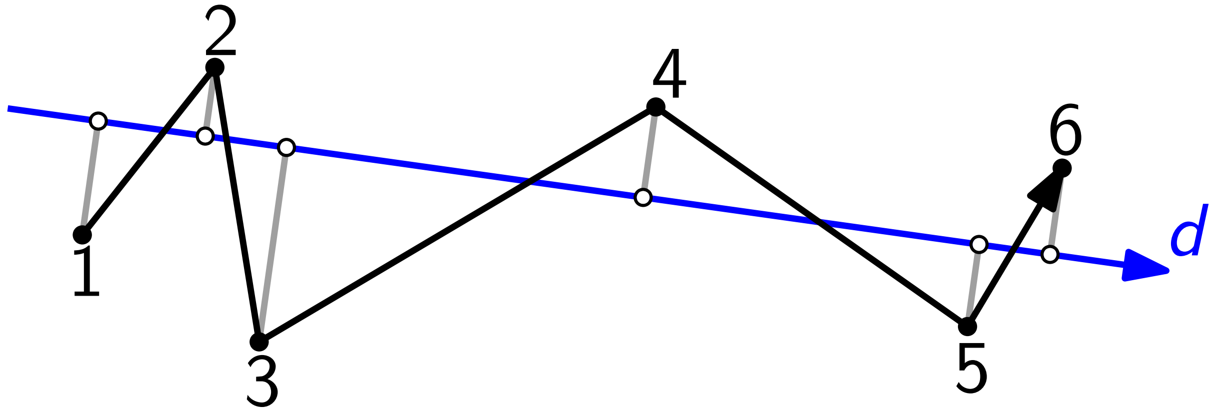

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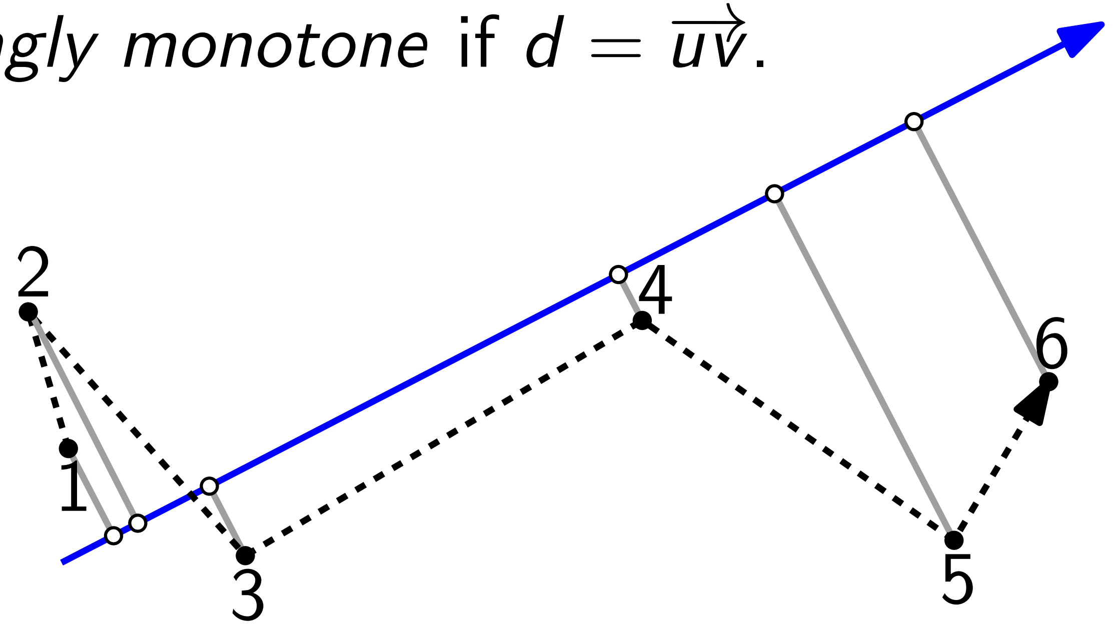

Known Results

[Angelini et al. JGAA'12]

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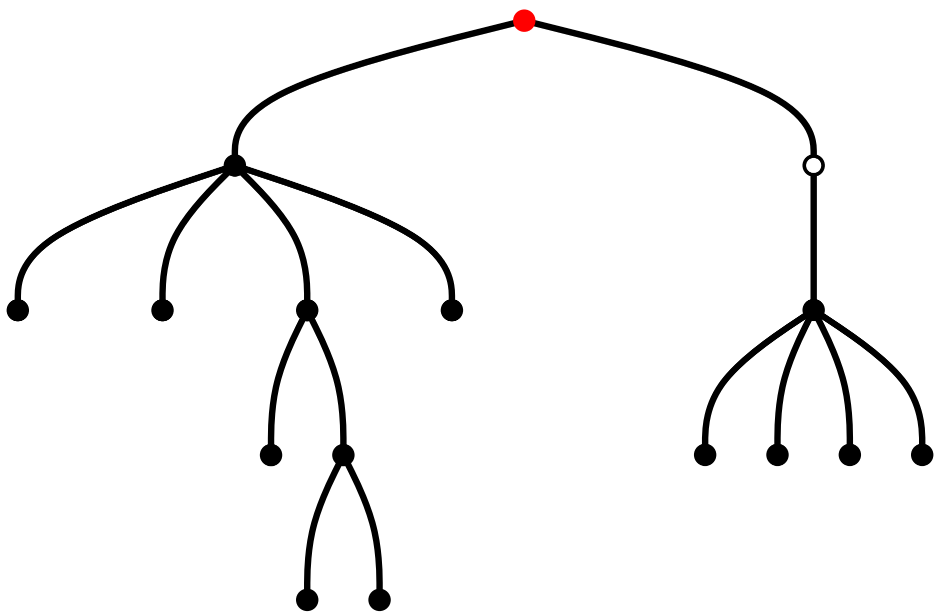

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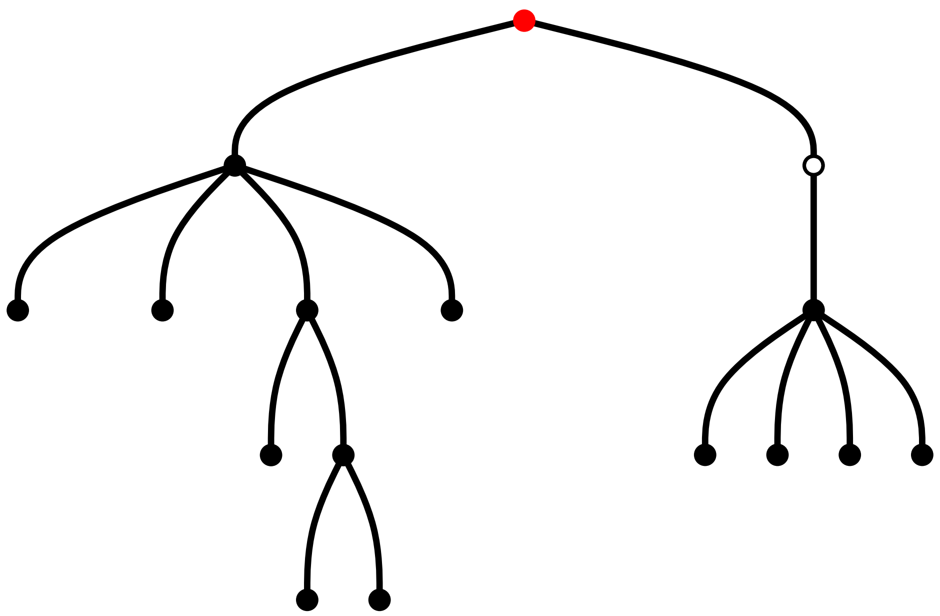

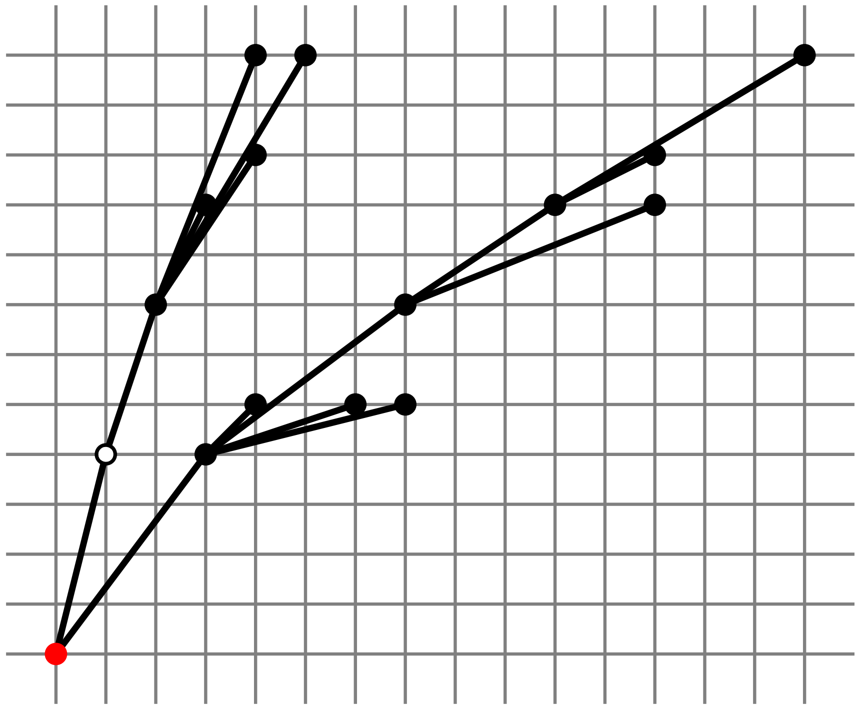

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[Hossain and Rahman FAW'14]

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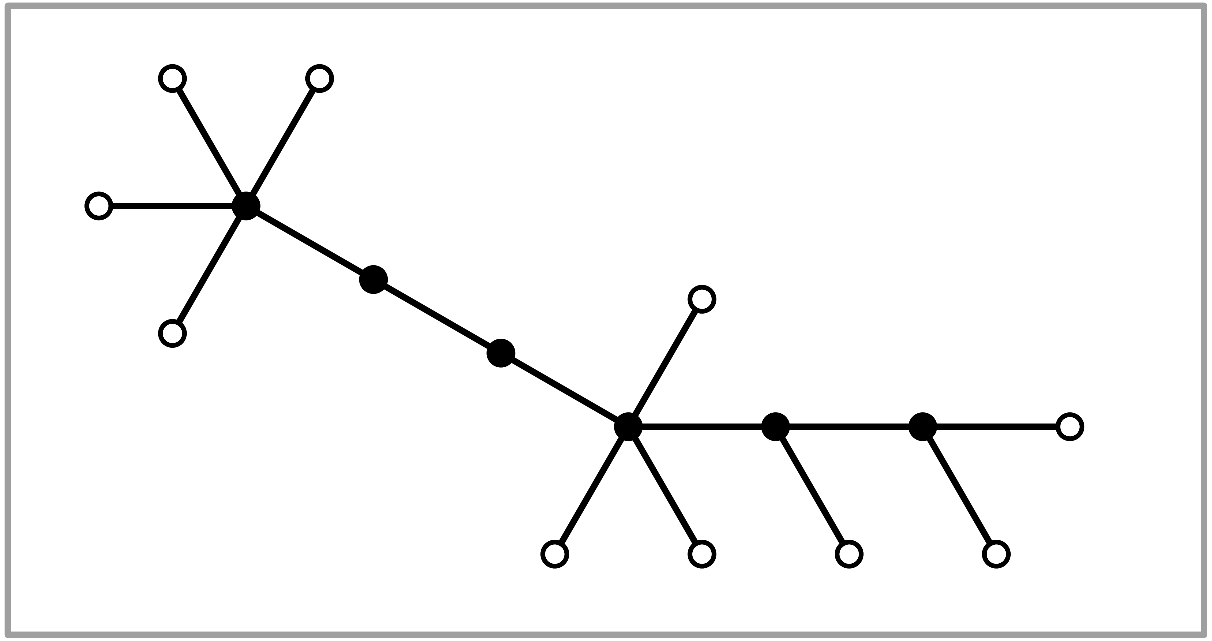

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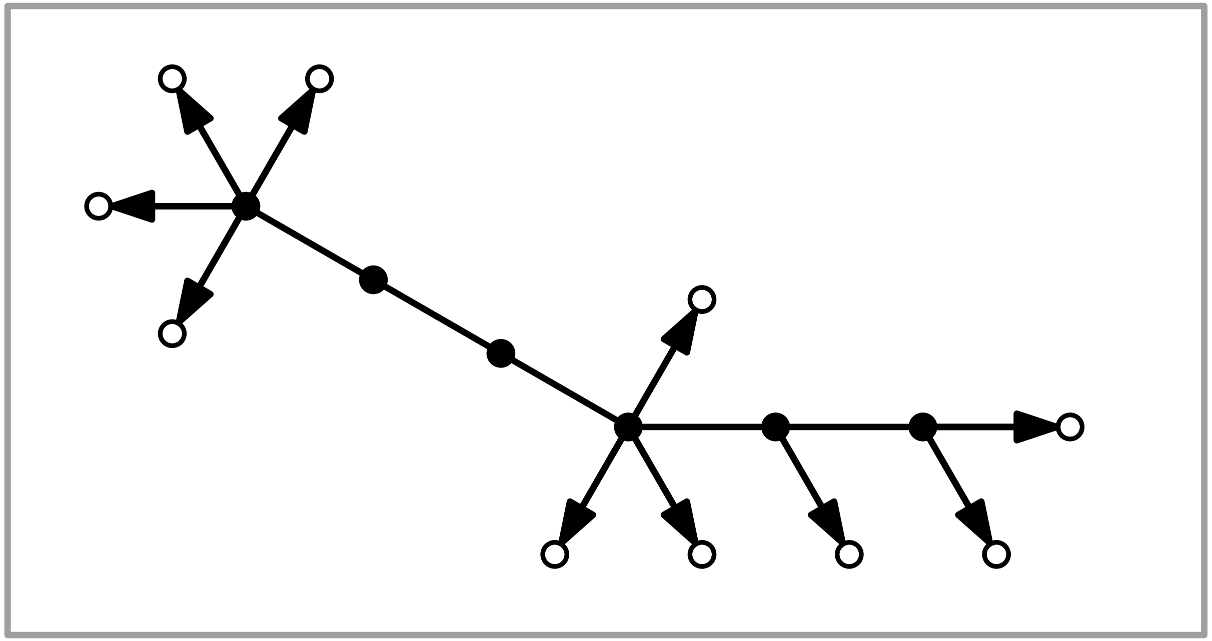

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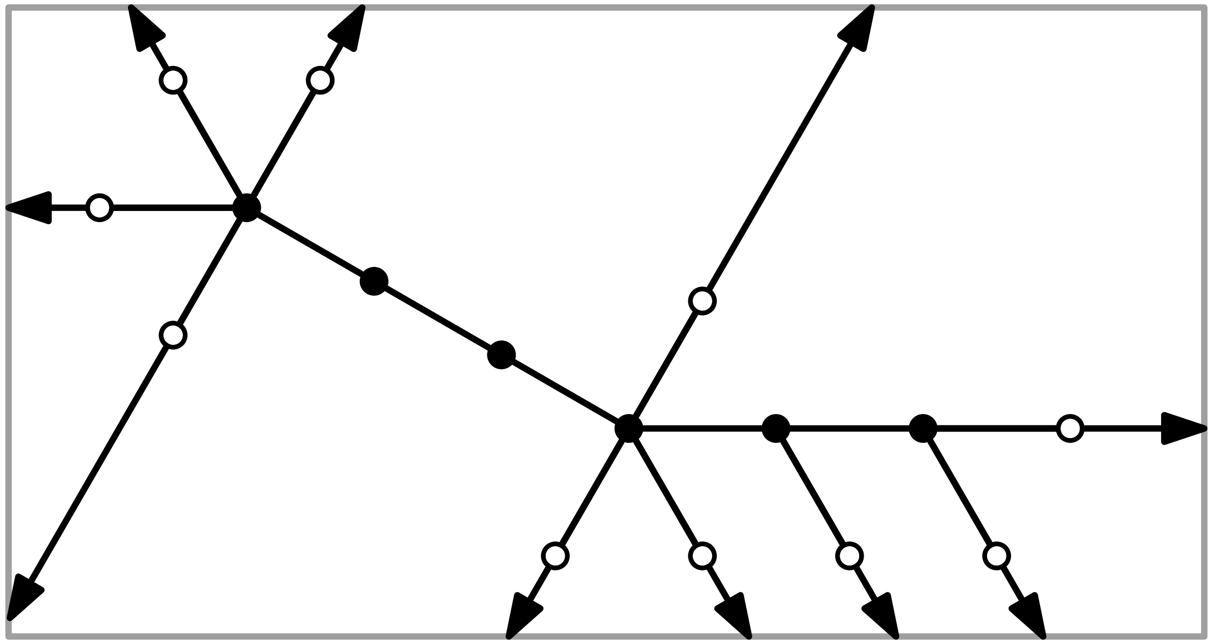

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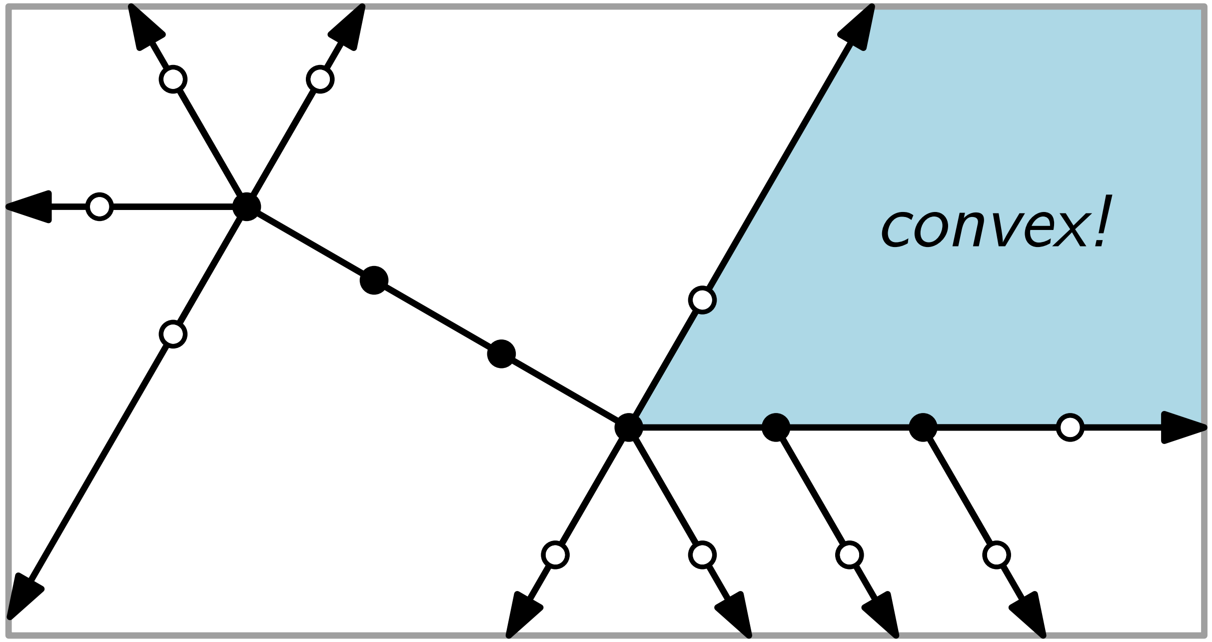

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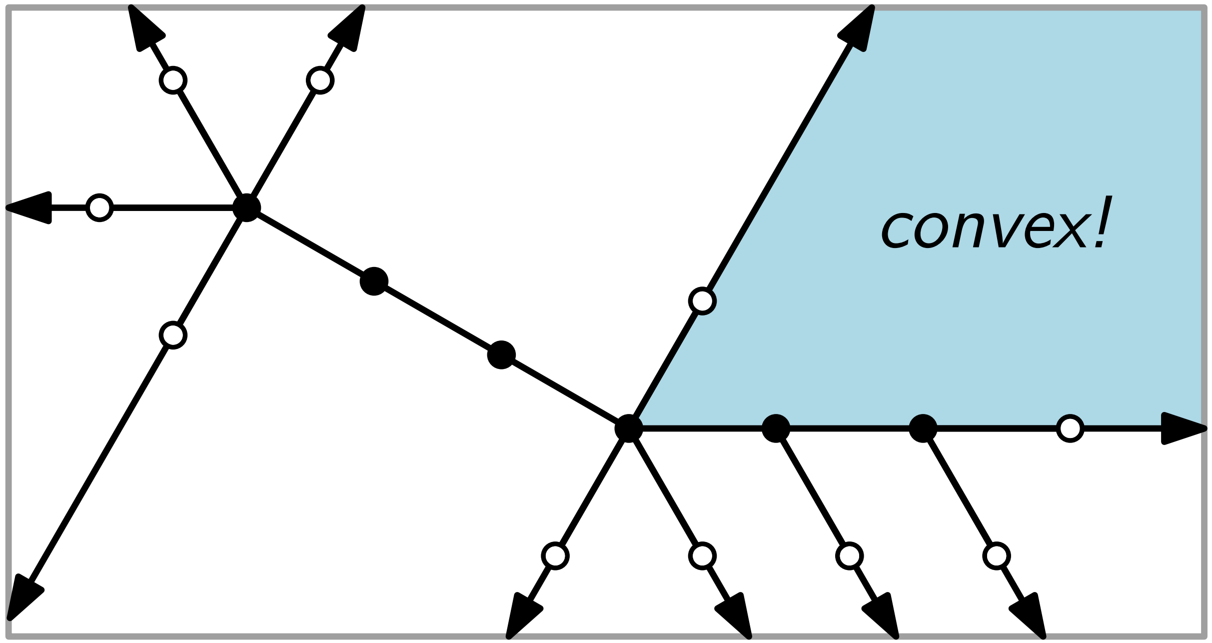

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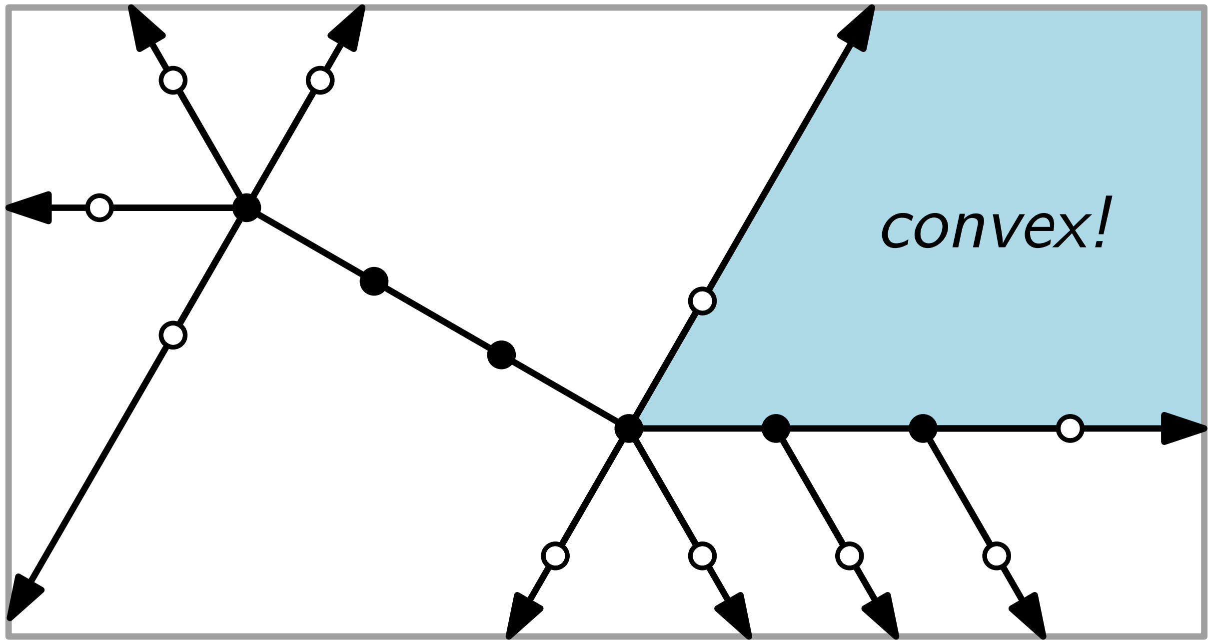

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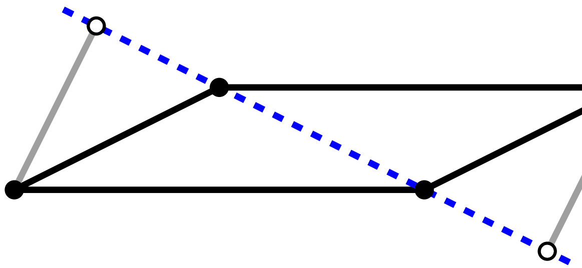

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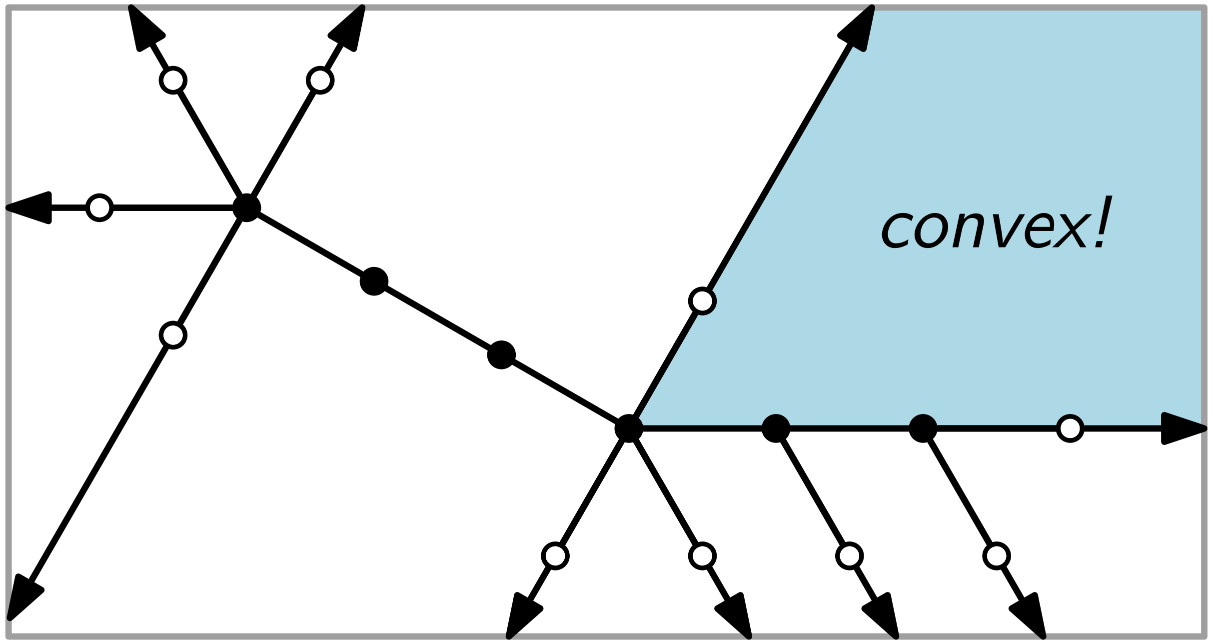

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[Carlson & Eppstein GD'06]
Can compute convex drawings of trees,
with optimal *angular resolution*.

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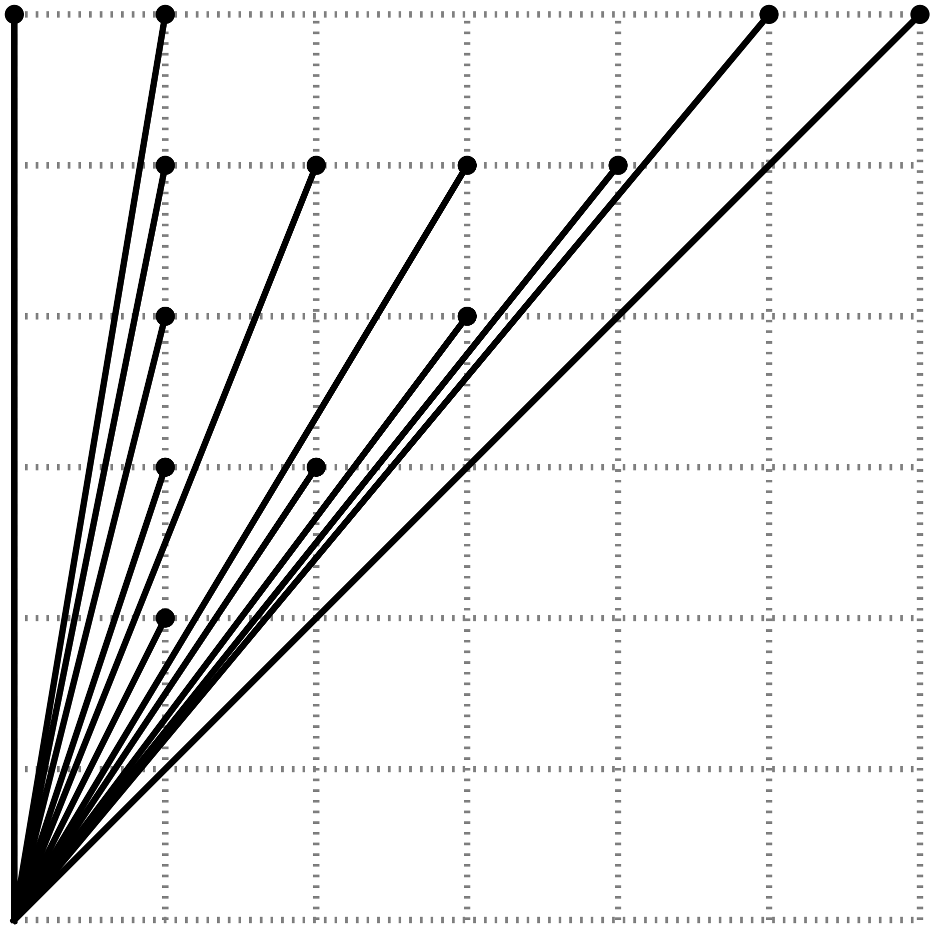

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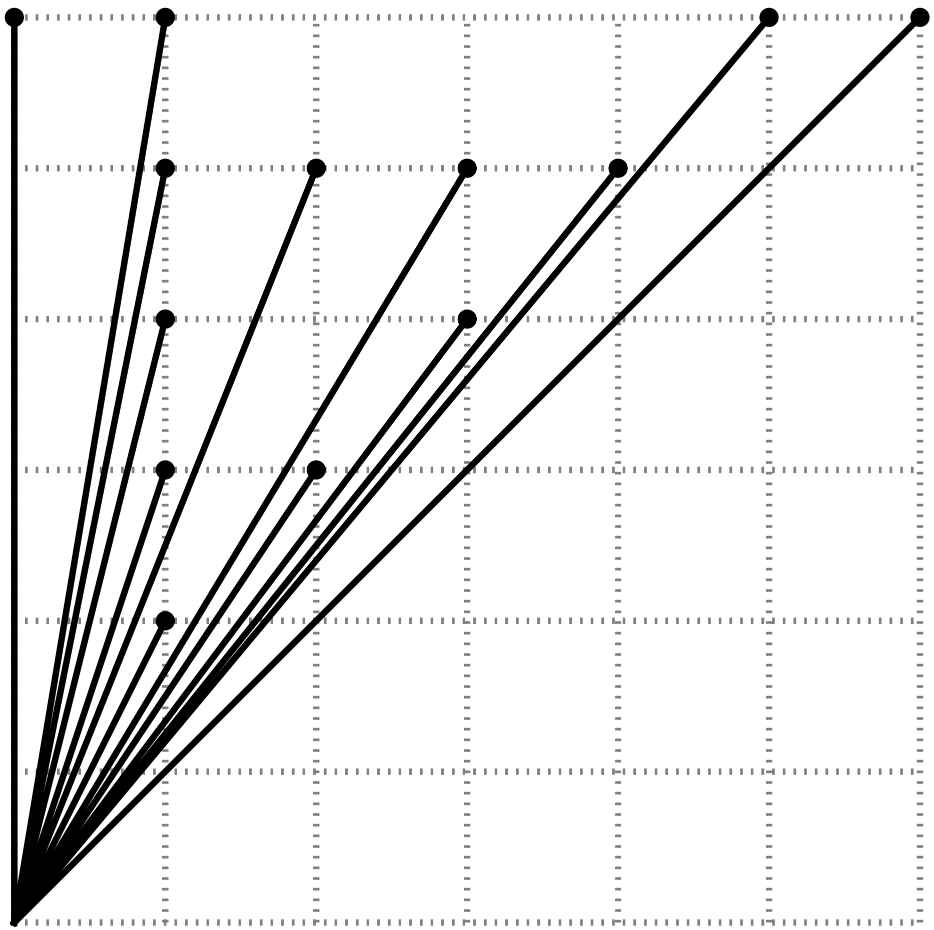

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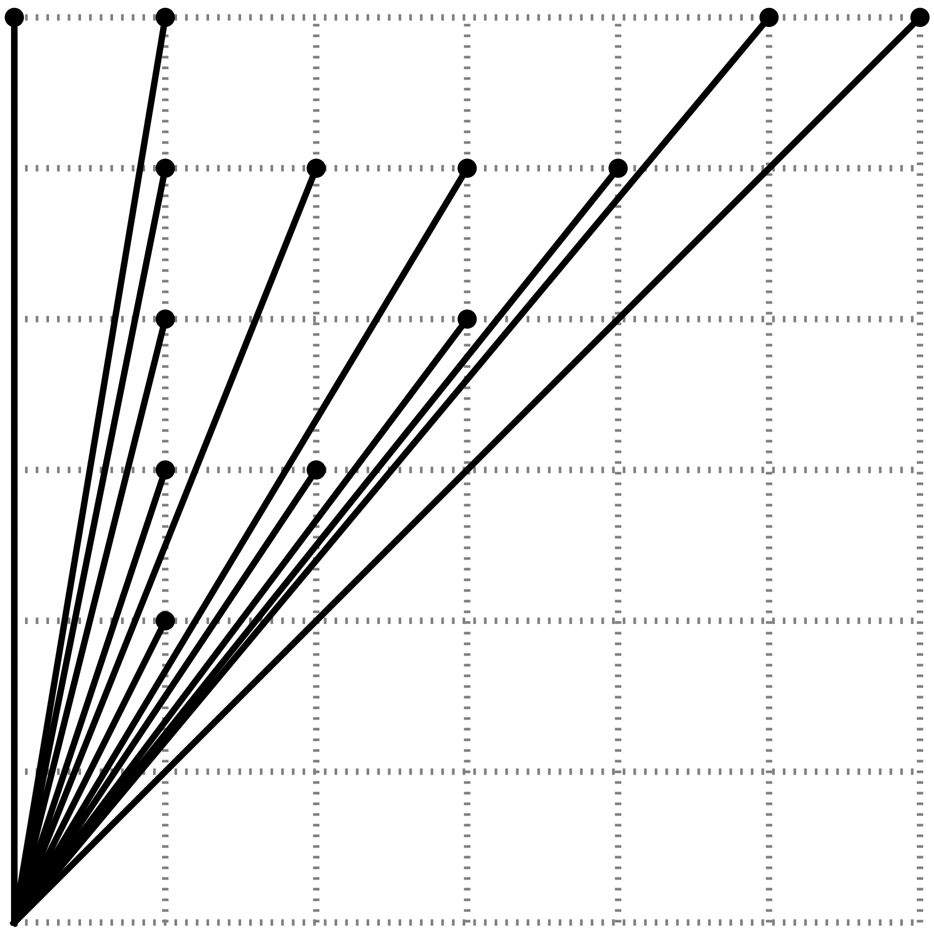

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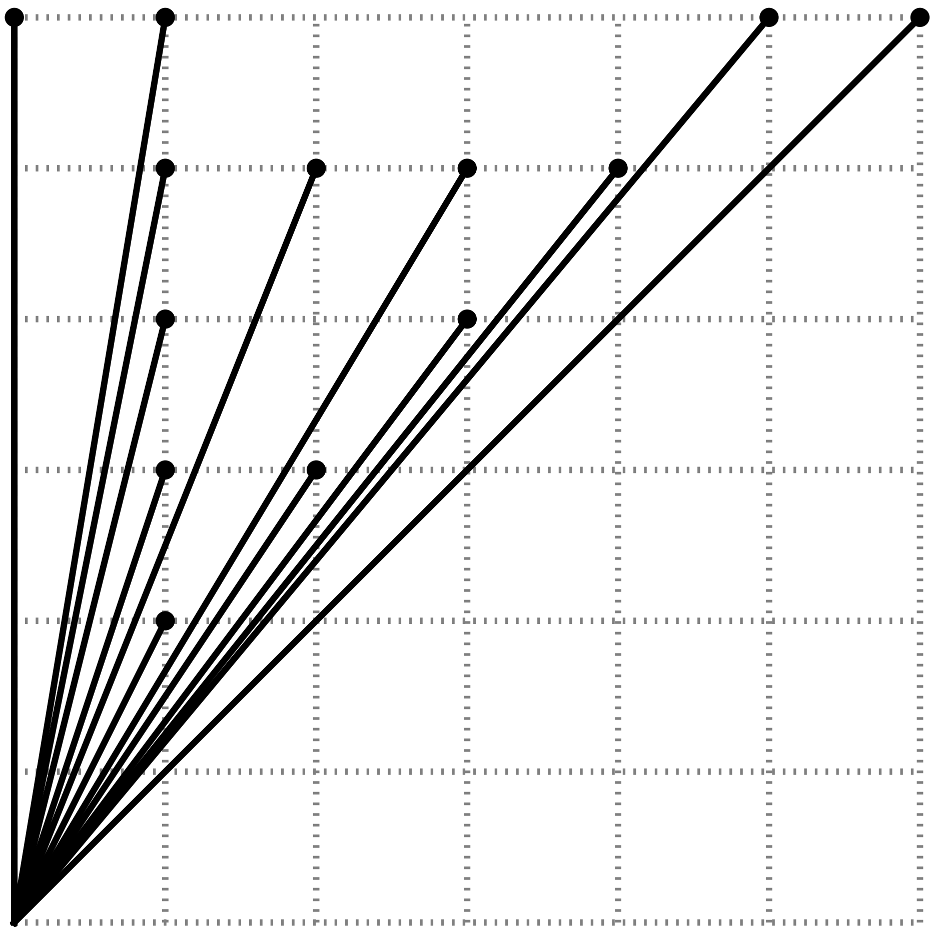

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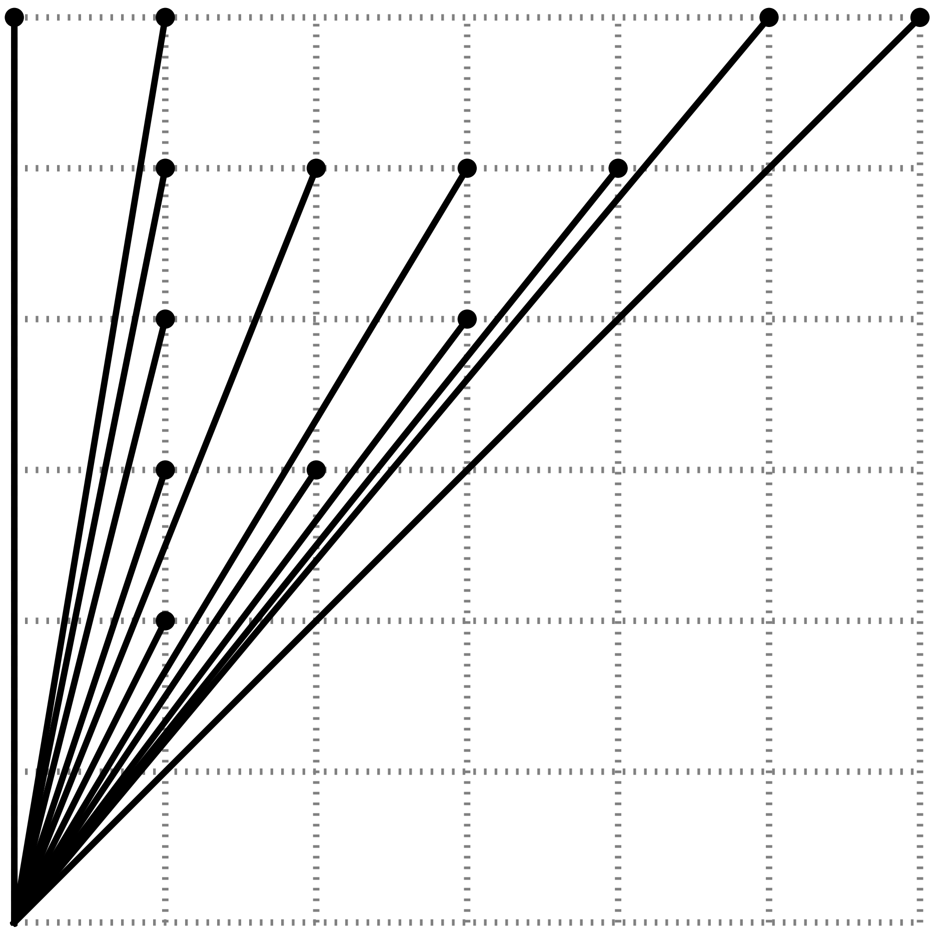

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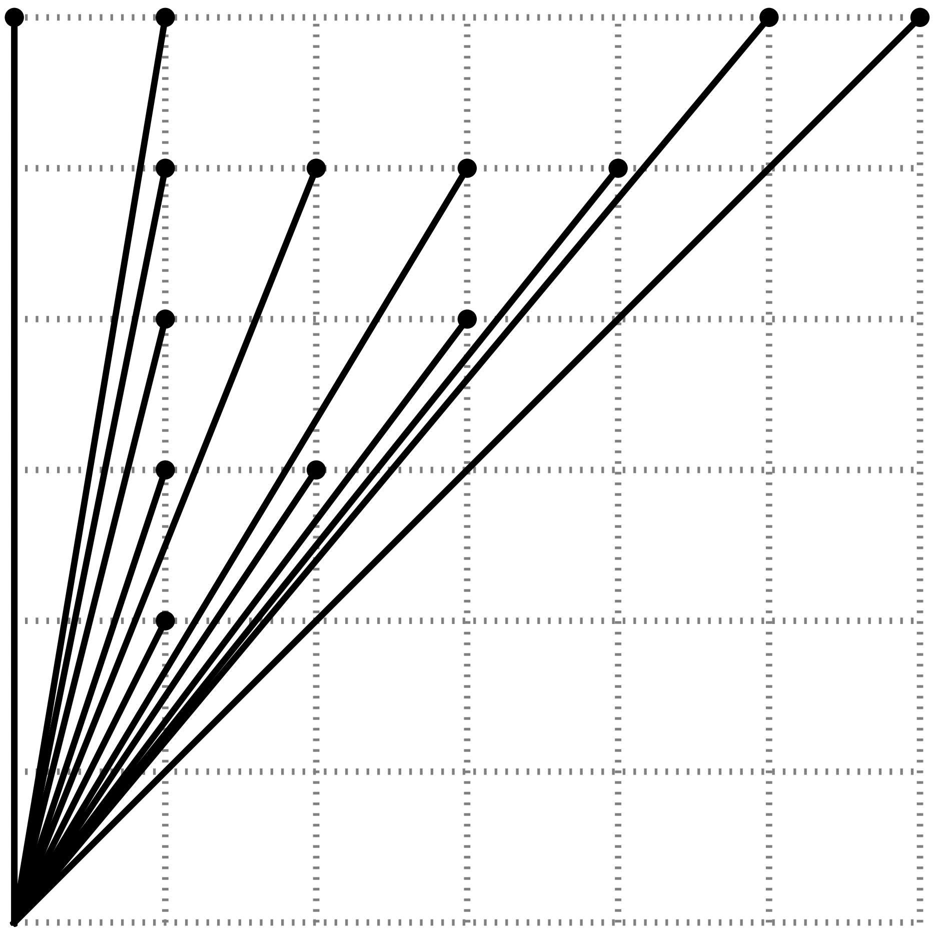

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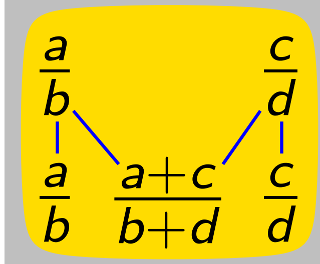

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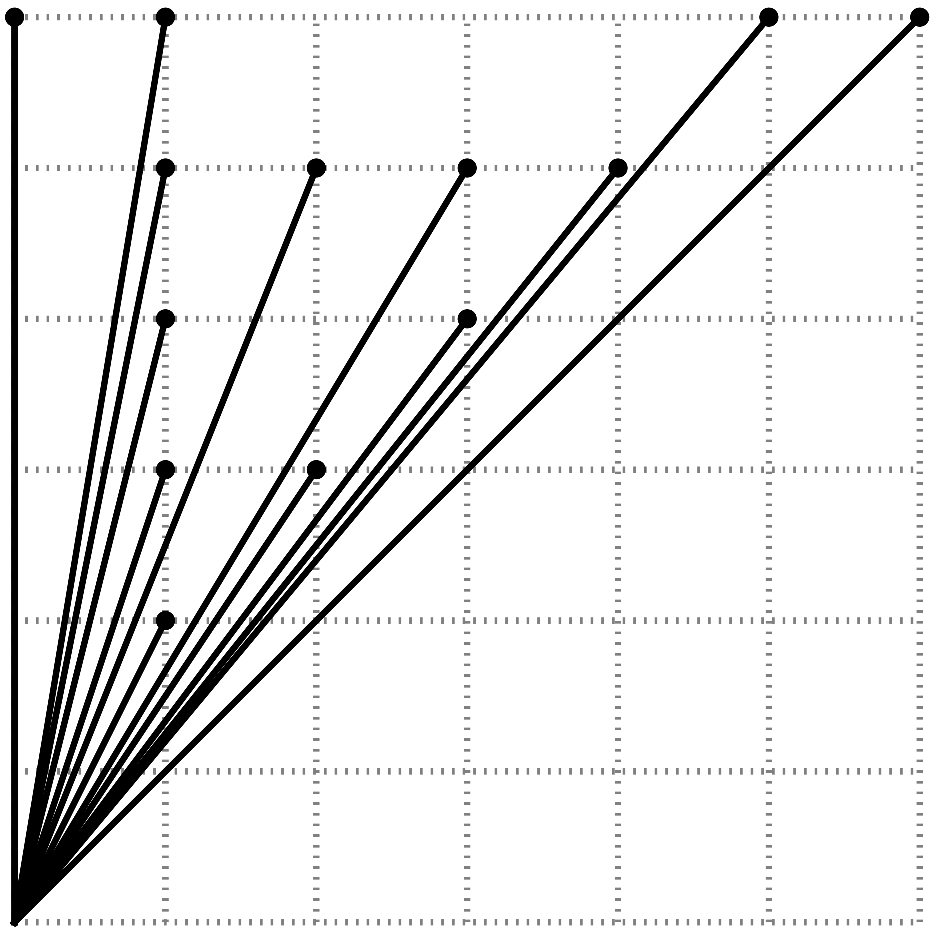

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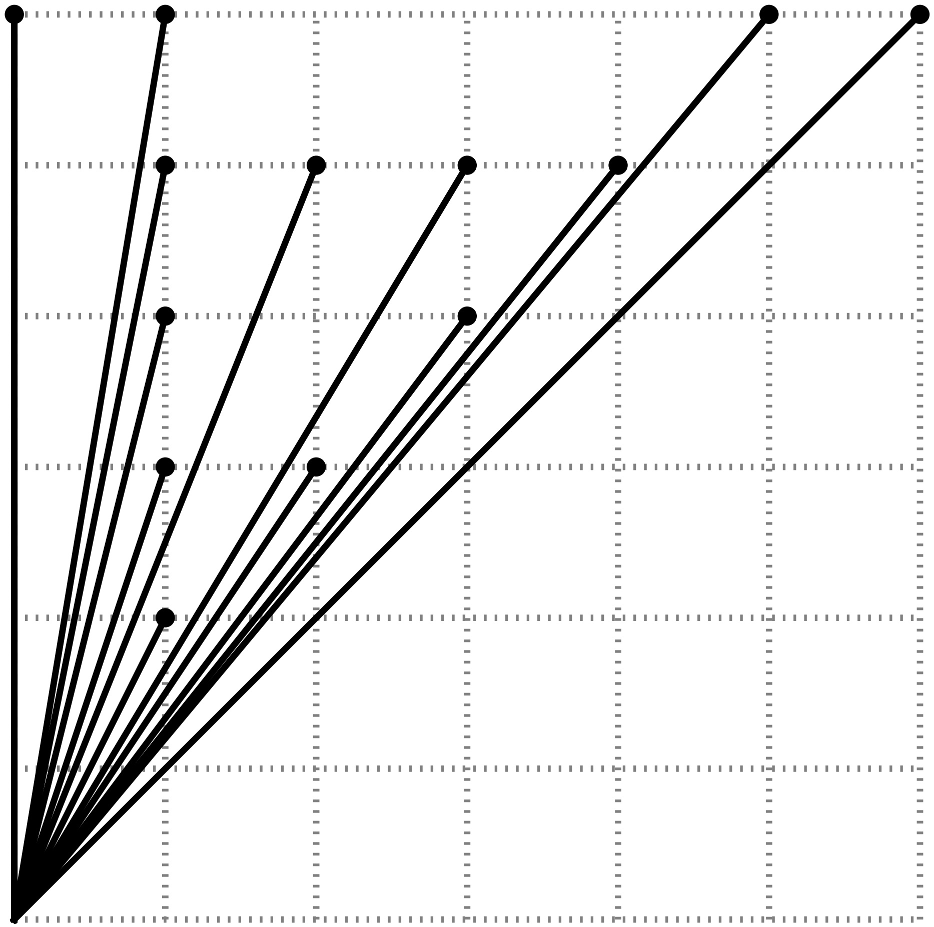

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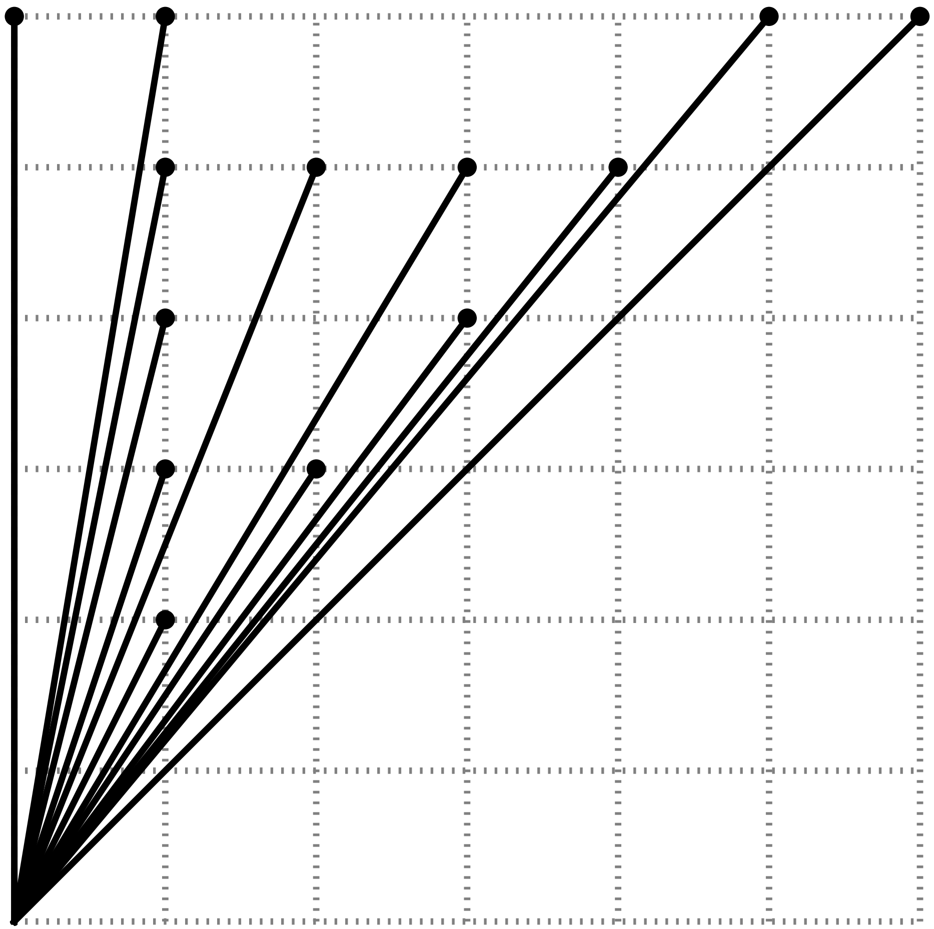

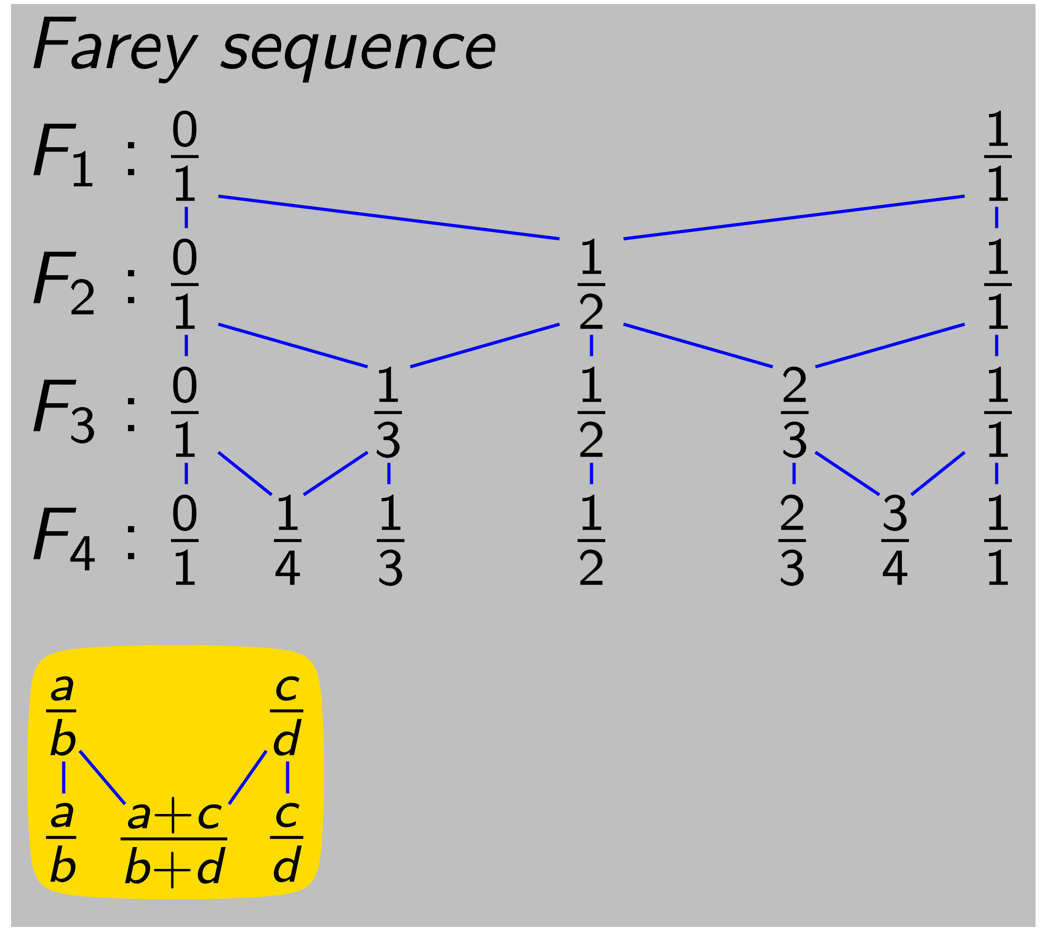

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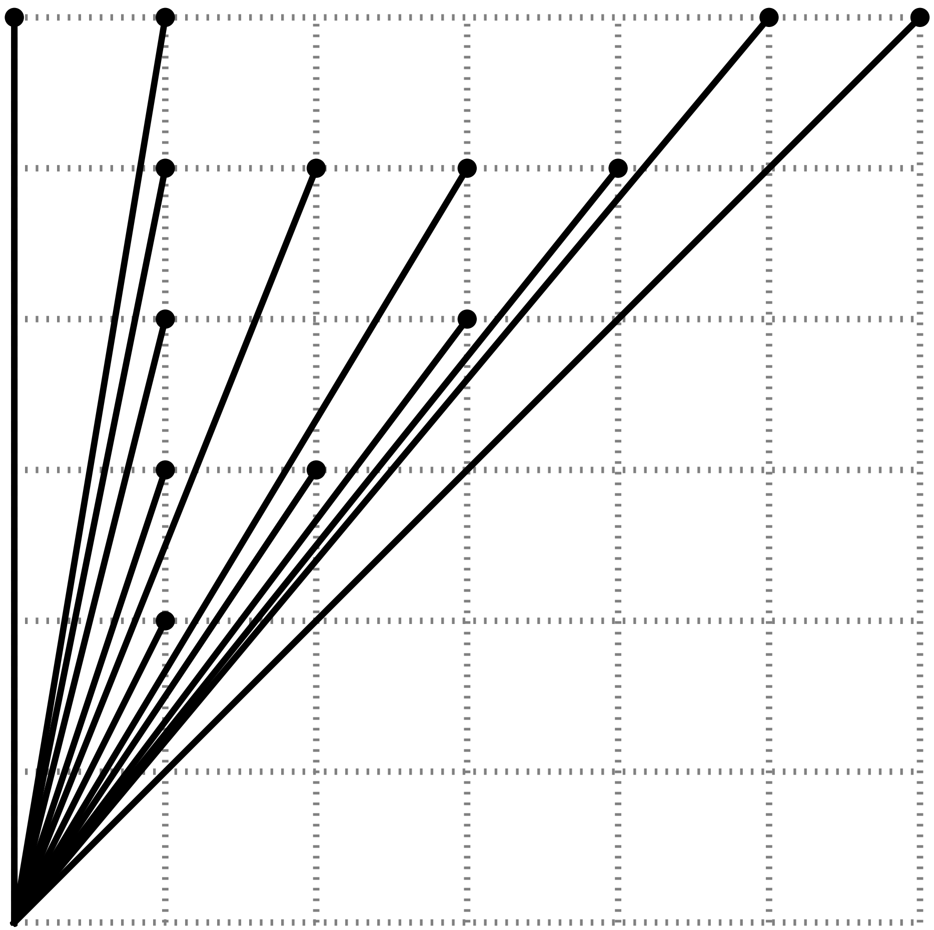

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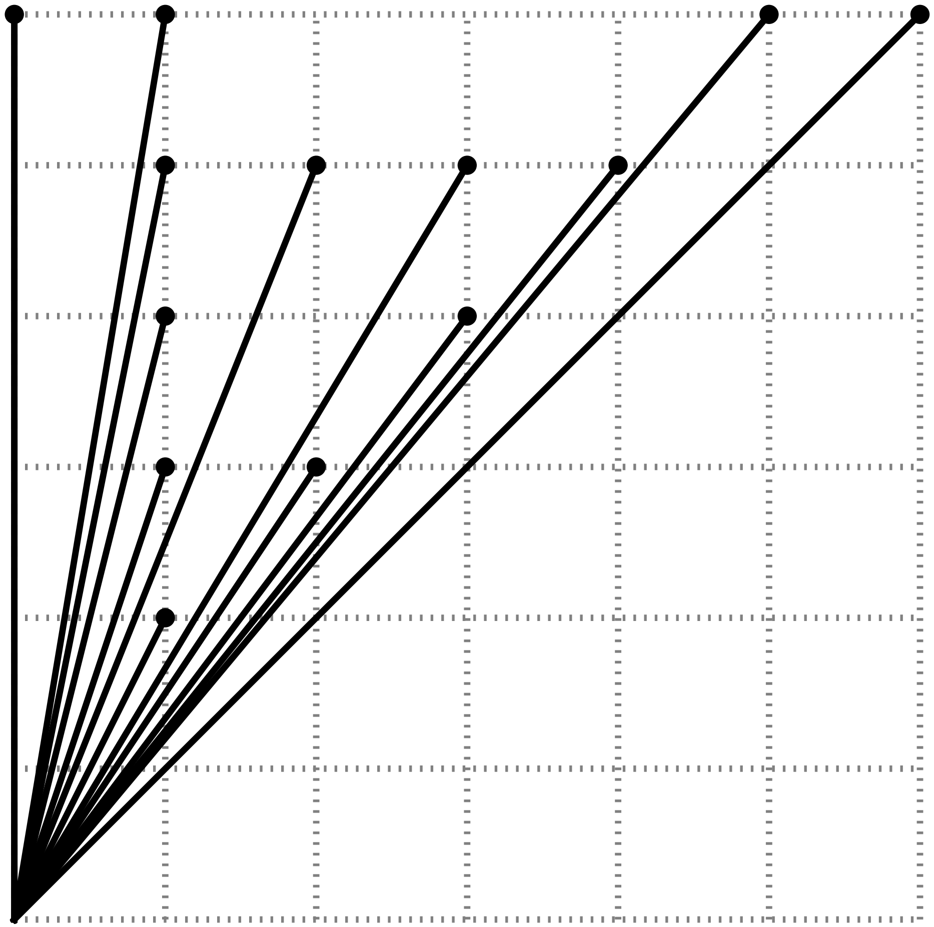

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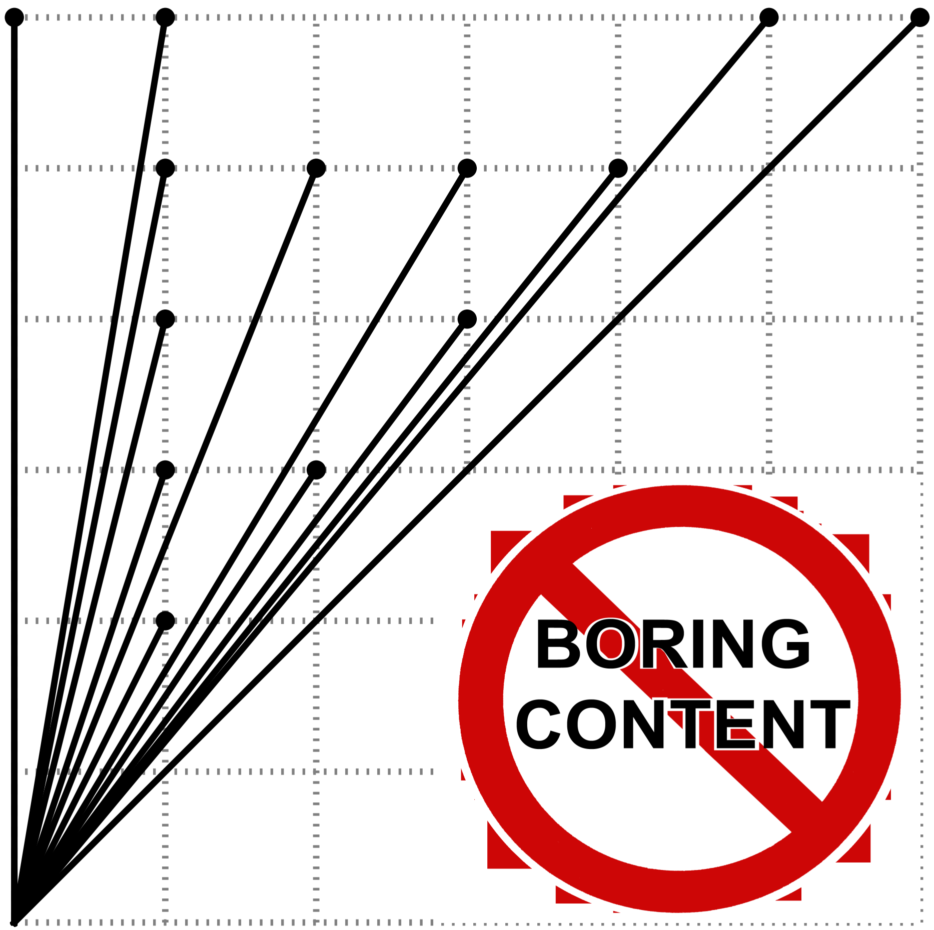

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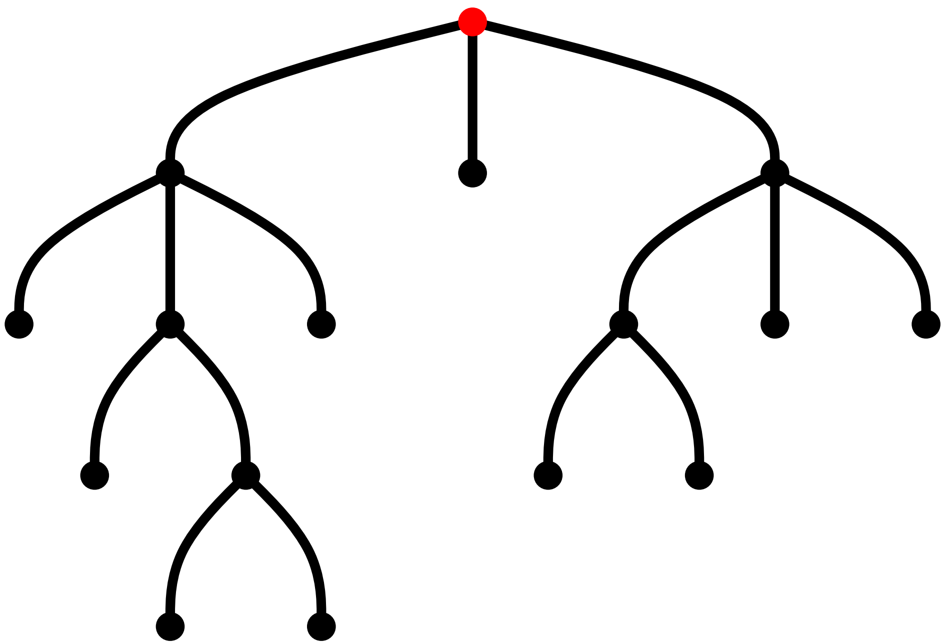

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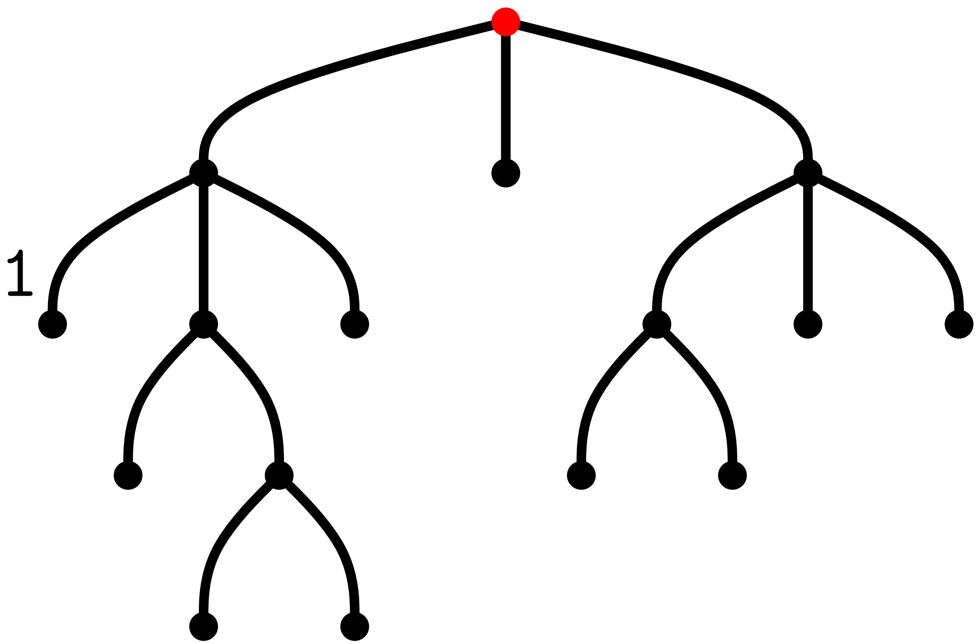

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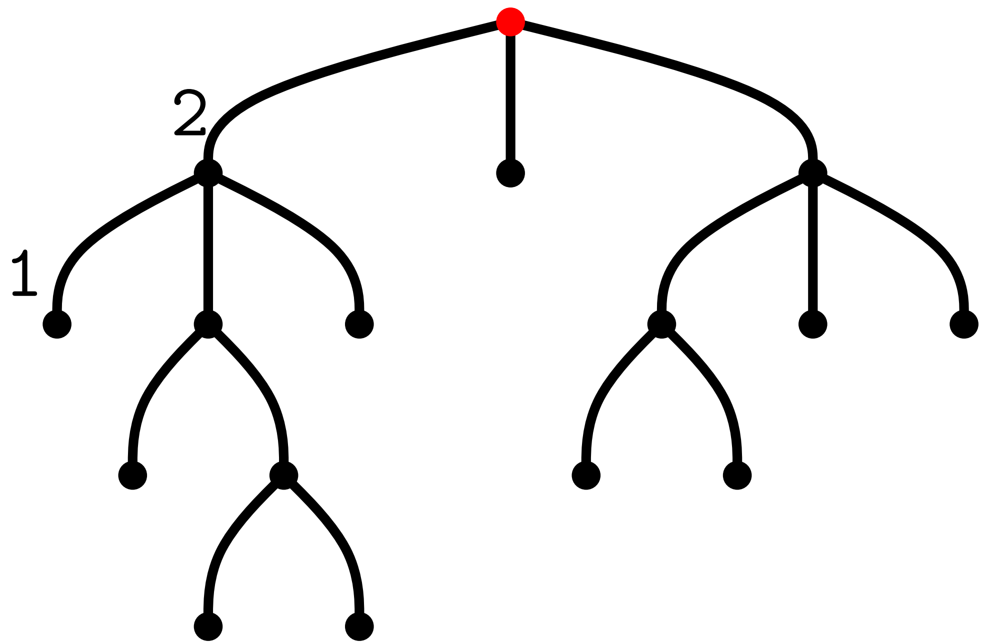

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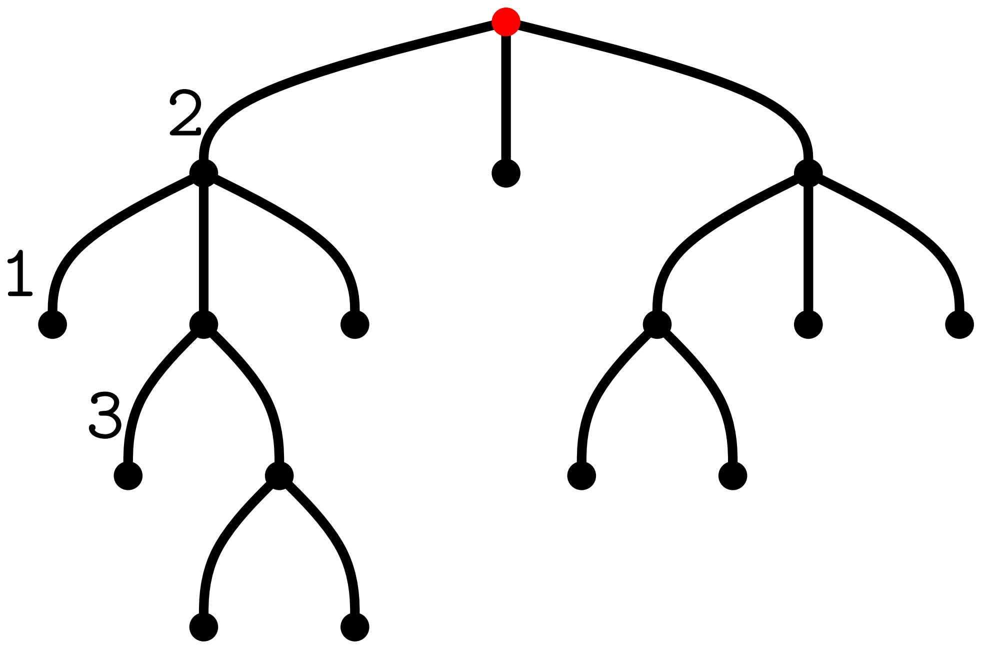

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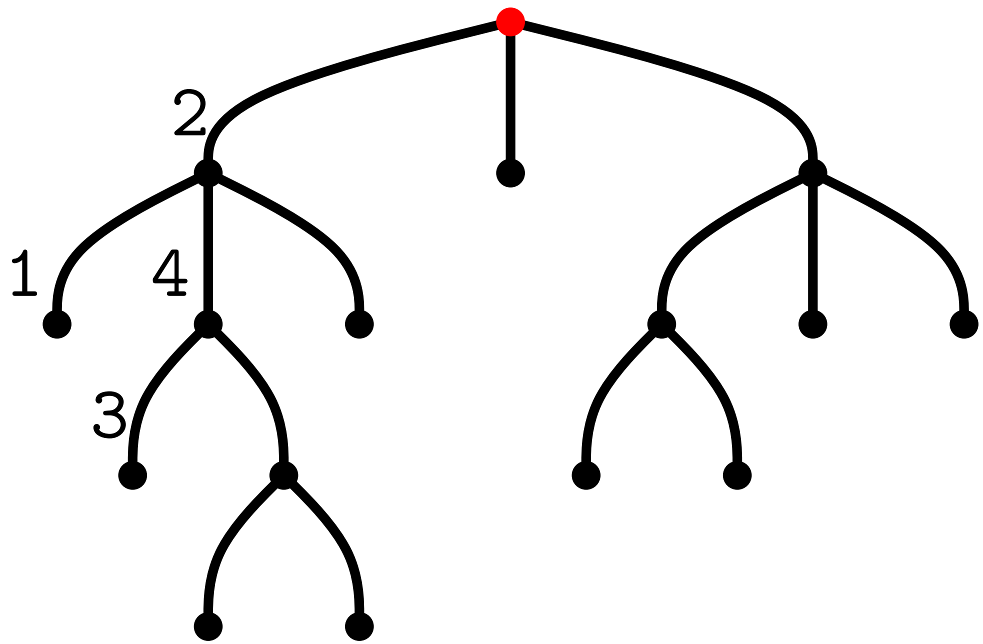

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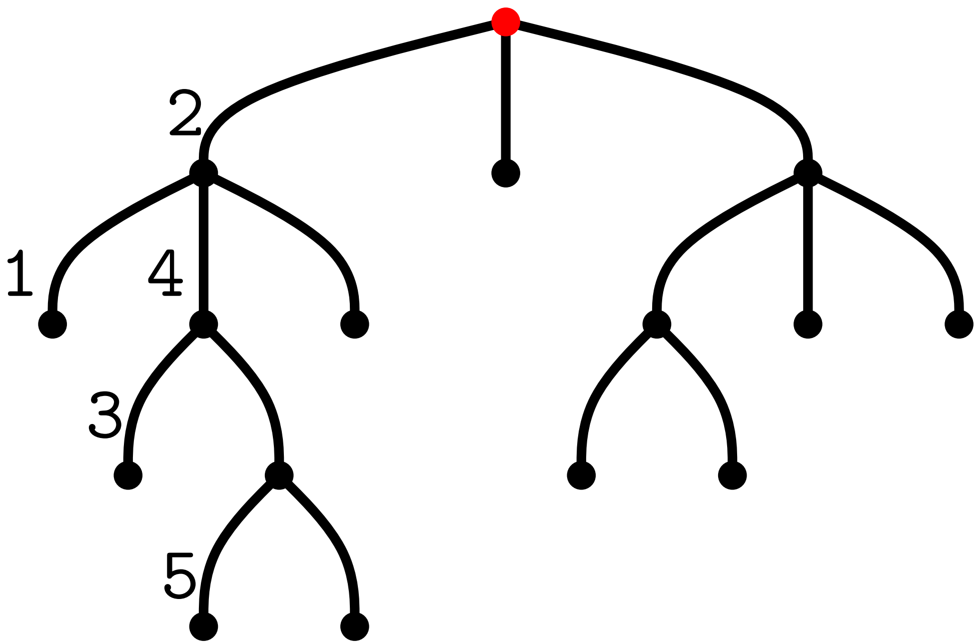

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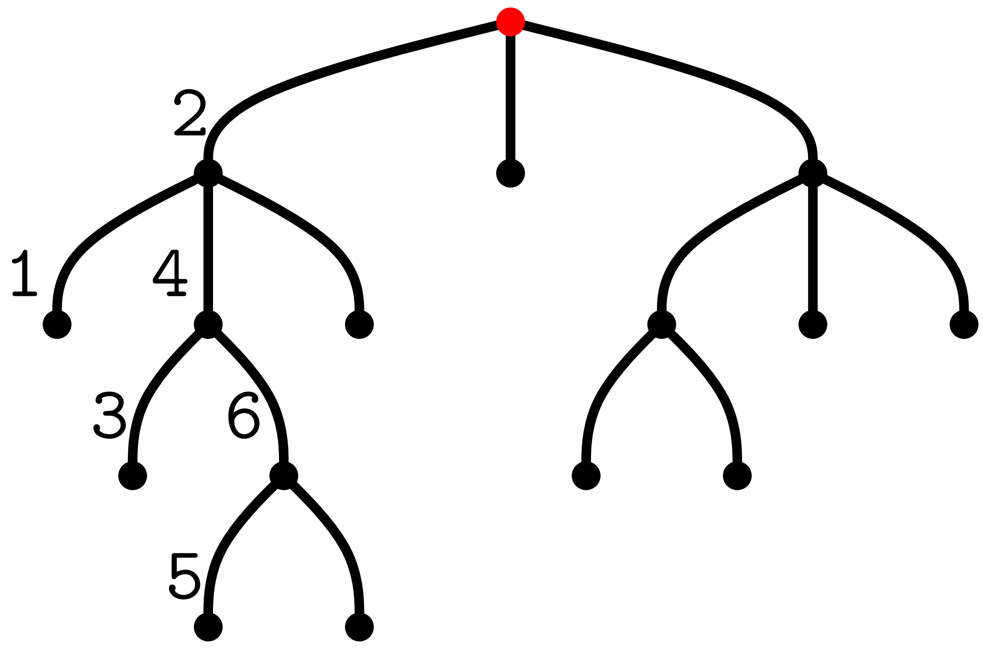

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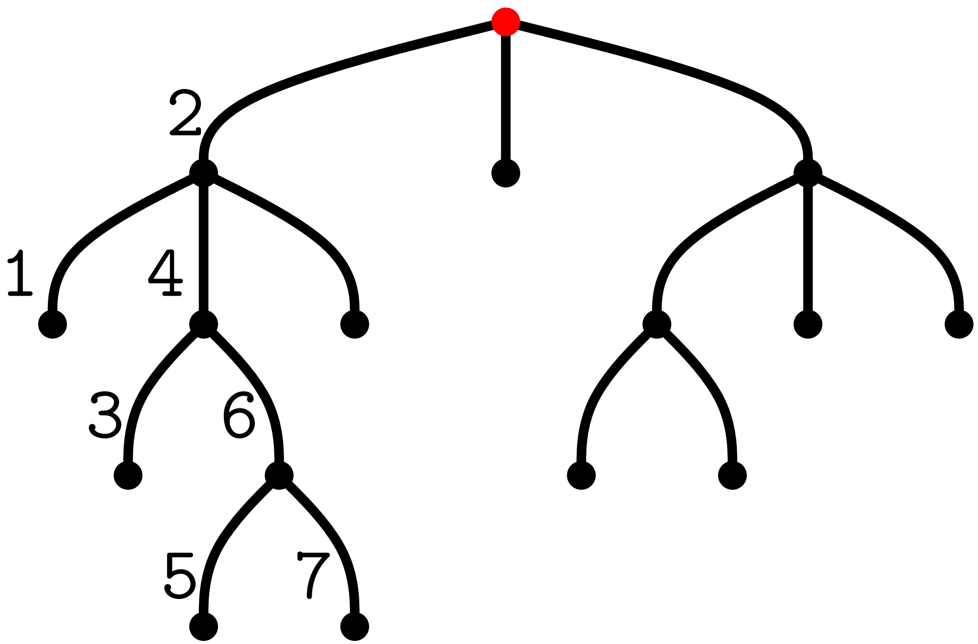

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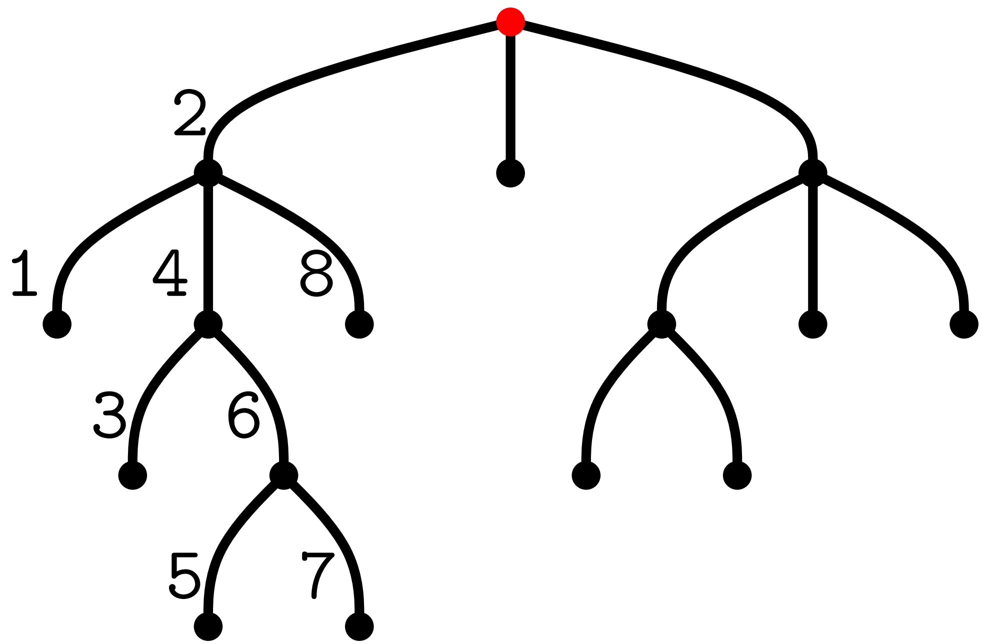

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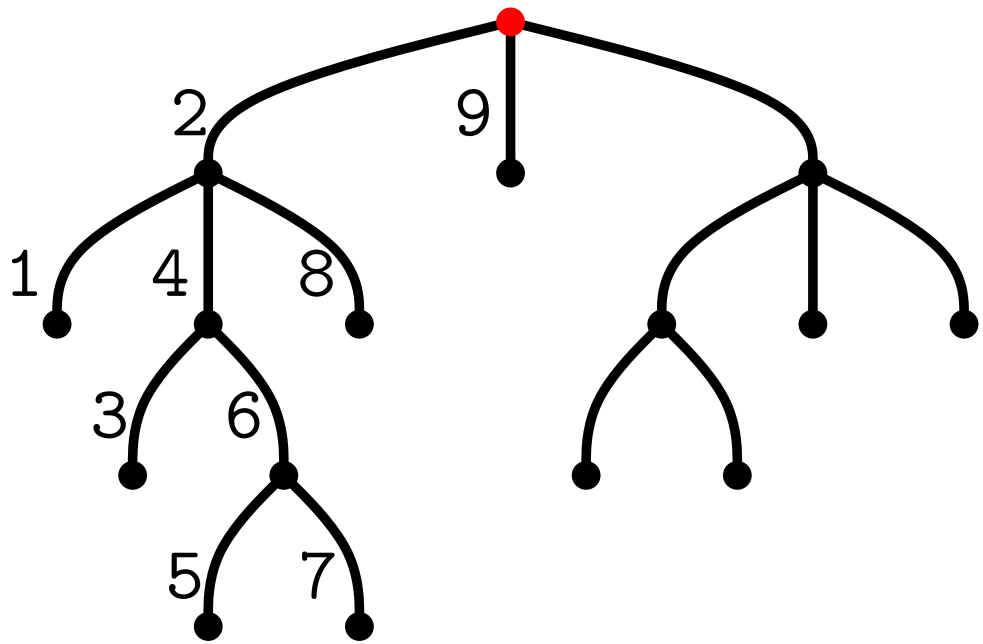

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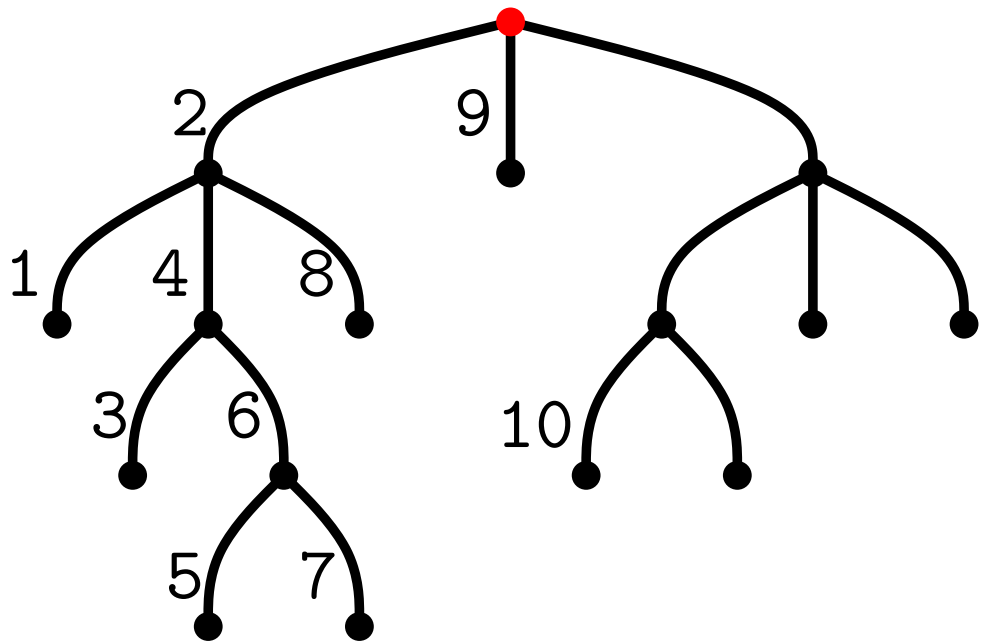

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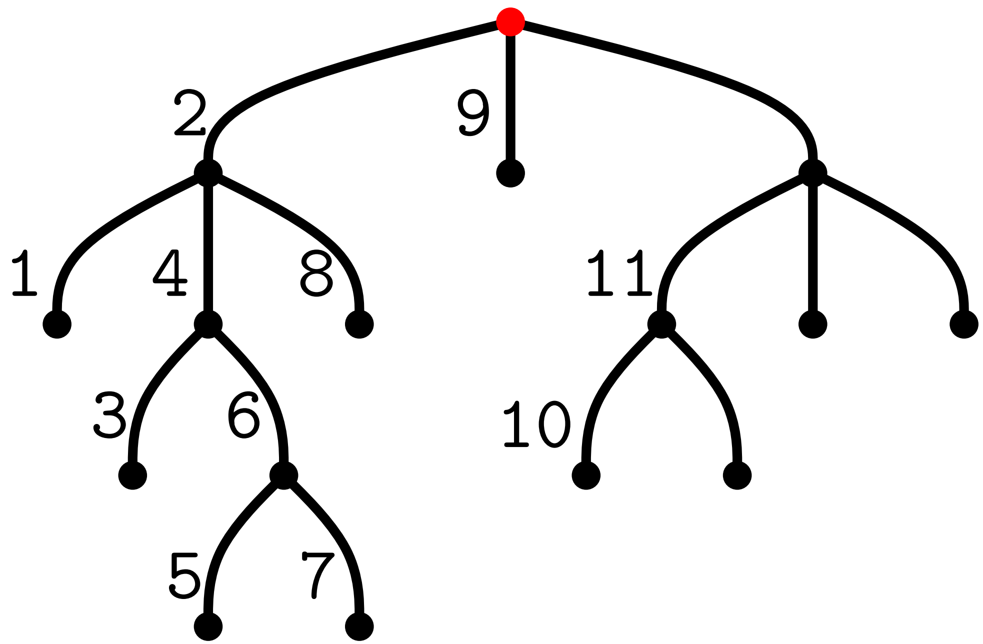

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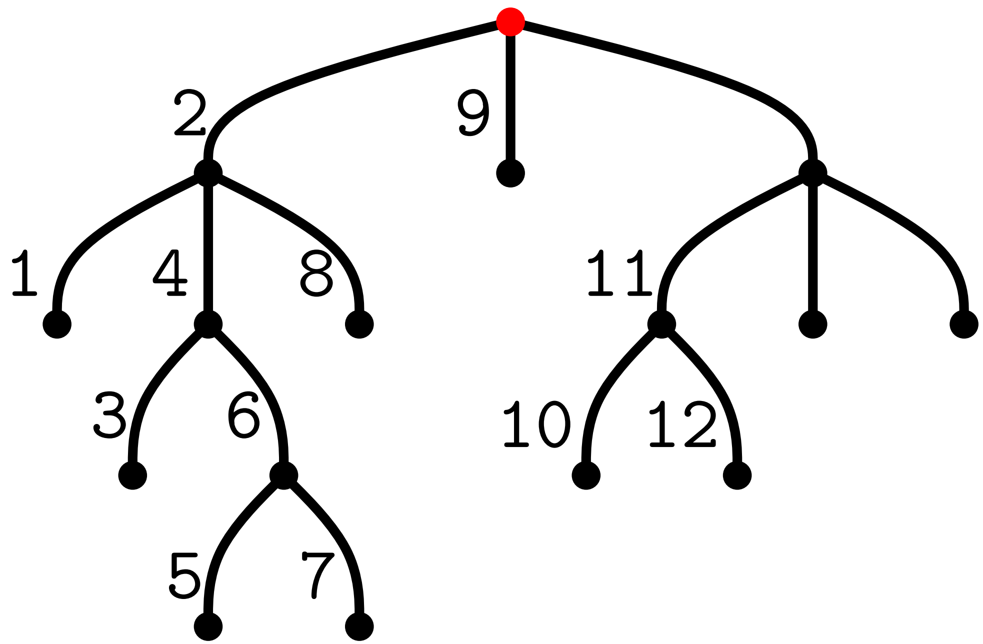

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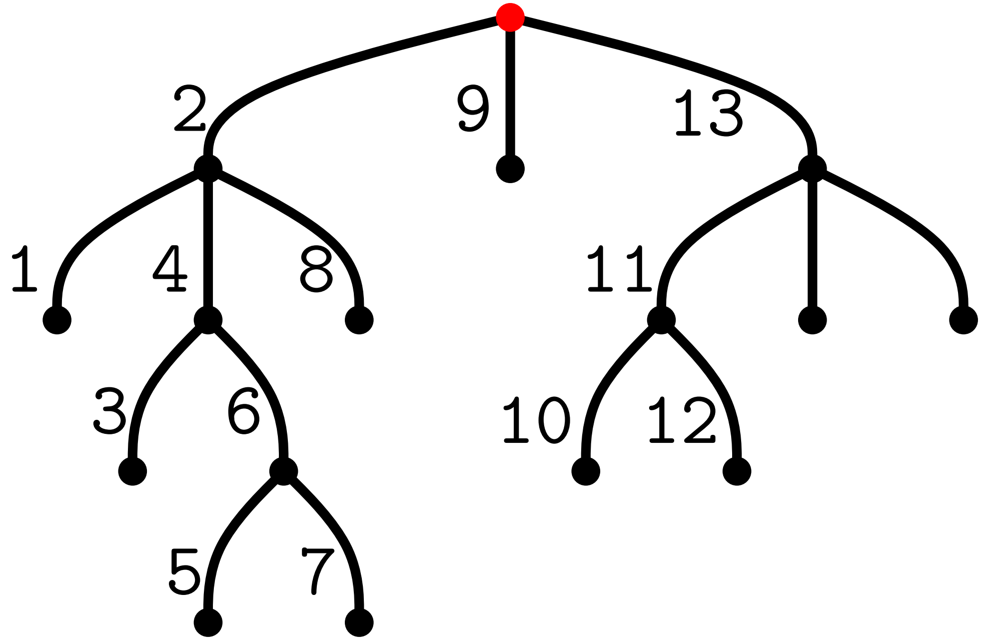

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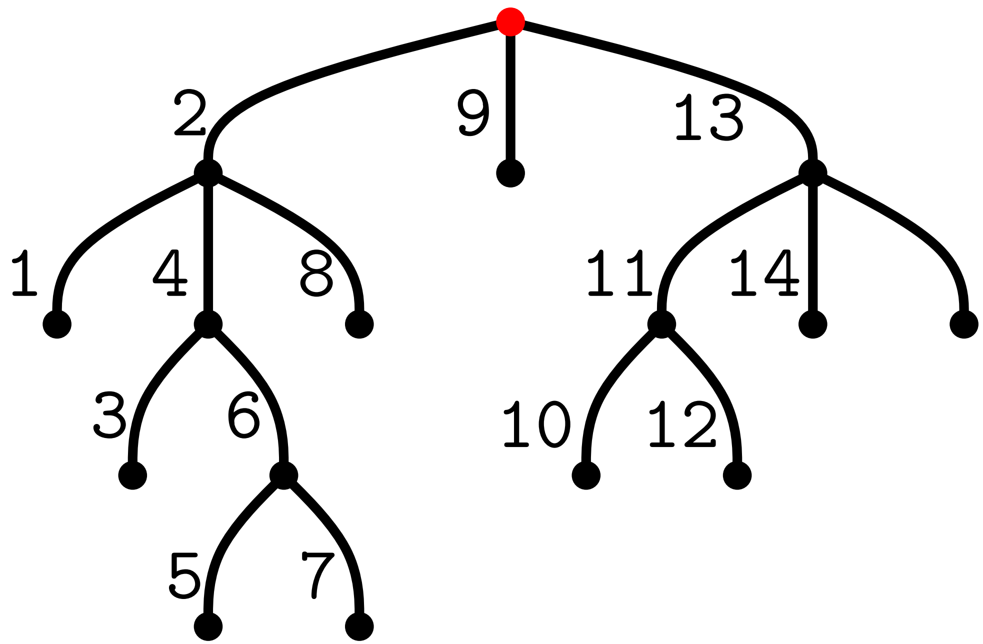

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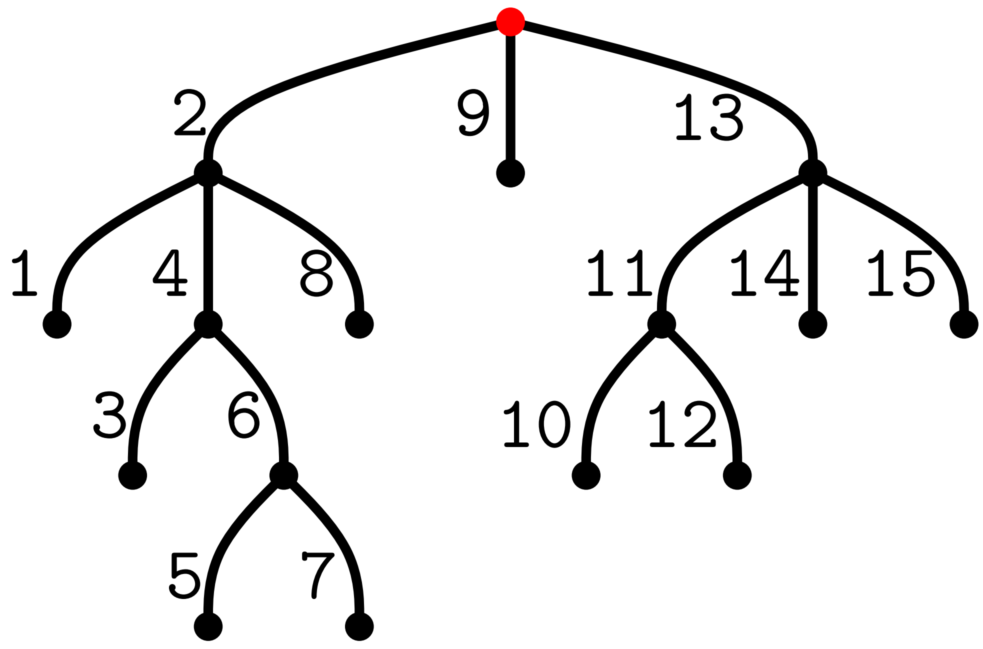

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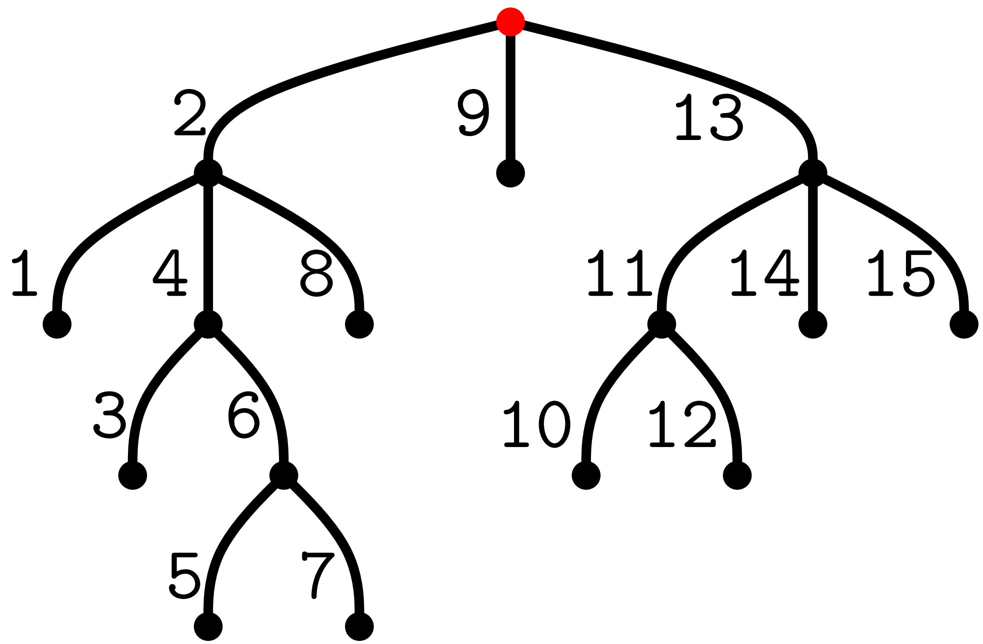
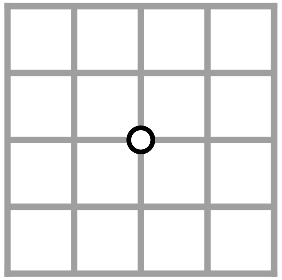


Step II: Primitive Vectors



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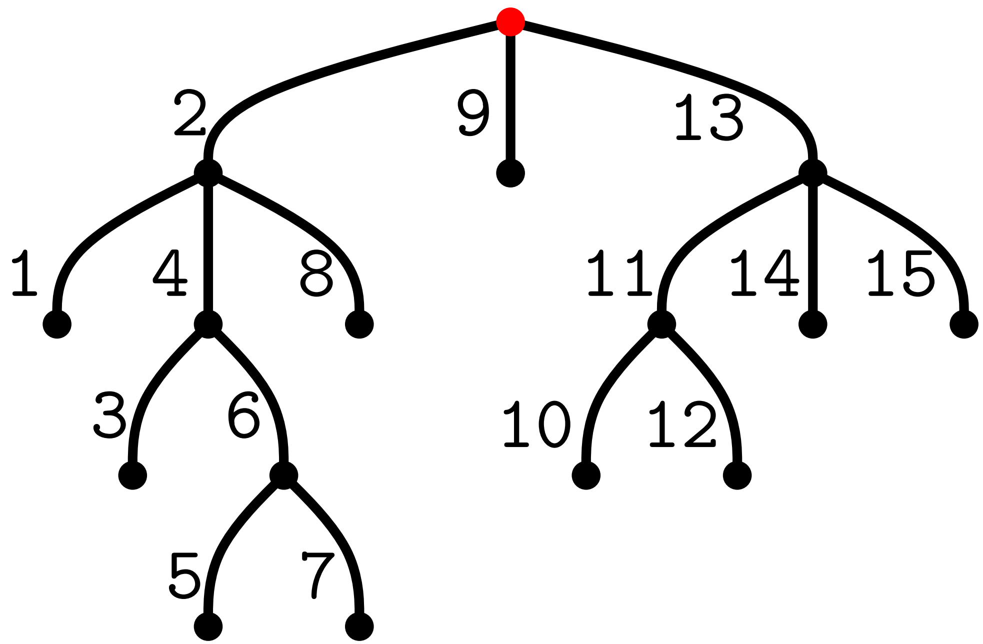

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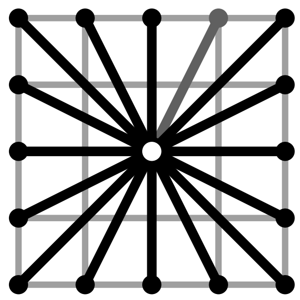

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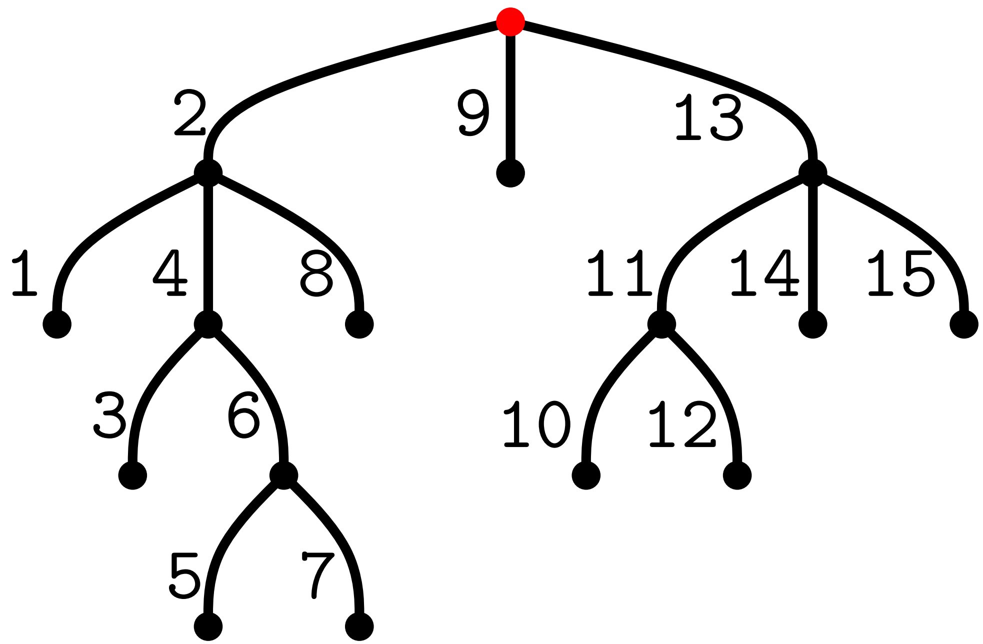

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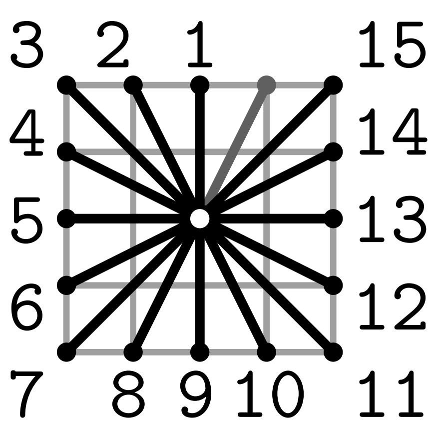

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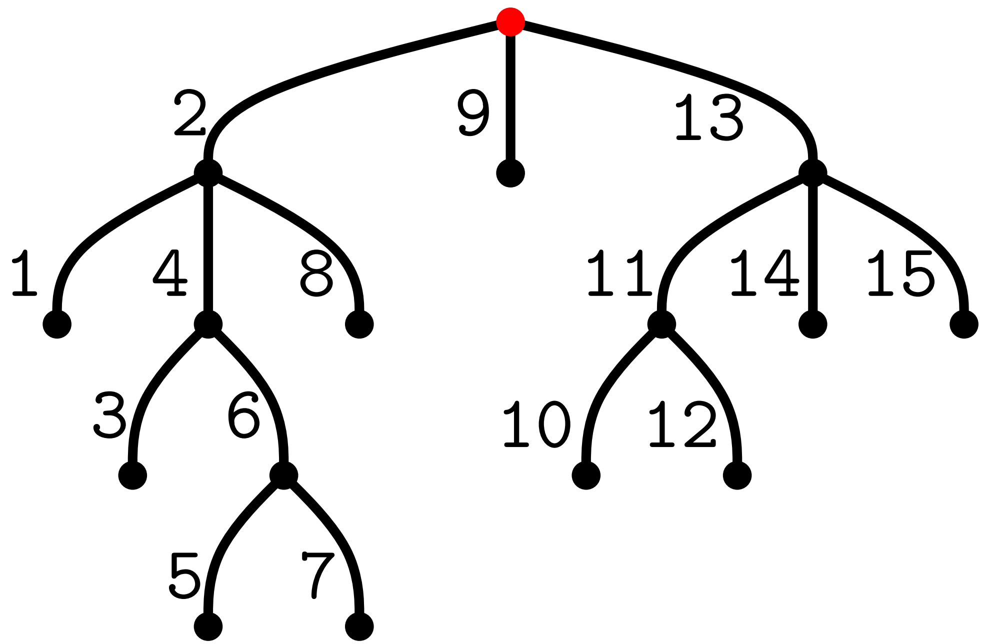
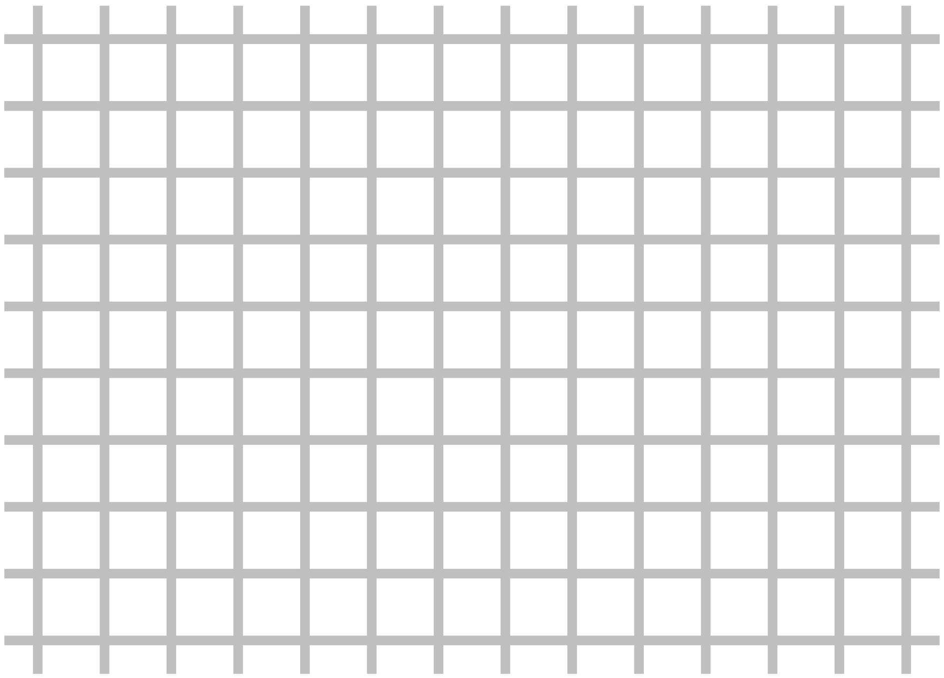


Step III: Draw Tree



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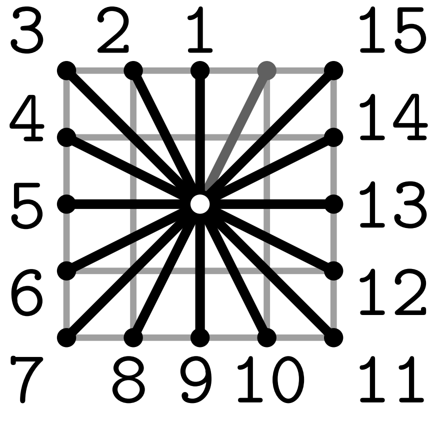

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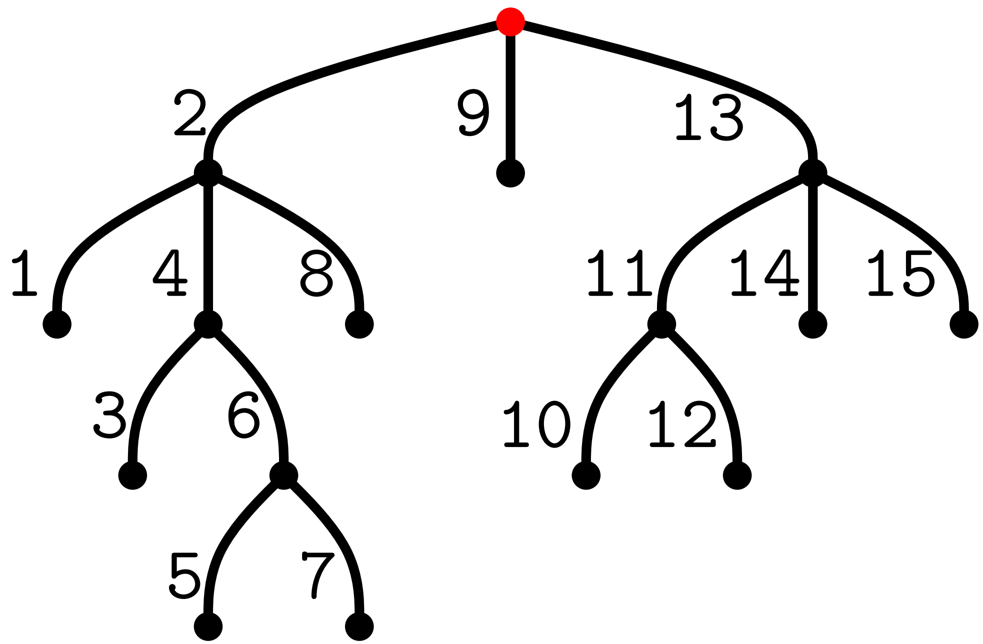

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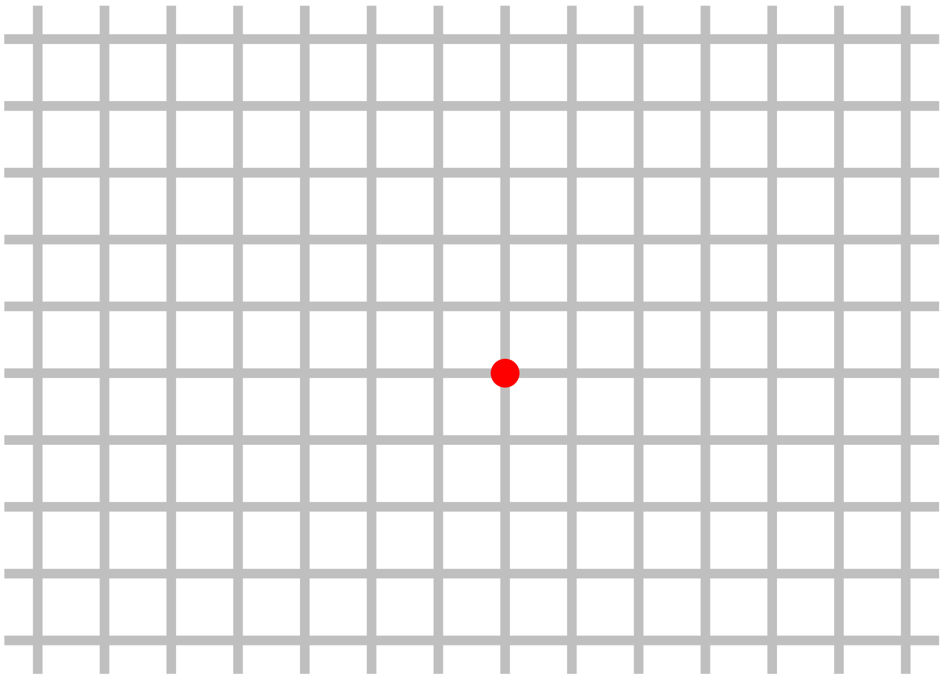

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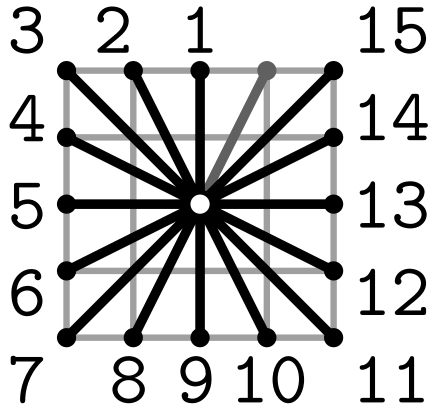

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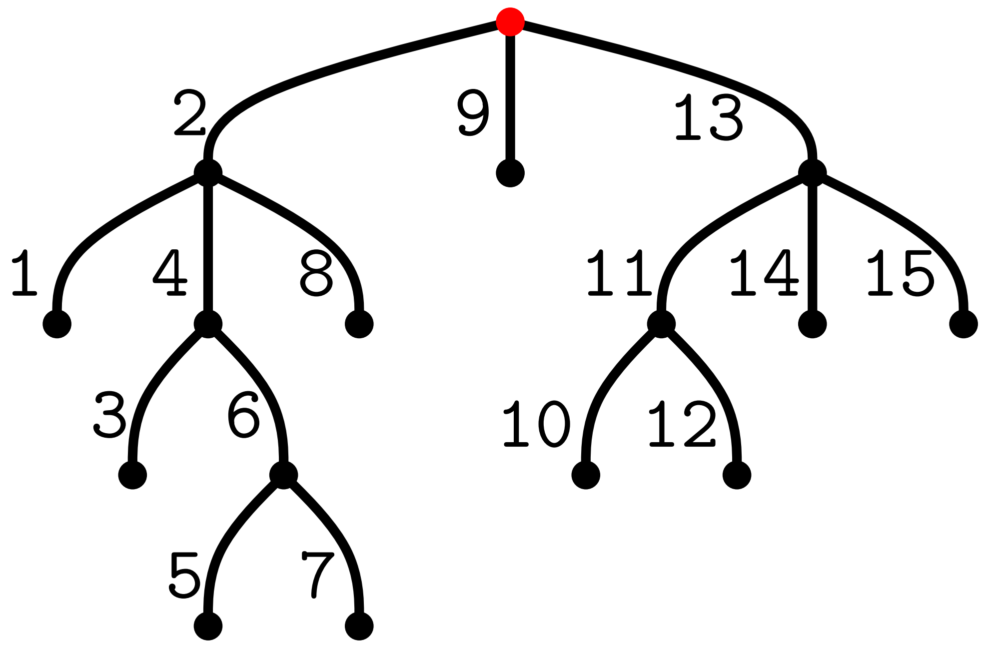

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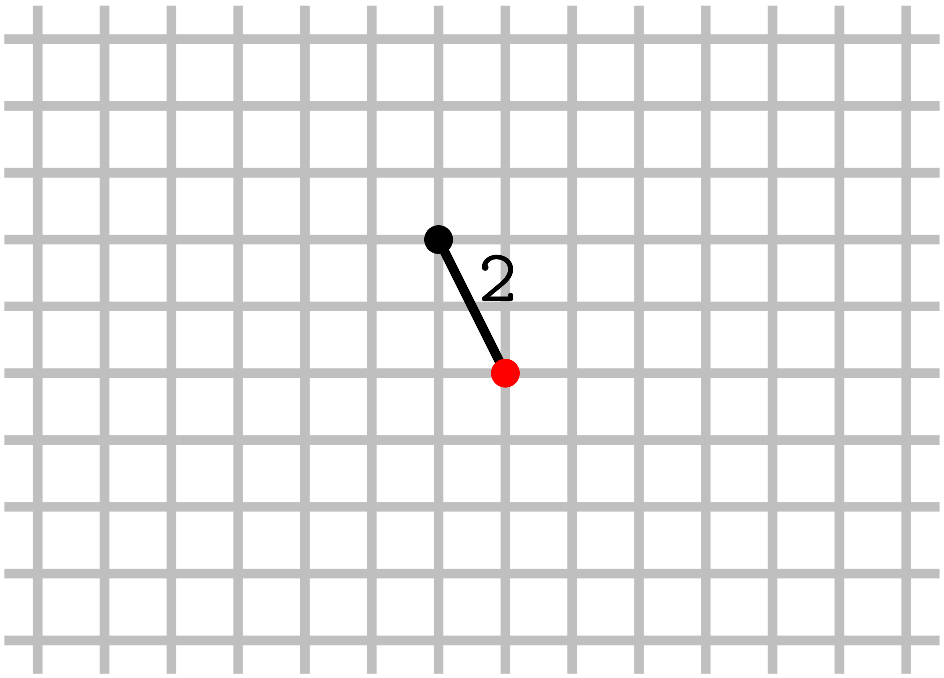

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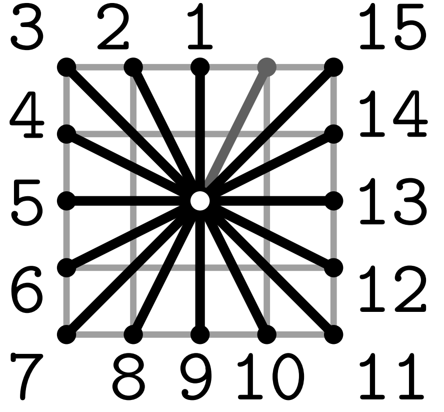

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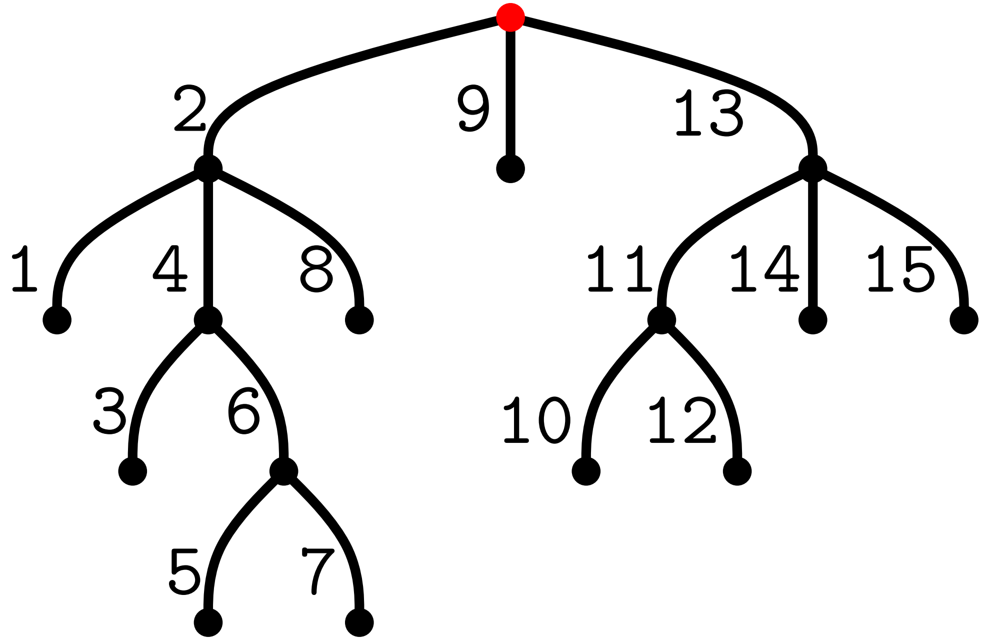

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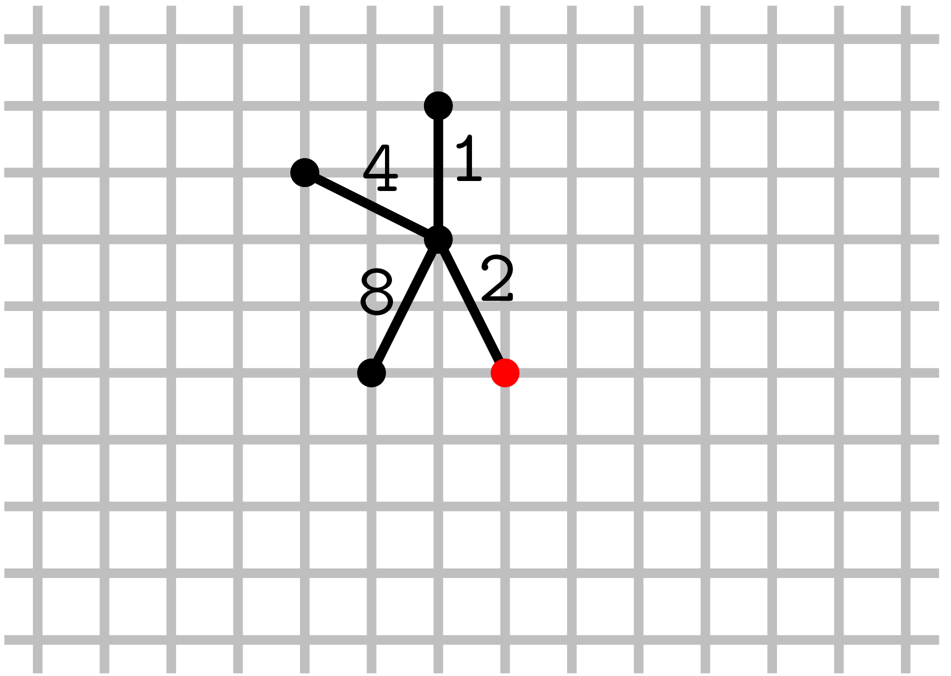

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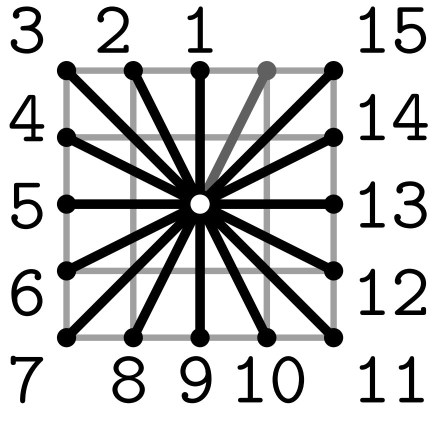

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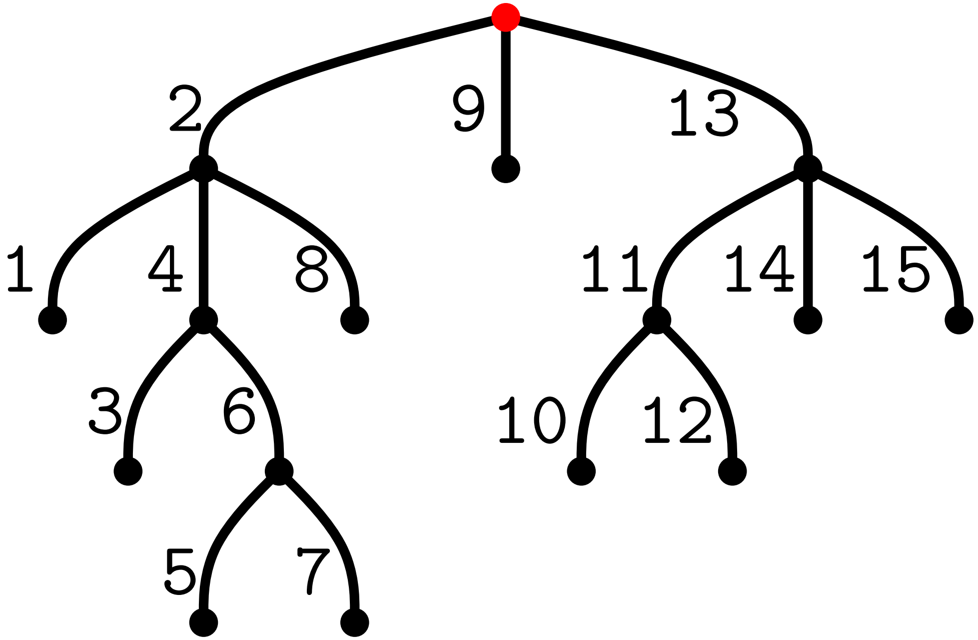

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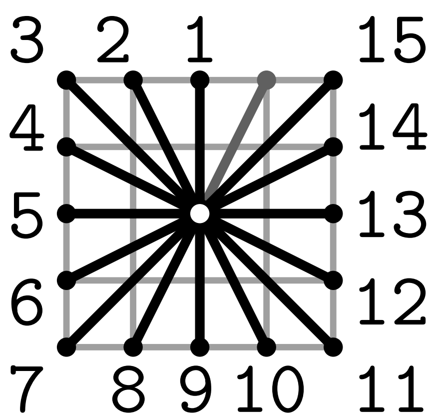

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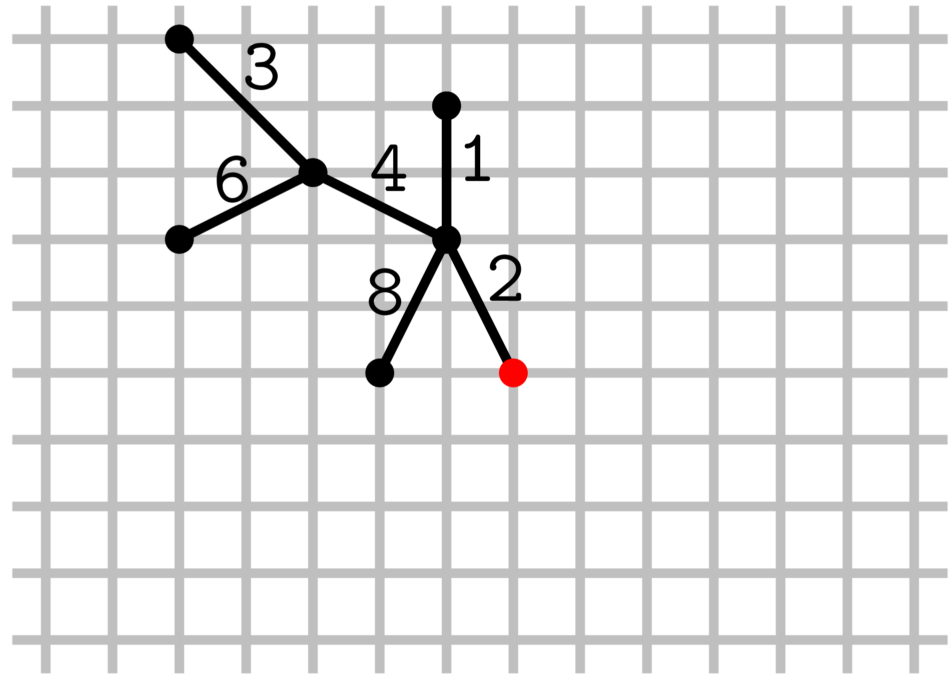

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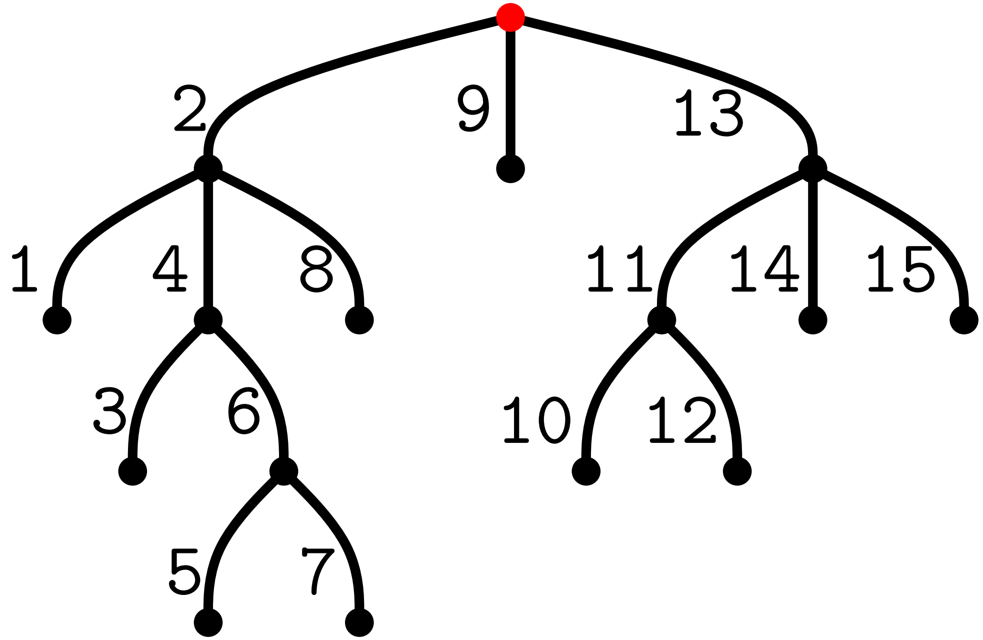

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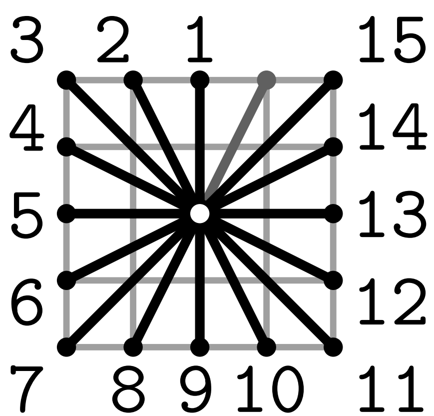

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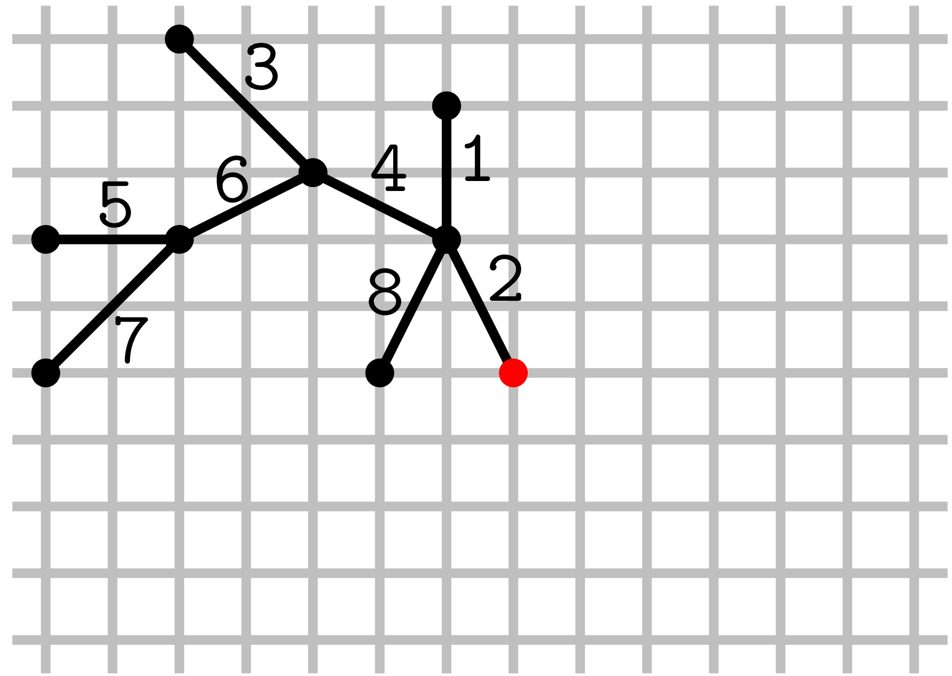

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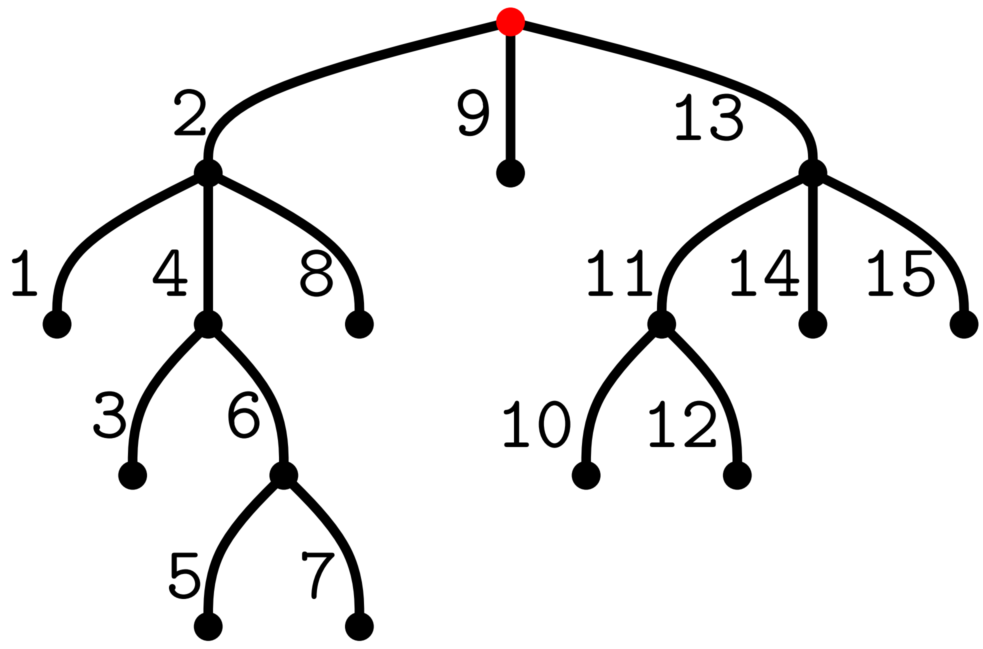

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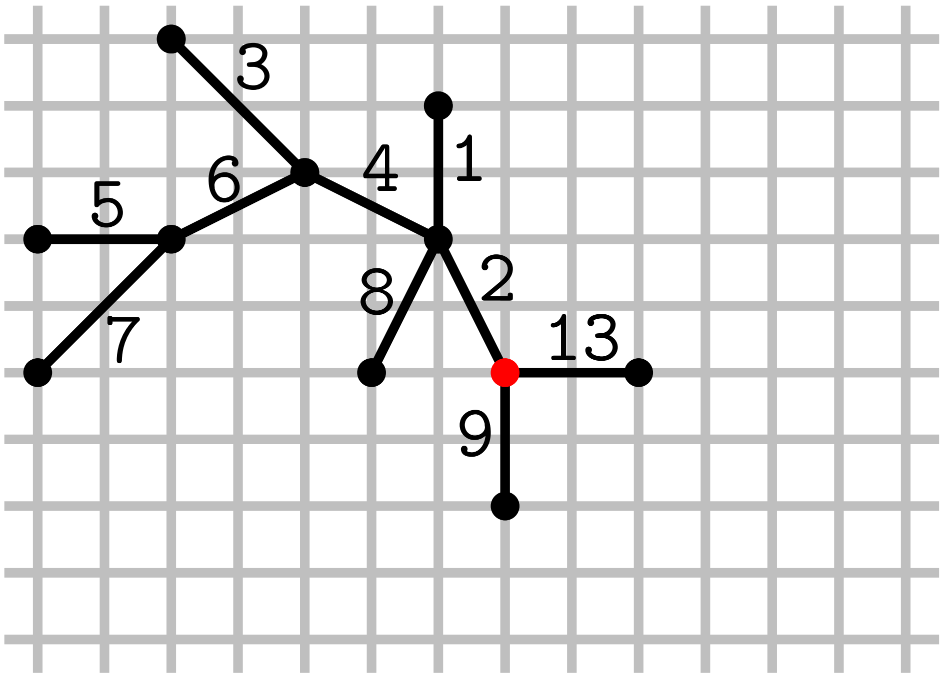

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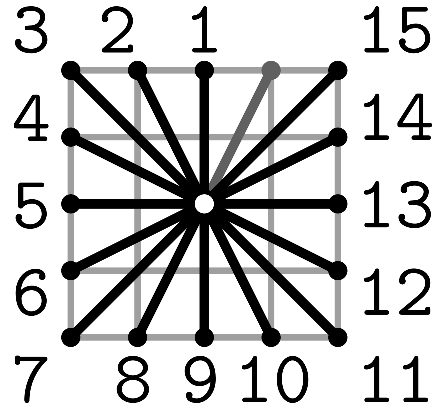

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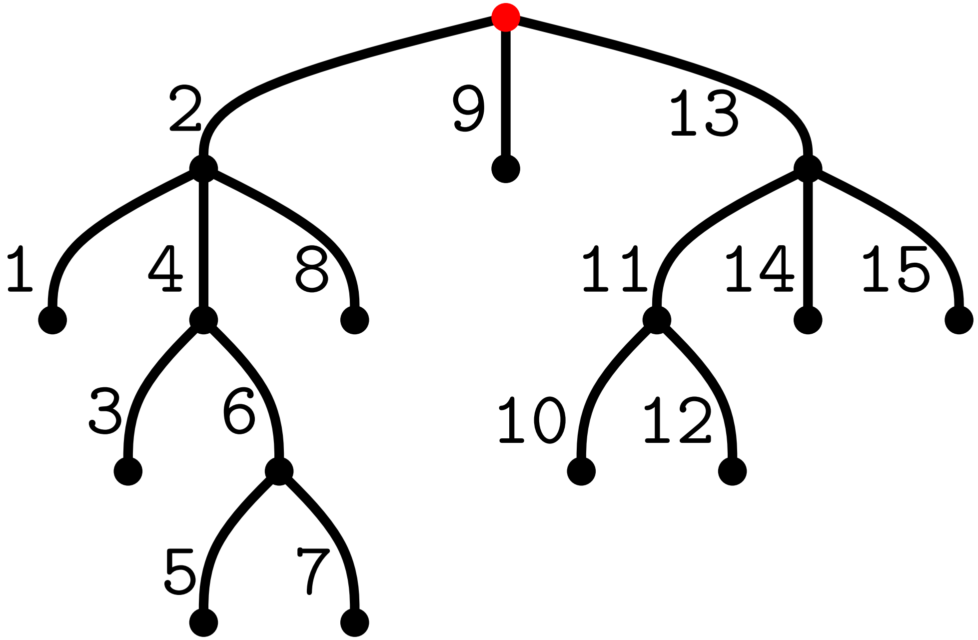

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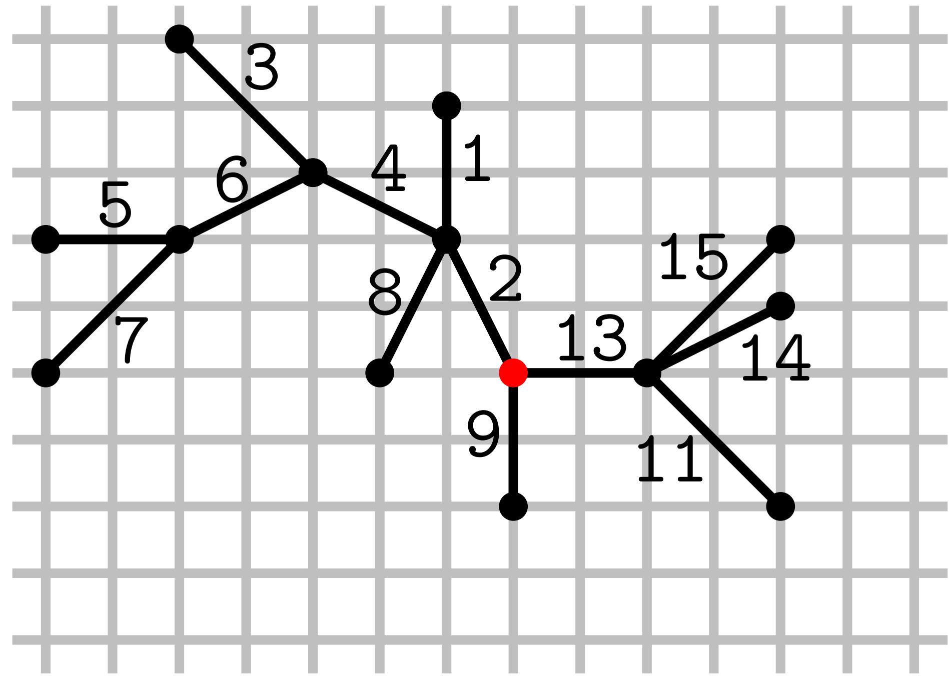

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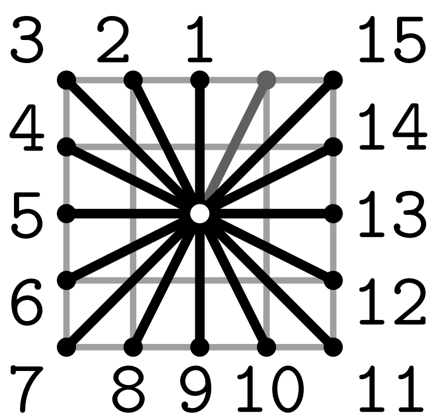

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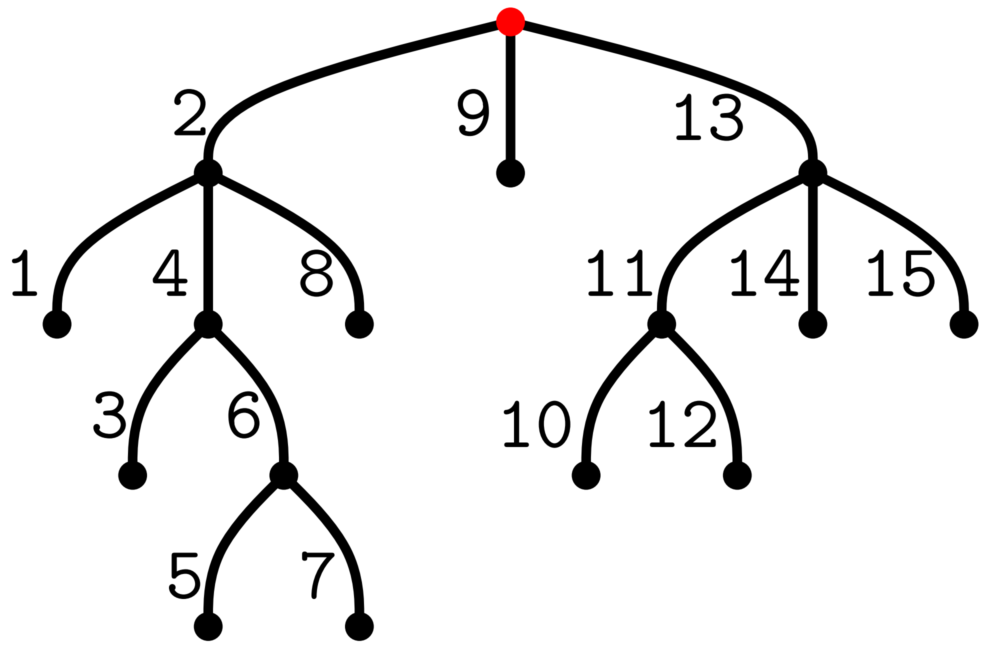

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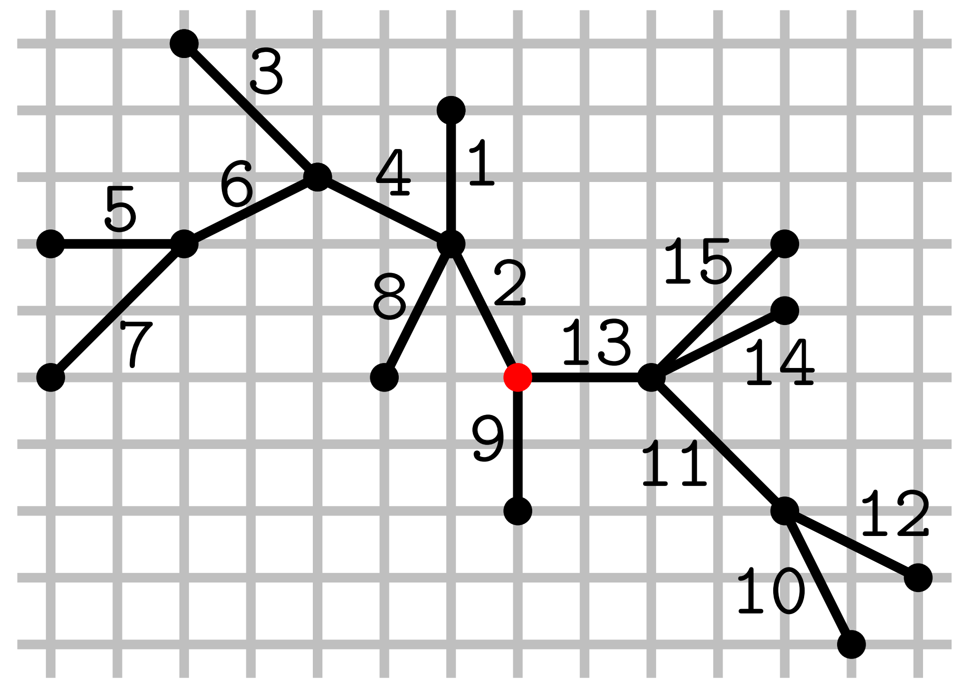

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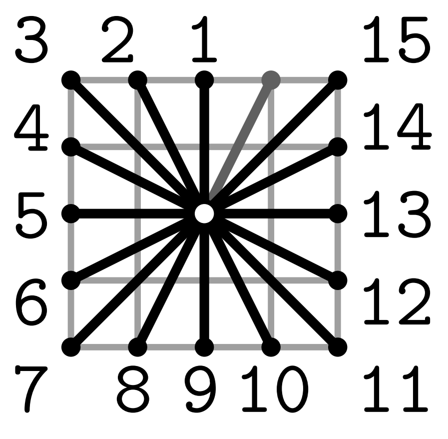

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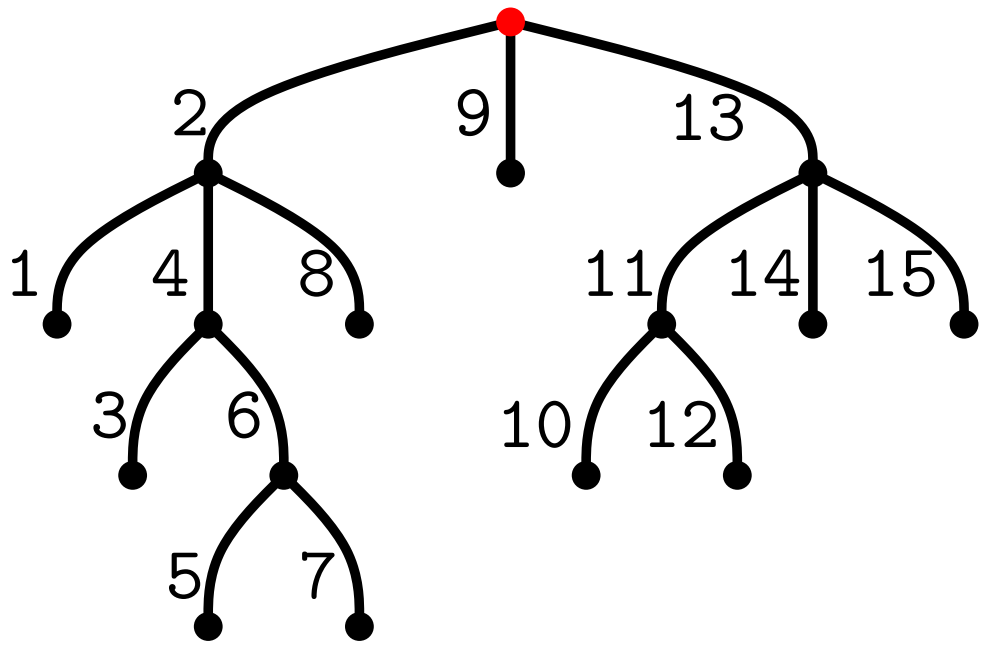

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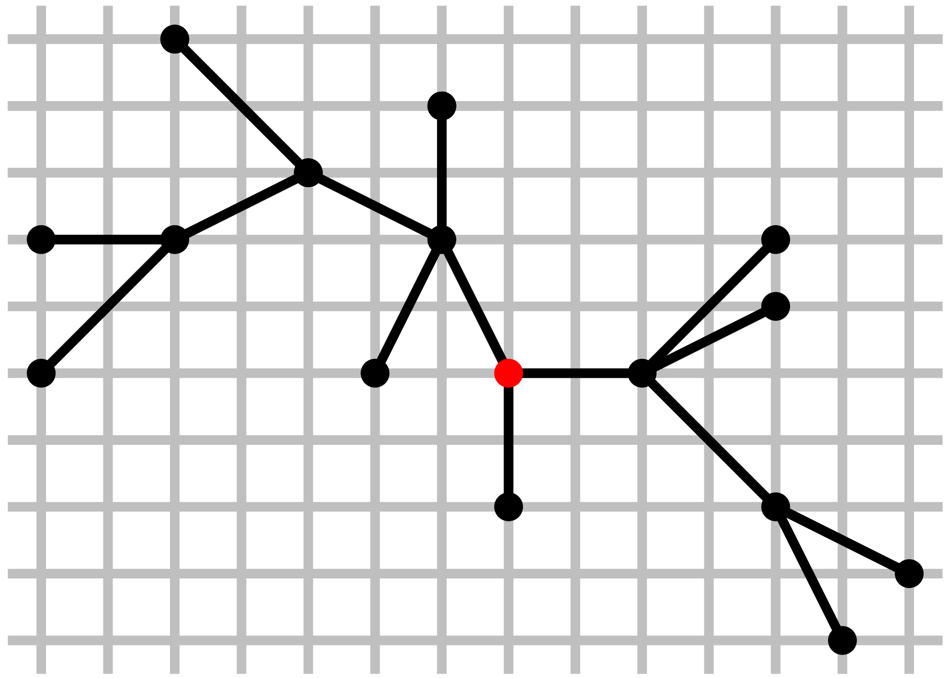

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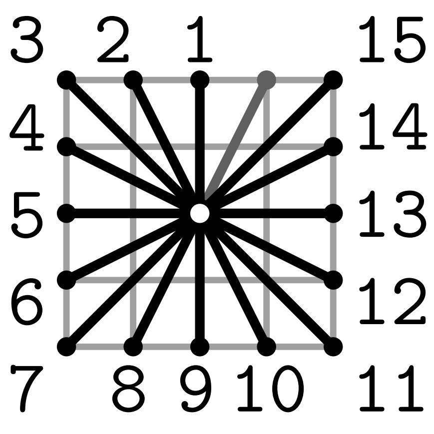

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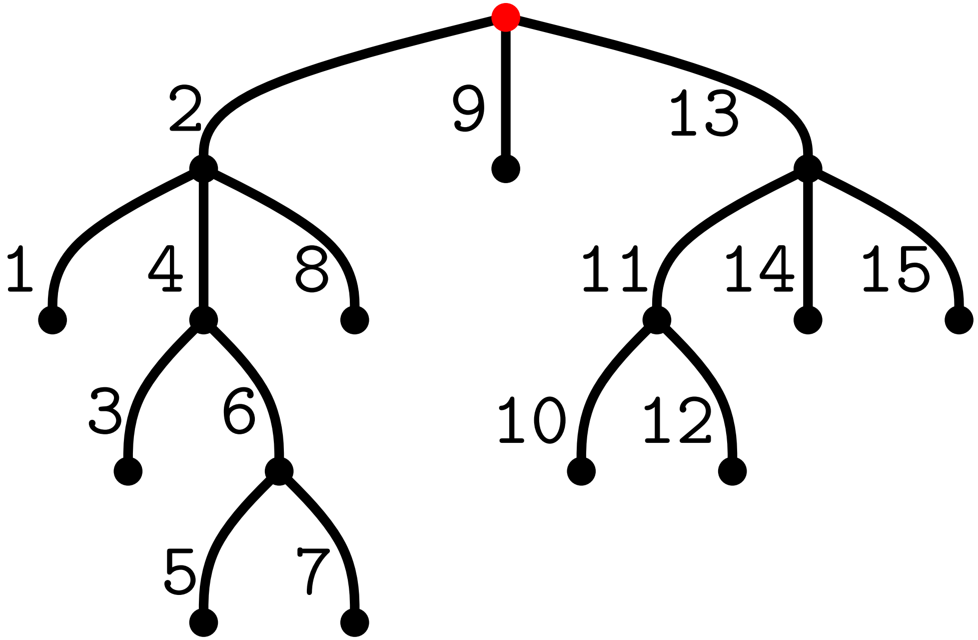

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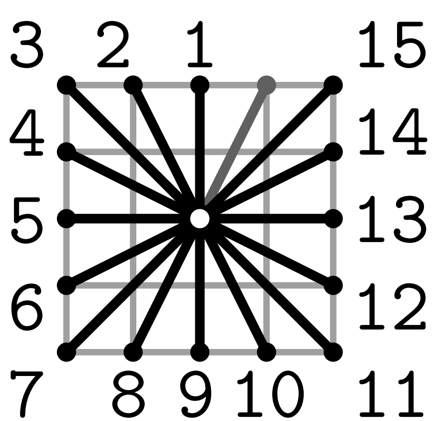

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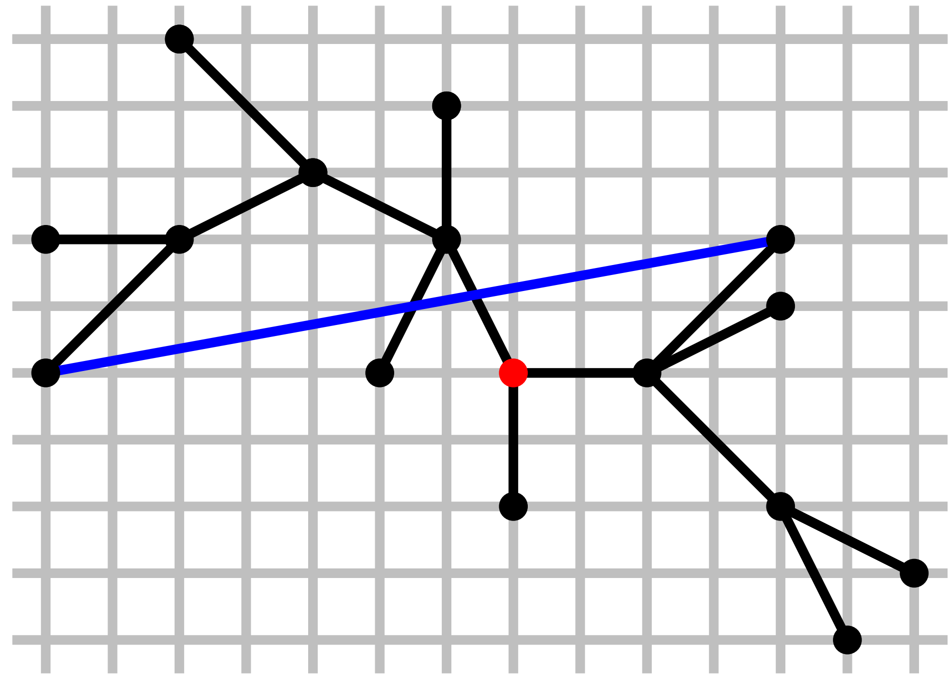

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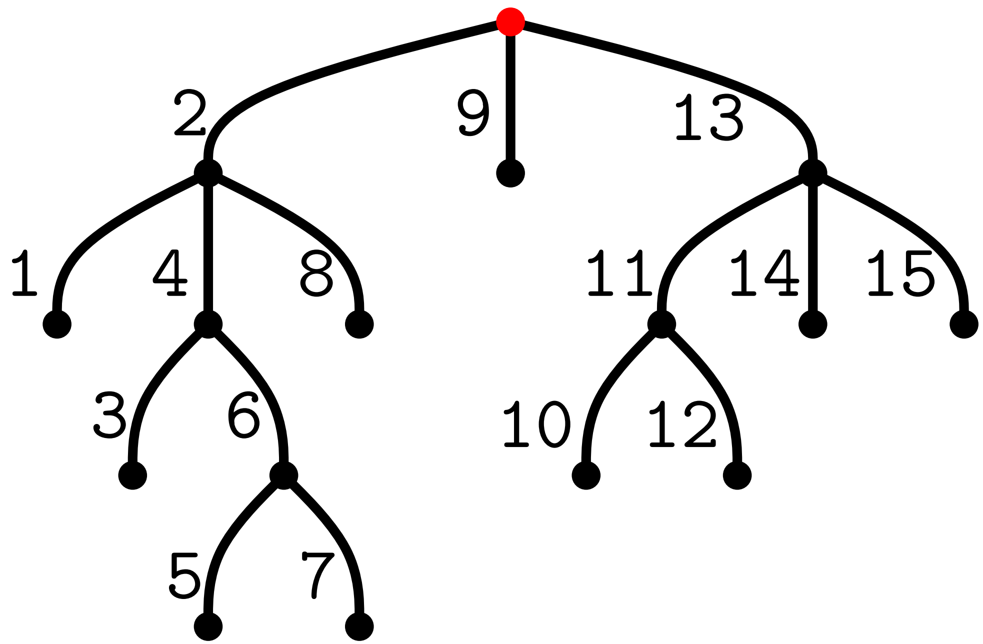

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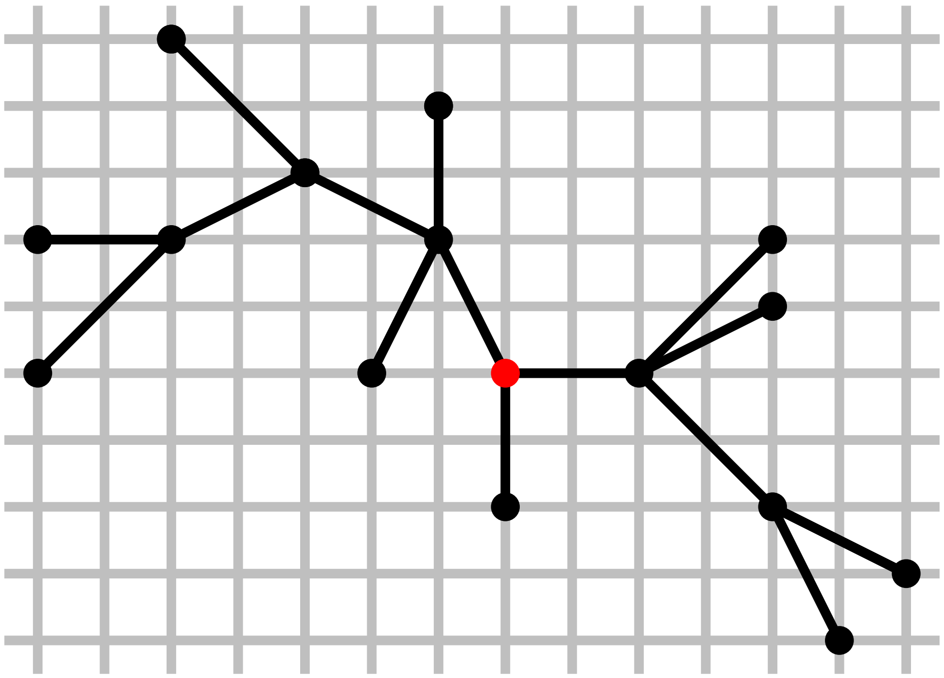

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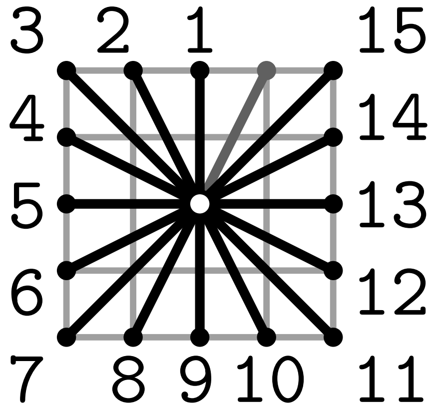

Theorem.
Every tree has a monotone and convex drawing on a grid of size $O(n^{1.5}) \times O(n^{1.5})$.

Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex

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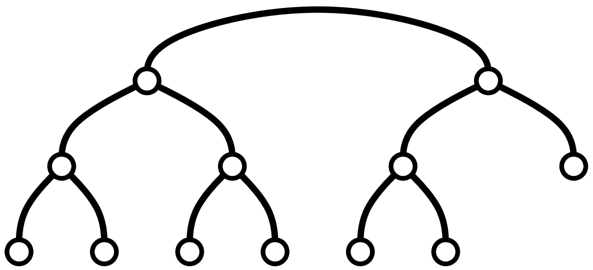
Proper Binary Trees: No degree-2 vertex



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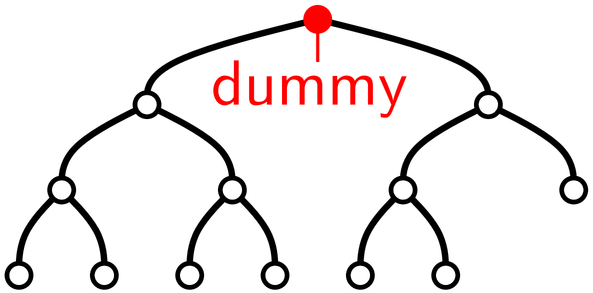

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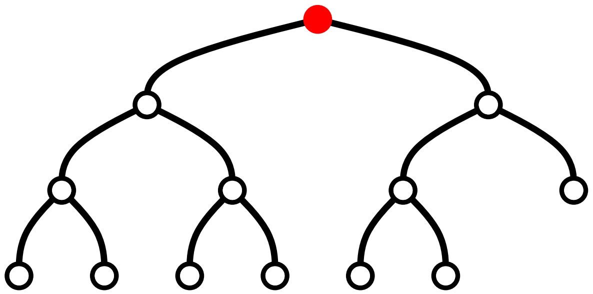

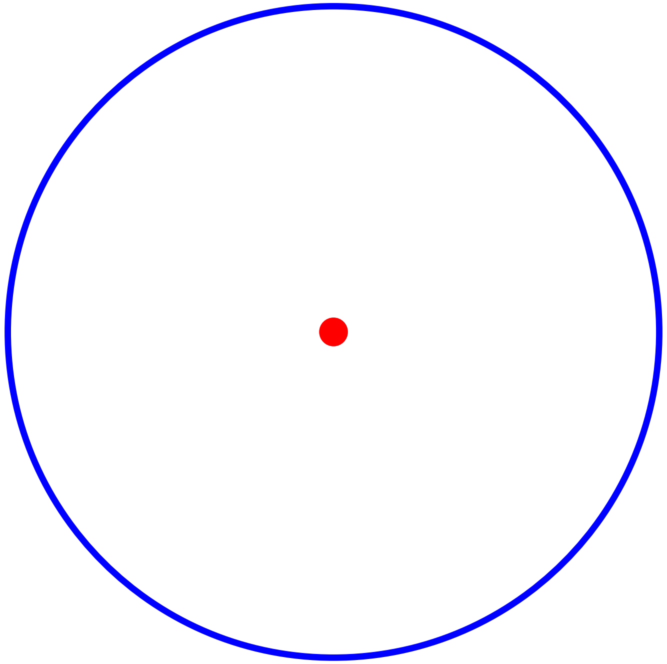

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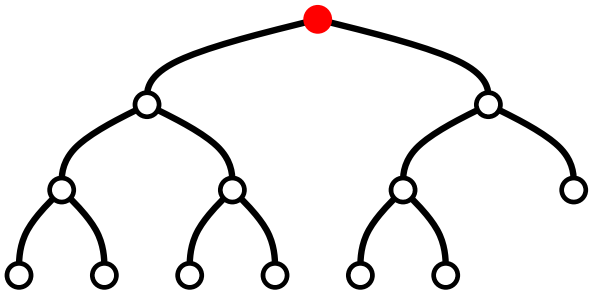

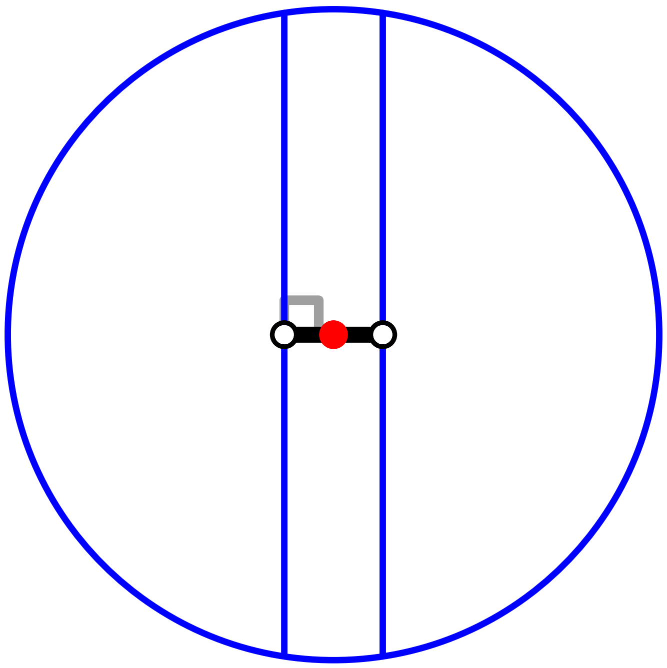

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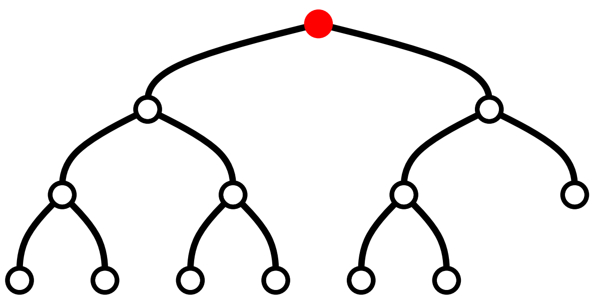

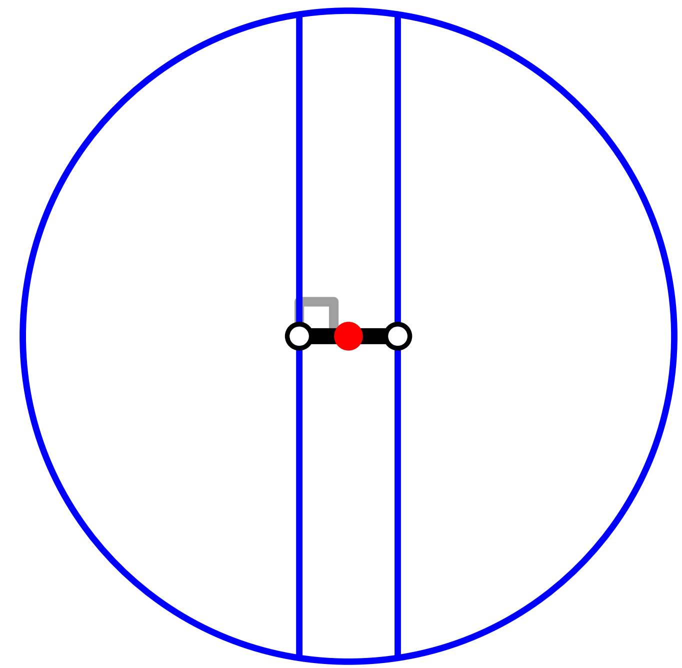

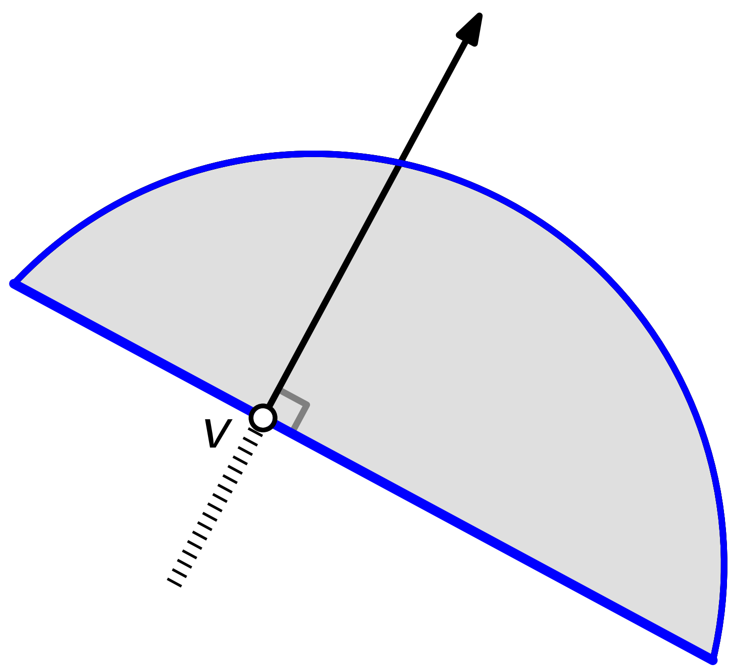

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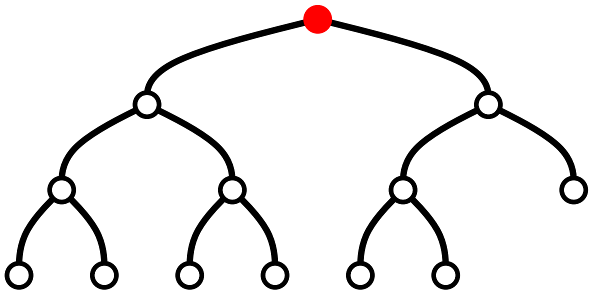

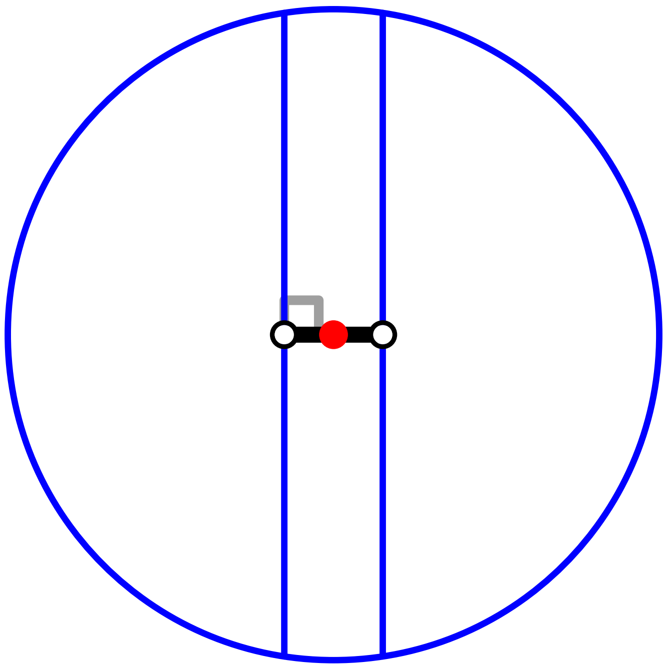

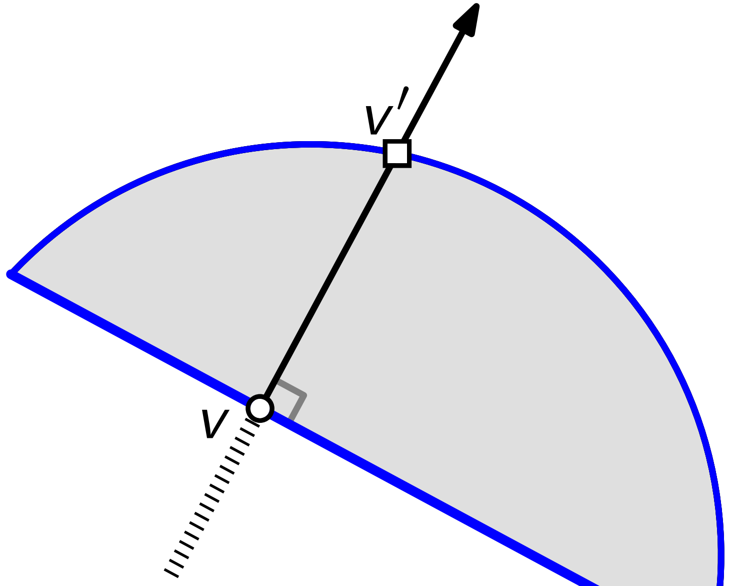

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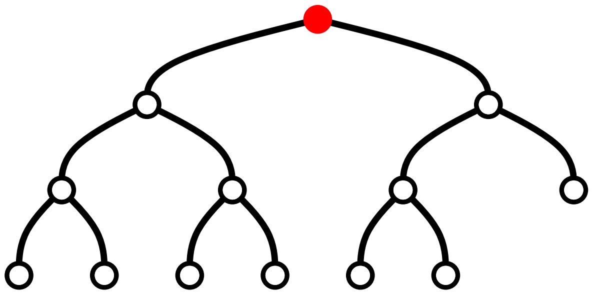

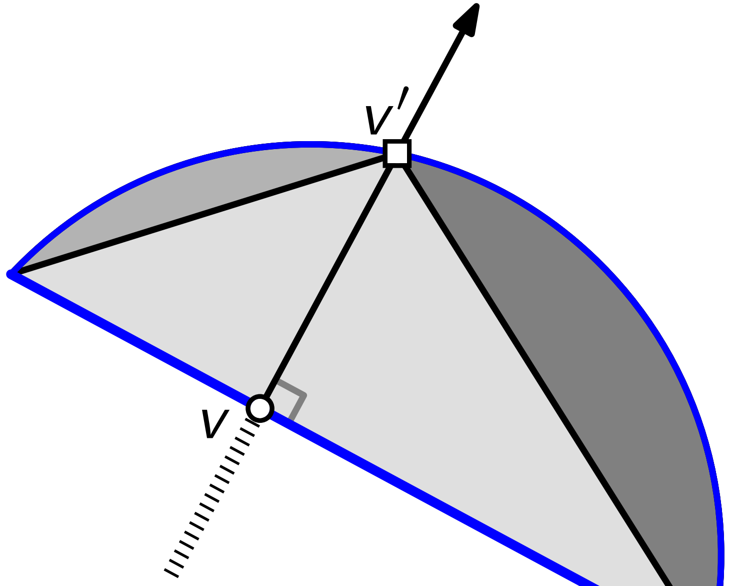

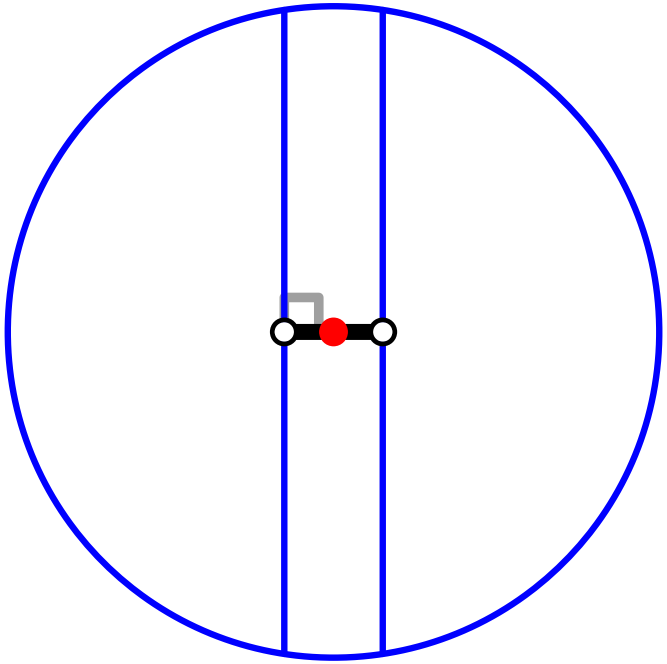

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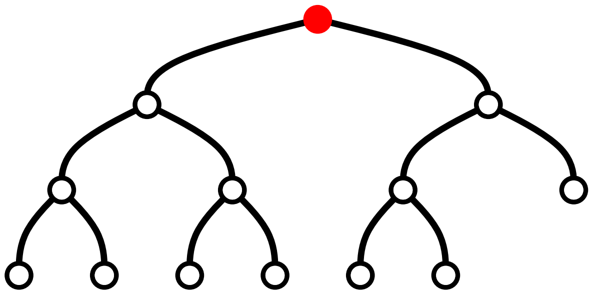

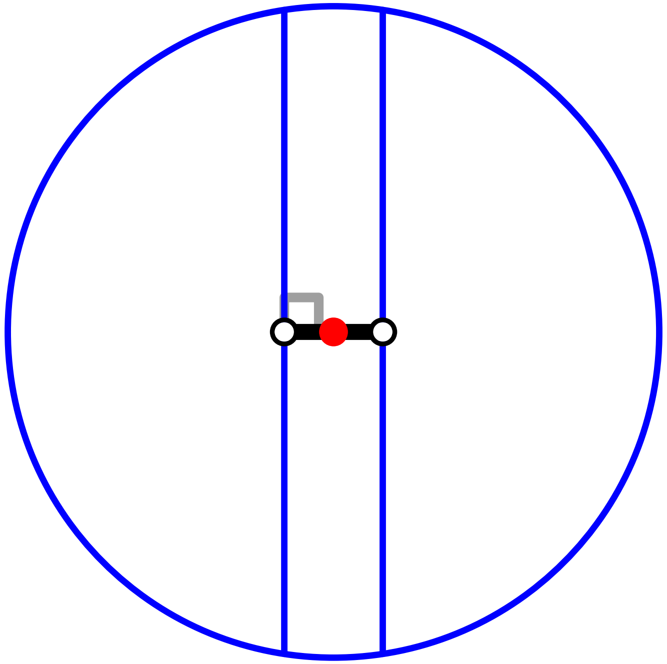

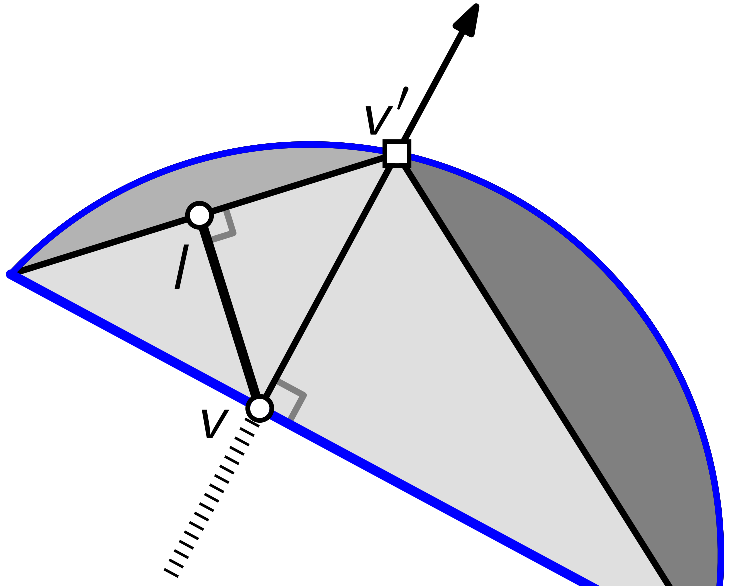

Strongly Monotone Drawings

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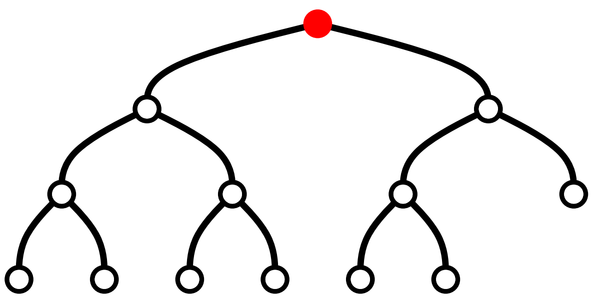

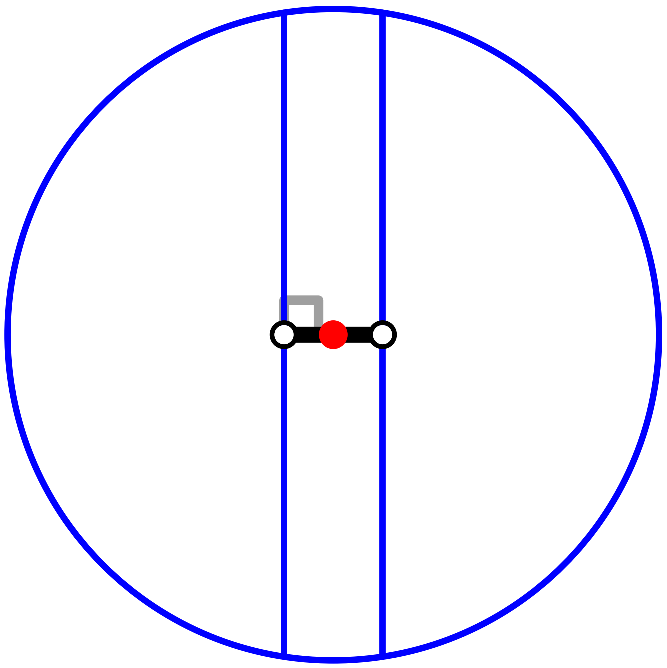

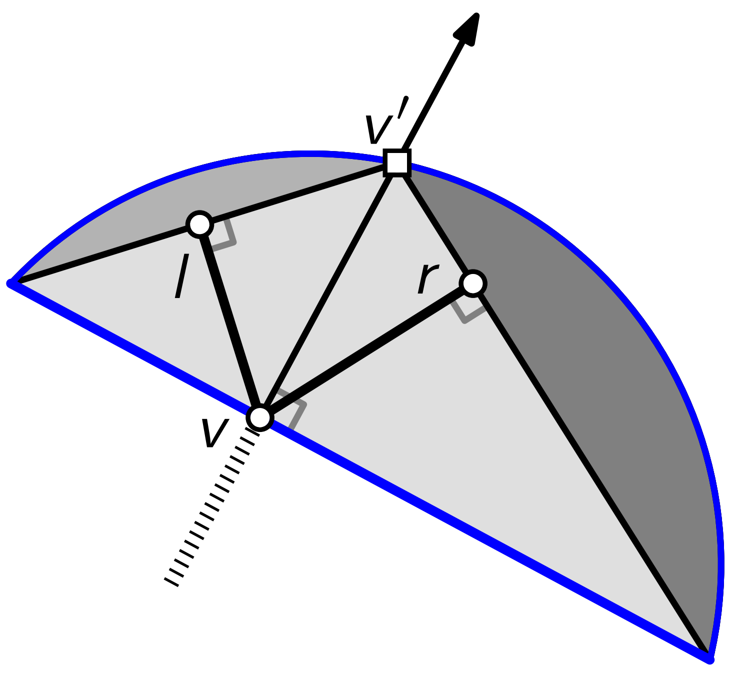

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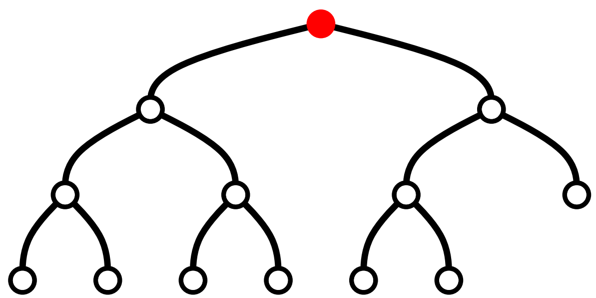

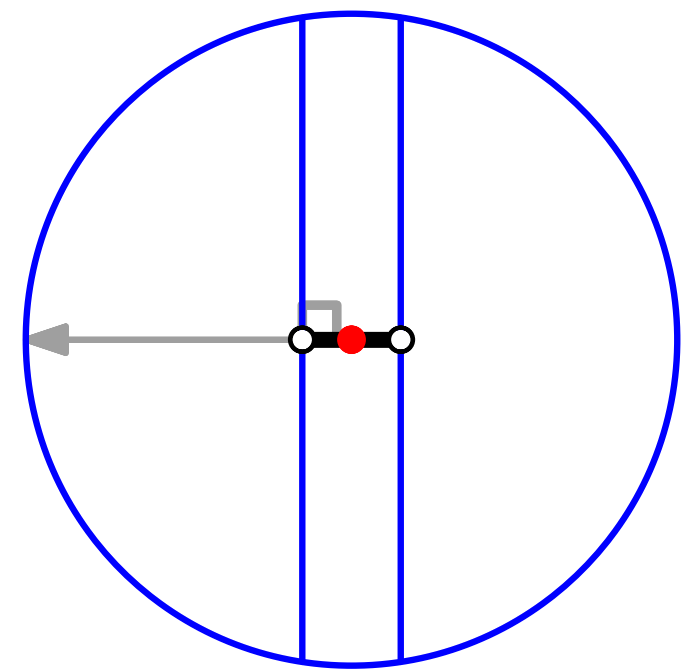

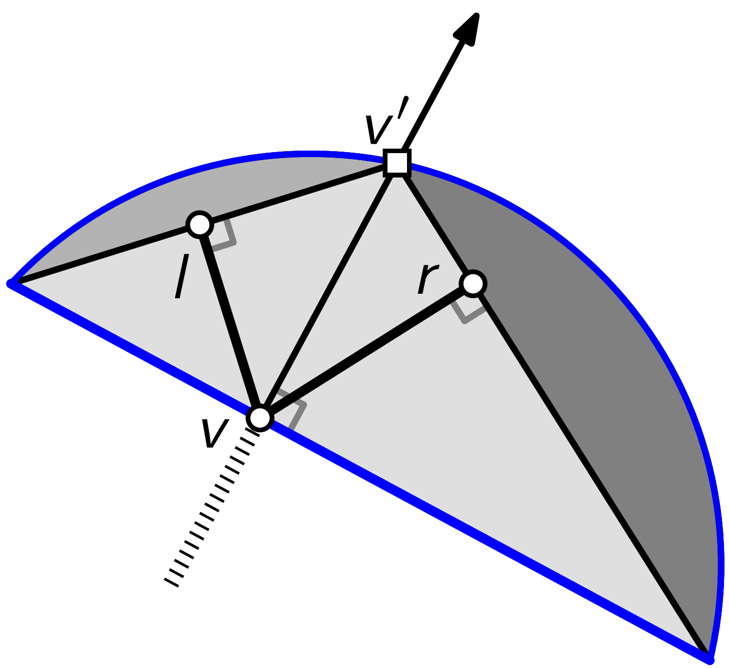

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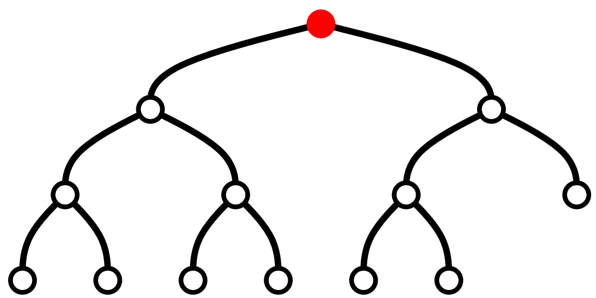

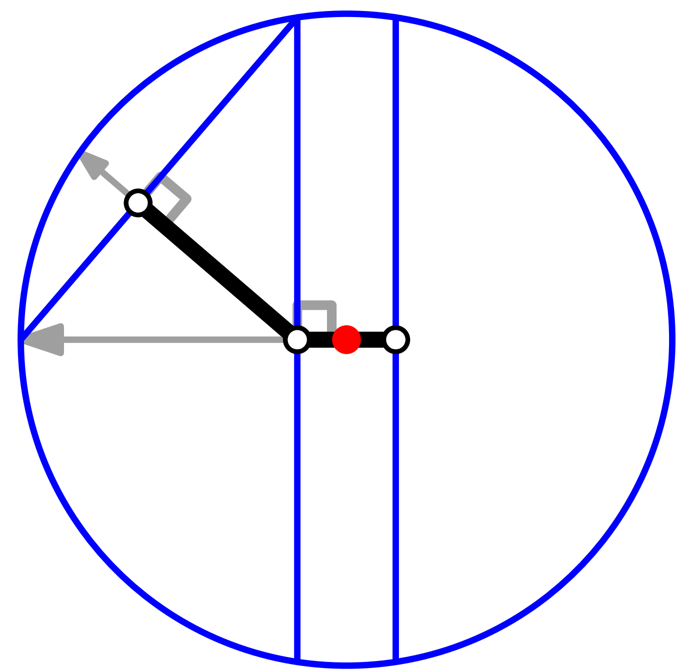

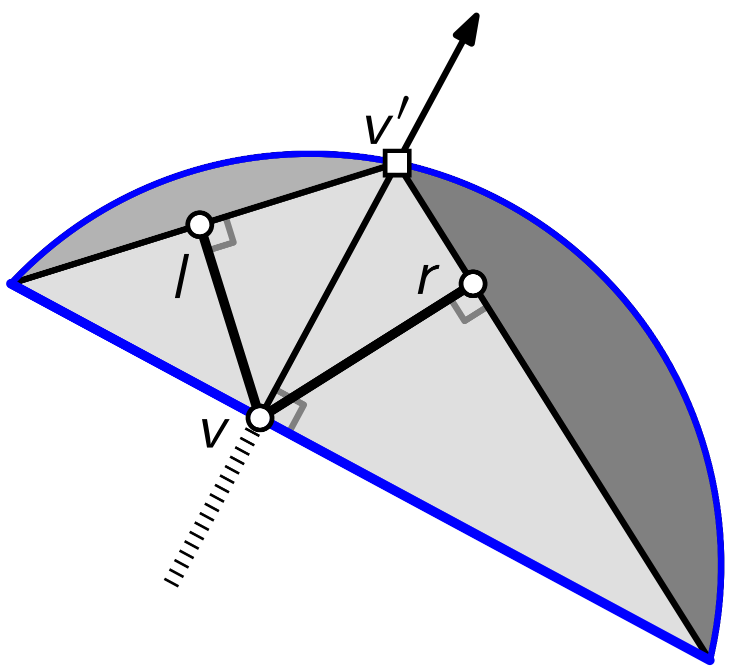

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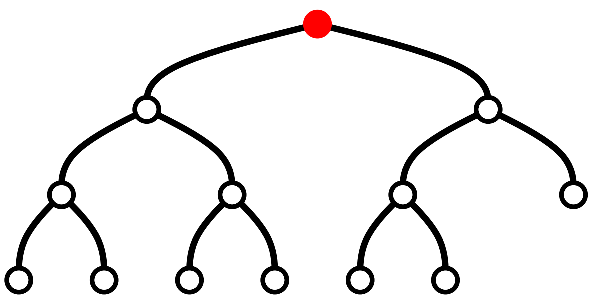

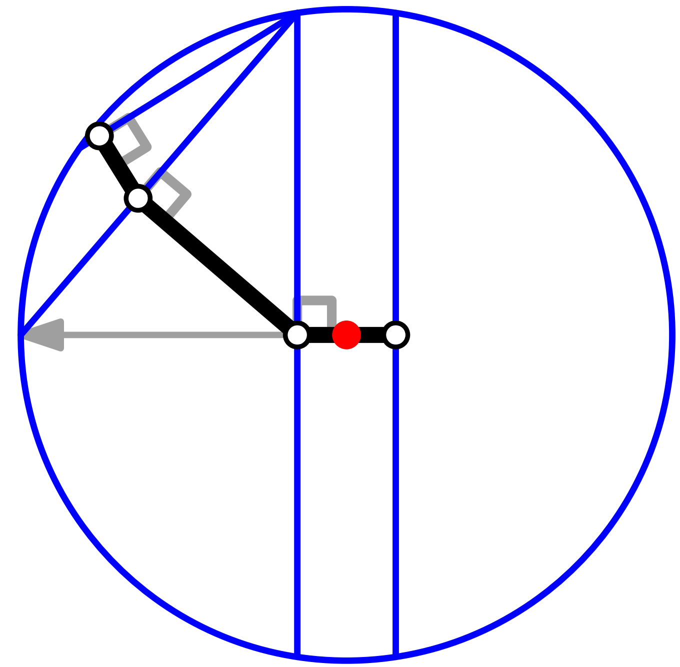

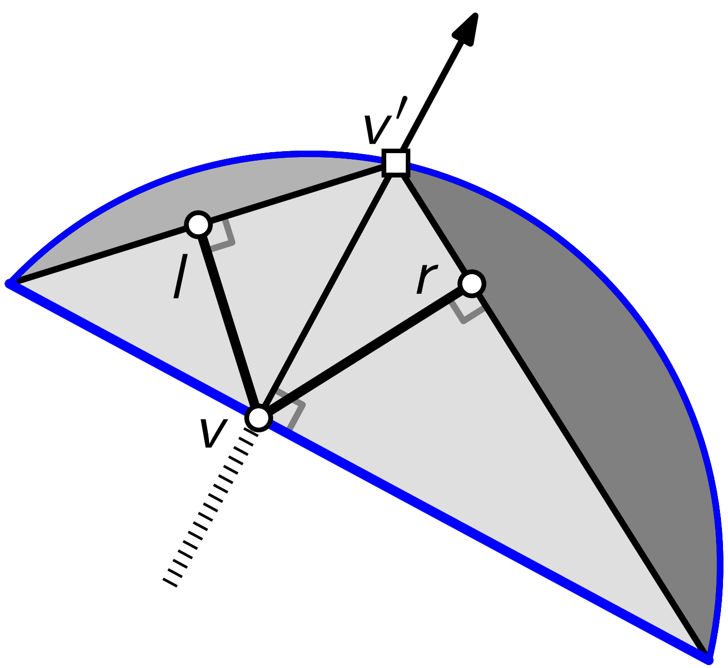

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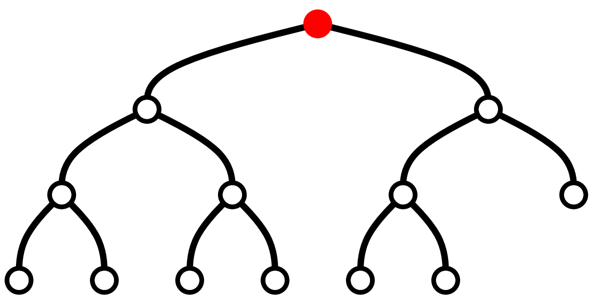

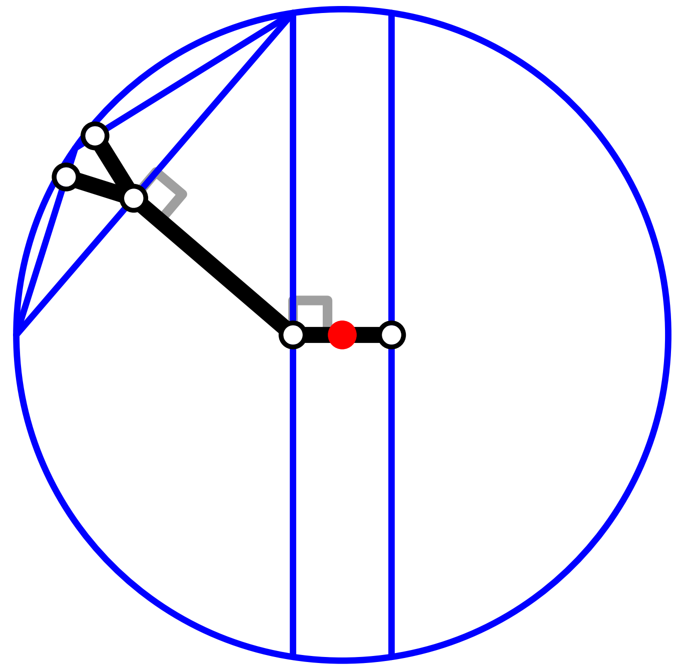

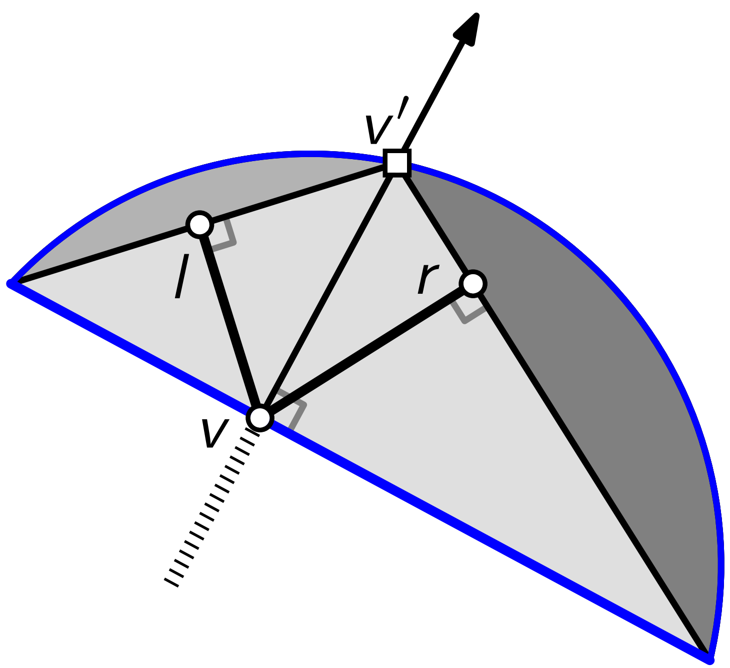

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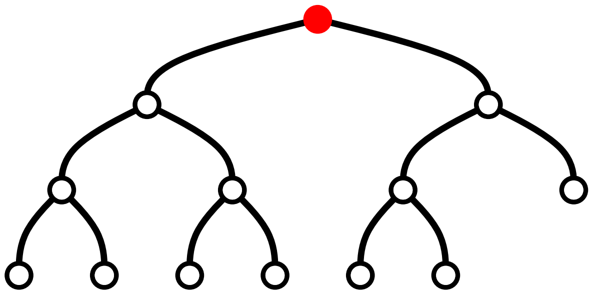

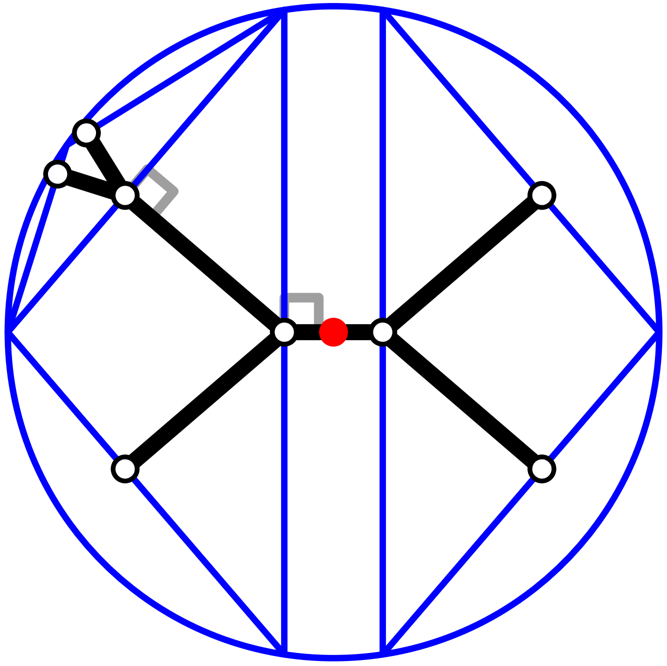

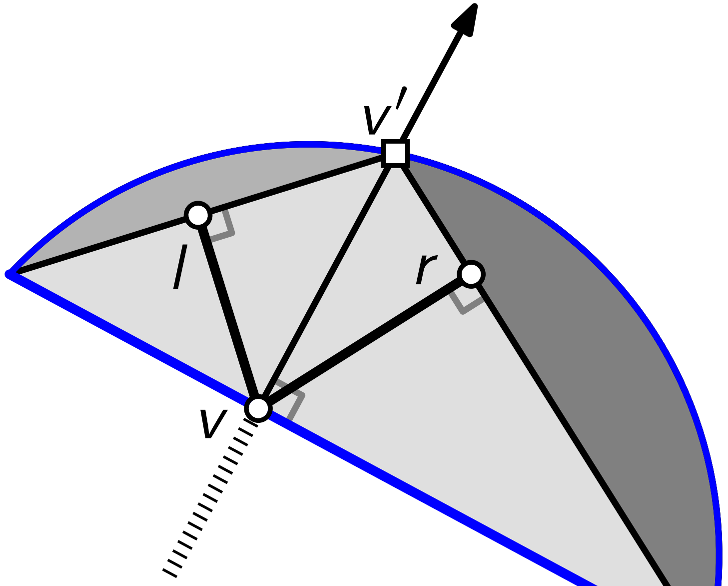

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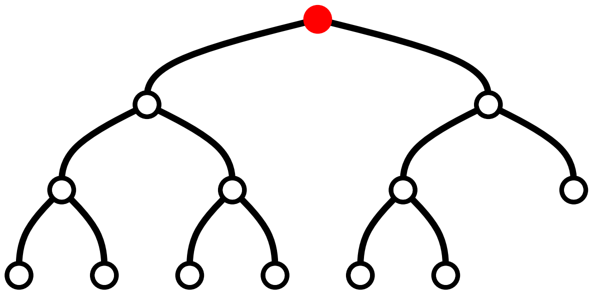

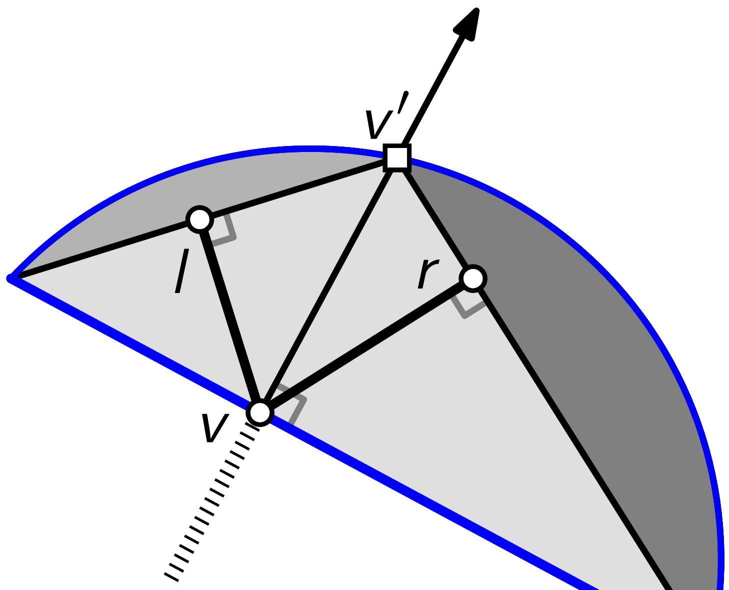

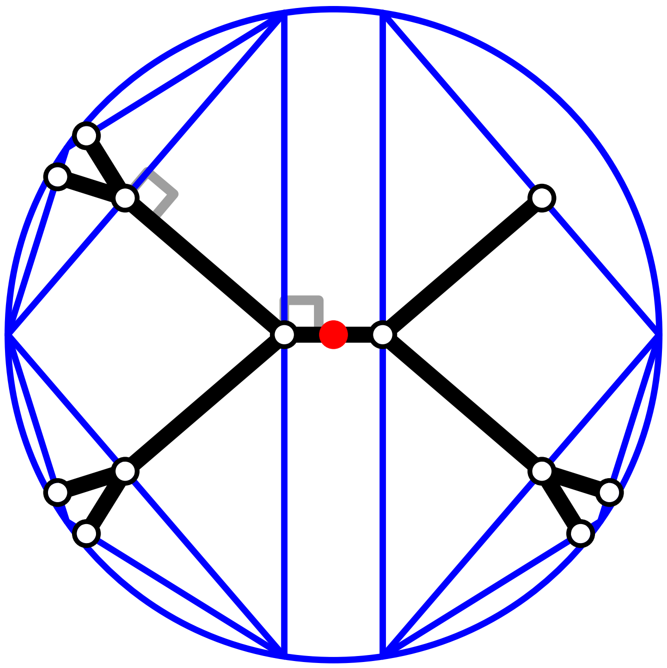

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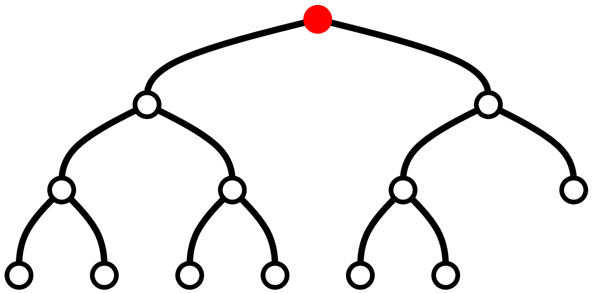

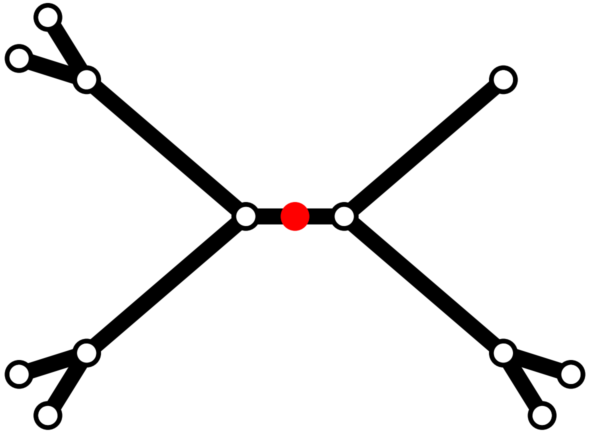

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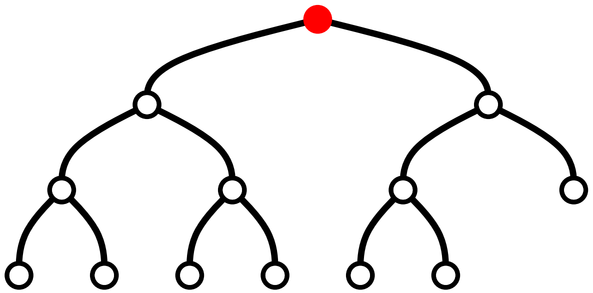

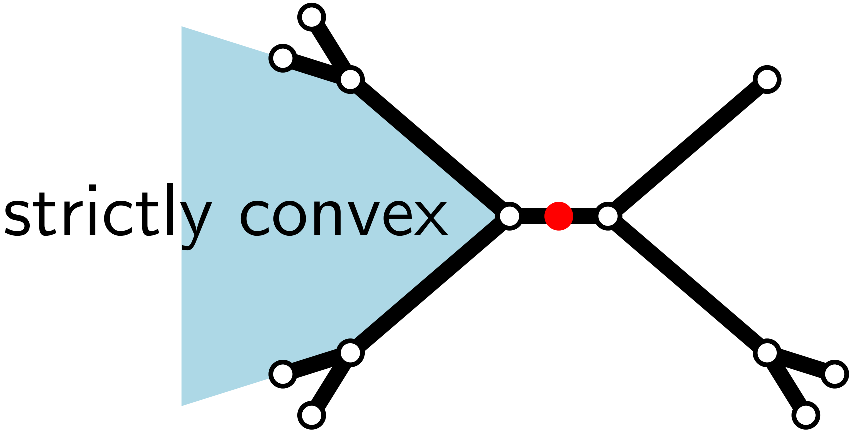

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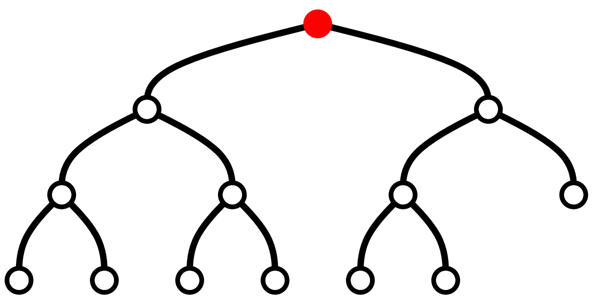

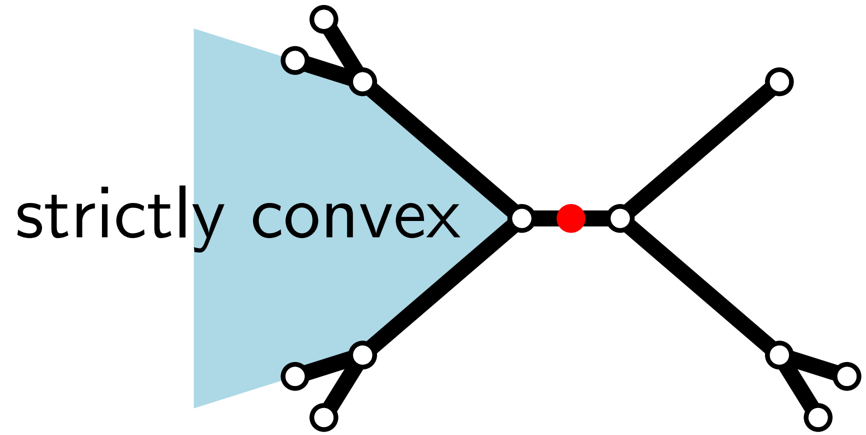

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Observation.
A u-v-path is *not* strongly monotone
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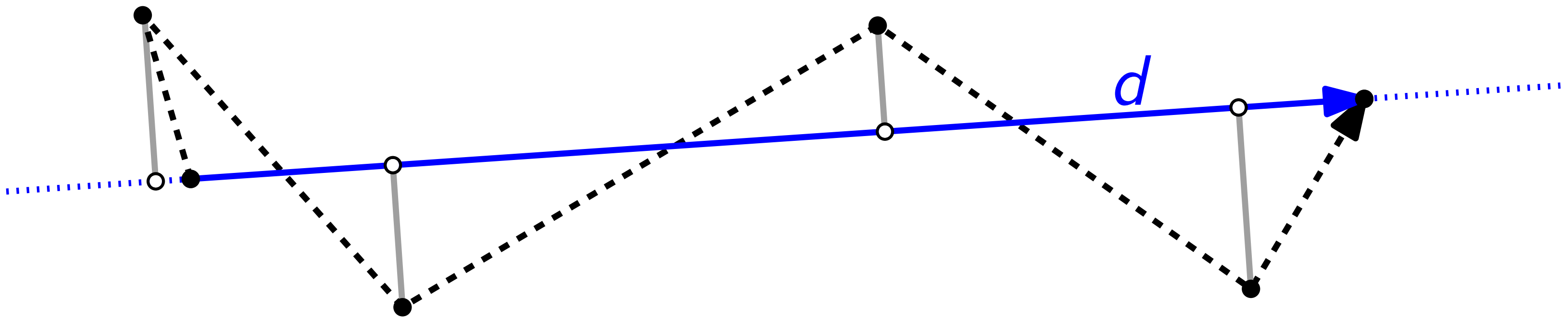

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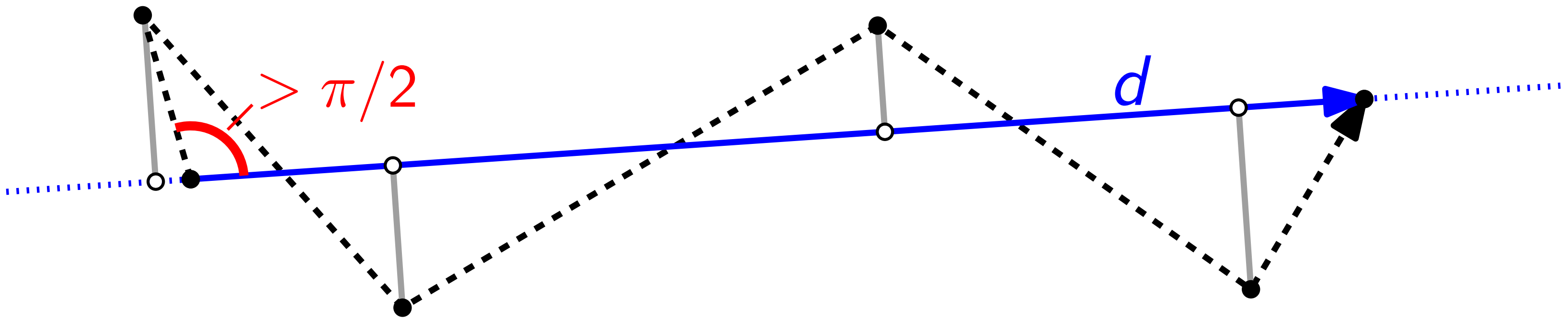

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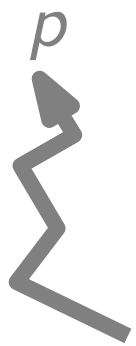

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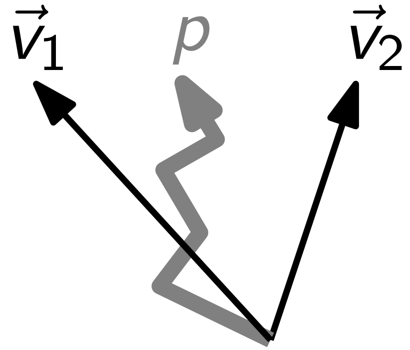

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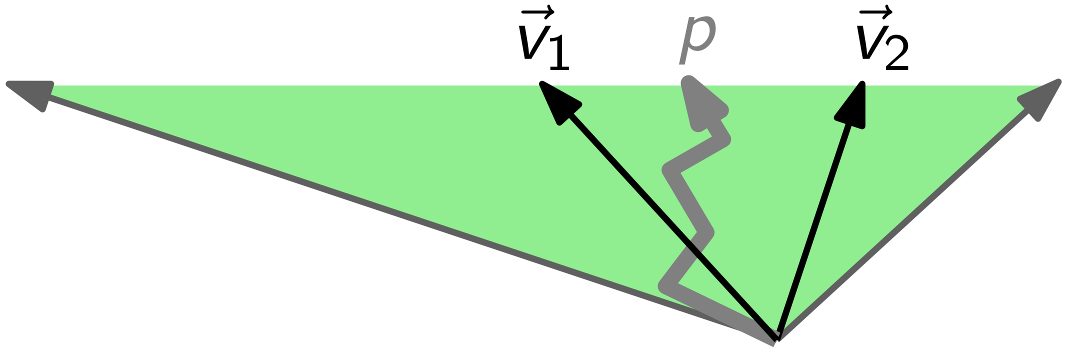

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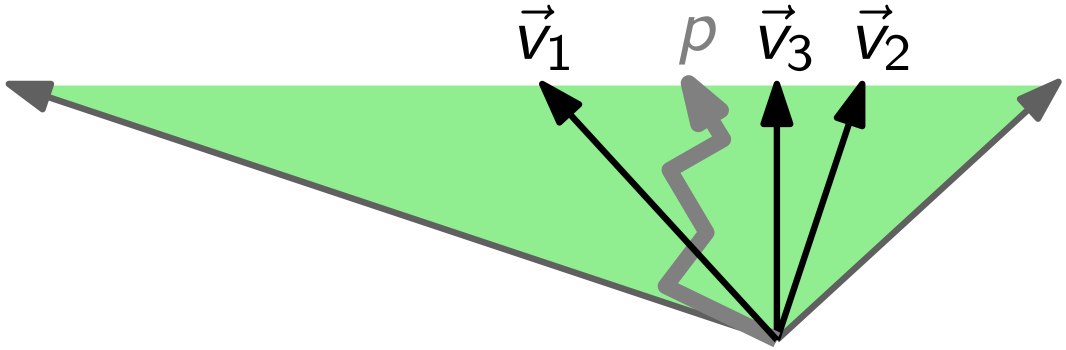
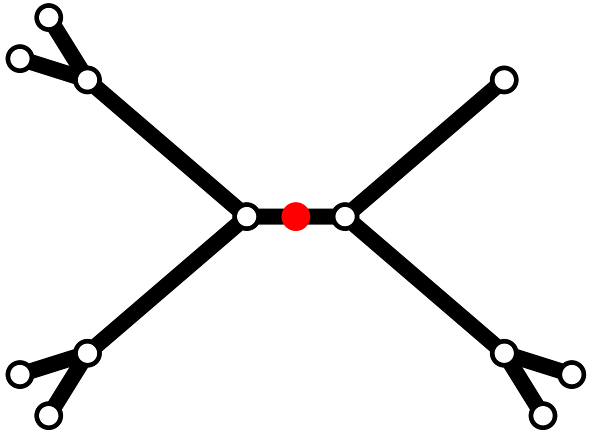


Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?



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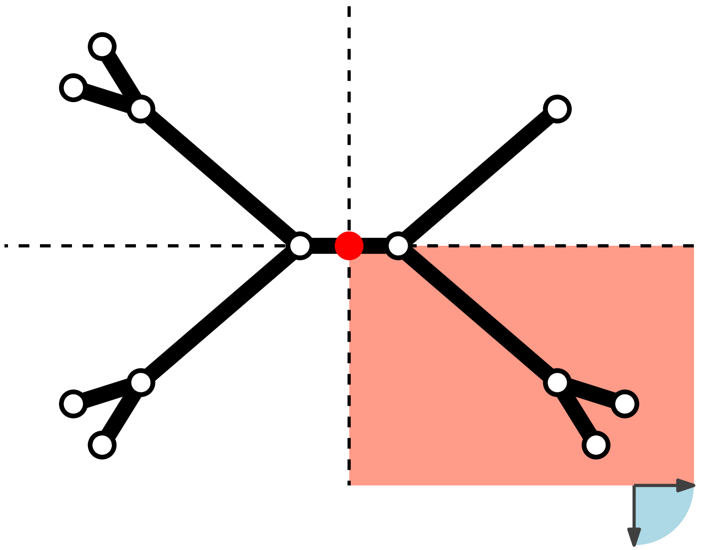

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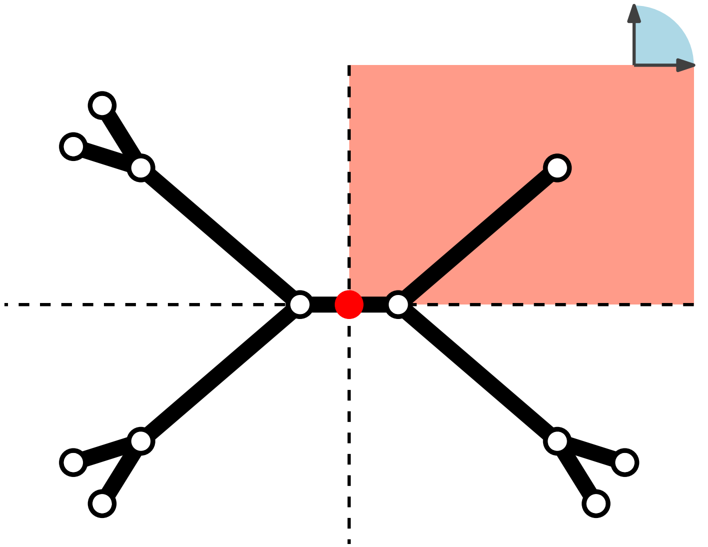

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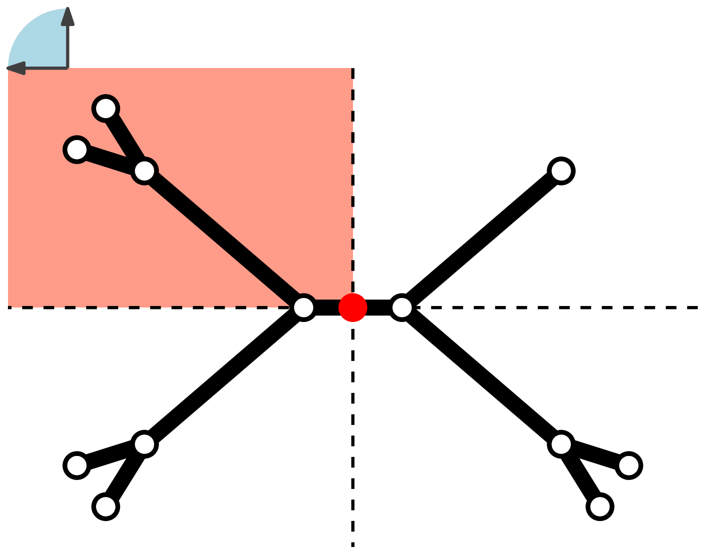

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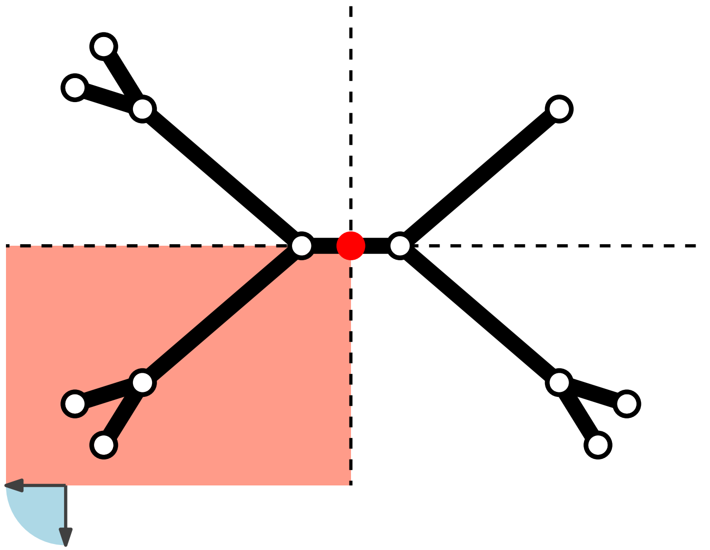

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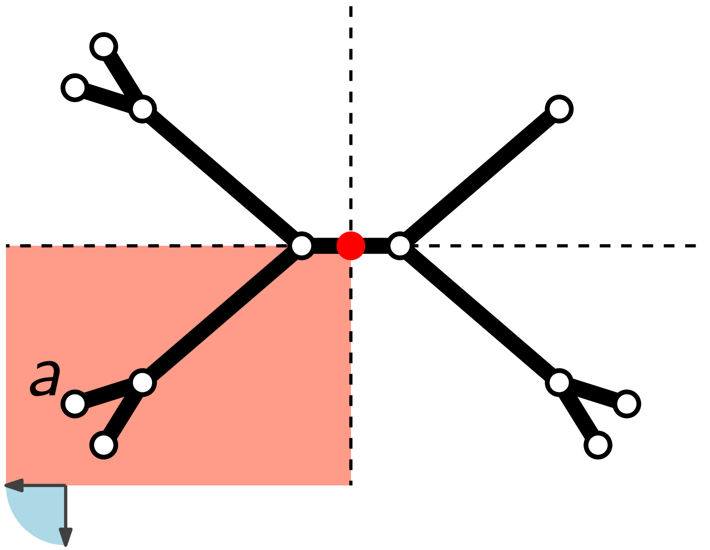

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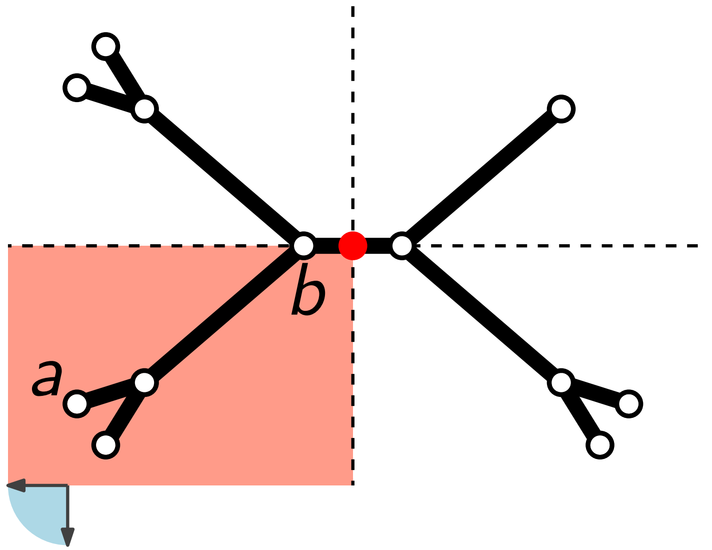

Case 1: a and b on common root-leaf path

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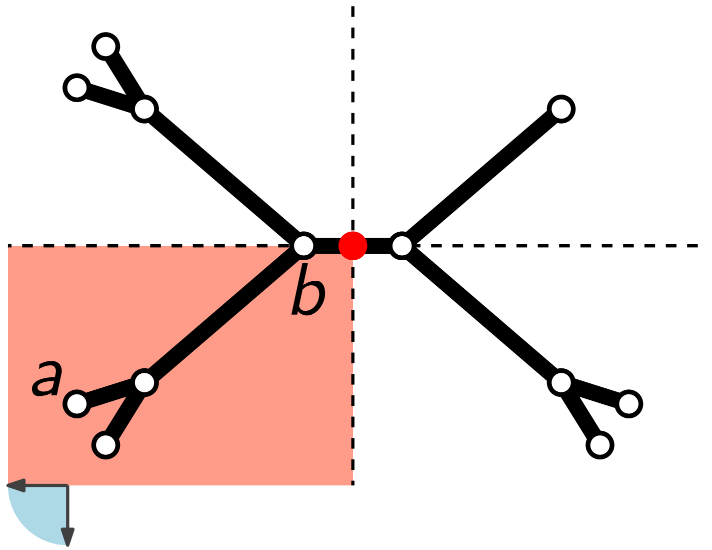
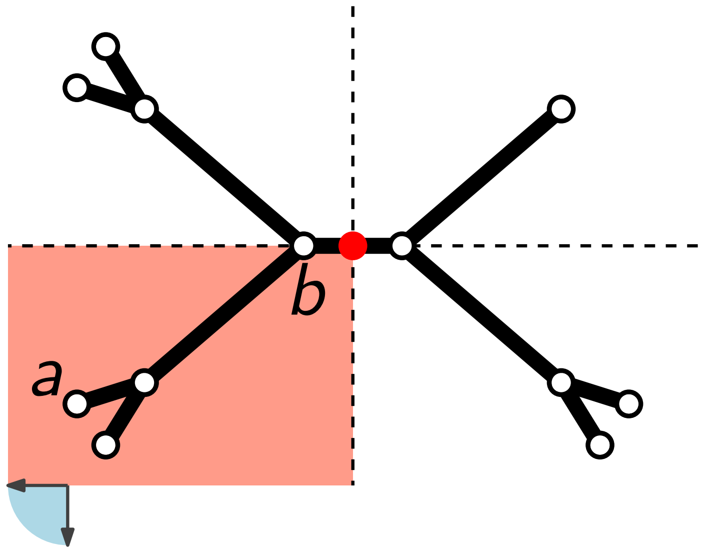


W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path

$$\Rightarrow \overrightarrow{ba} \in$$ 

Proper Binary Trees



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W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path

$\Rightarrow \overrightarrow{ba} \in$, path from b to $a \in$

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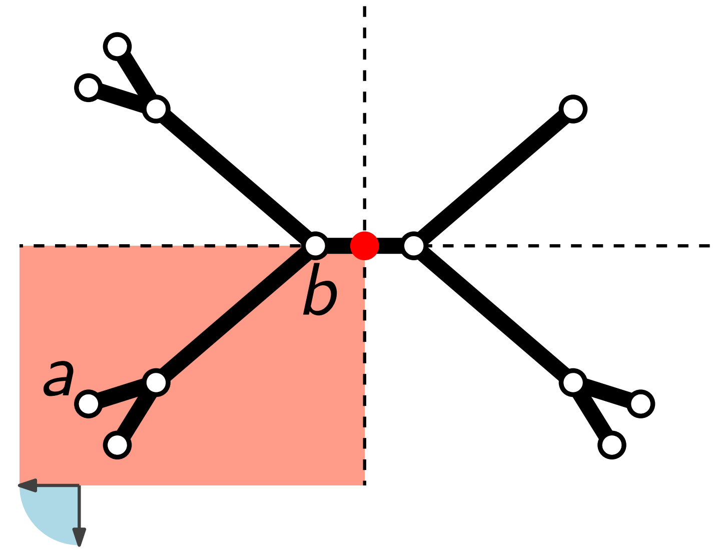

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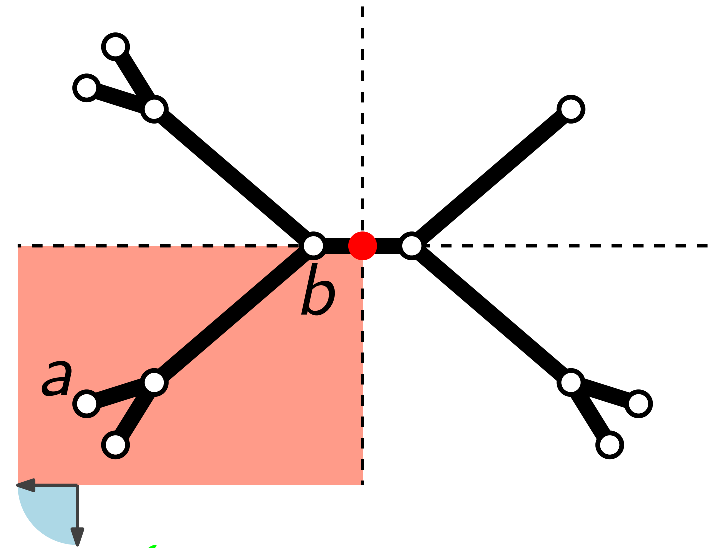
W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path

$\Rightarrow \overrightarrow{ba} \in$, path from b to $a \in$

$\Rightarrow$ path from b to a is strongly monotone

Proper Binary Trees



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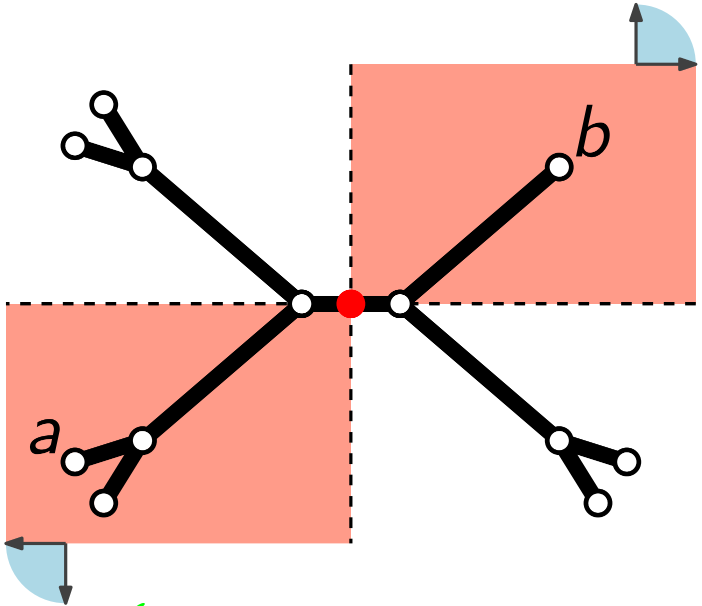
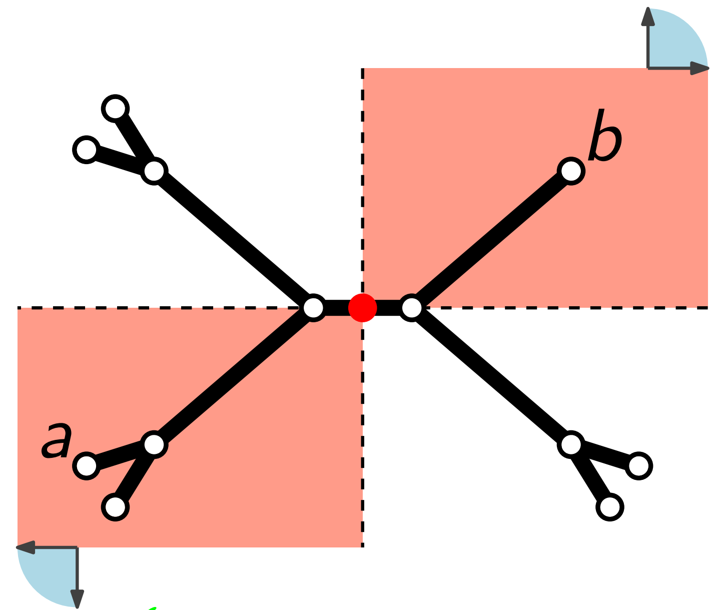


W.l.o.g. assume *a* lies bottom-left

Case 1: *a* and *b* on common root-leaf path ✓

Case 2: *a* and *b* in opposite sectors

Proper Binary Trees



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Case 1: a and b on common root-leaf path ✓

Case 2: a and b in opposite sectors ✓

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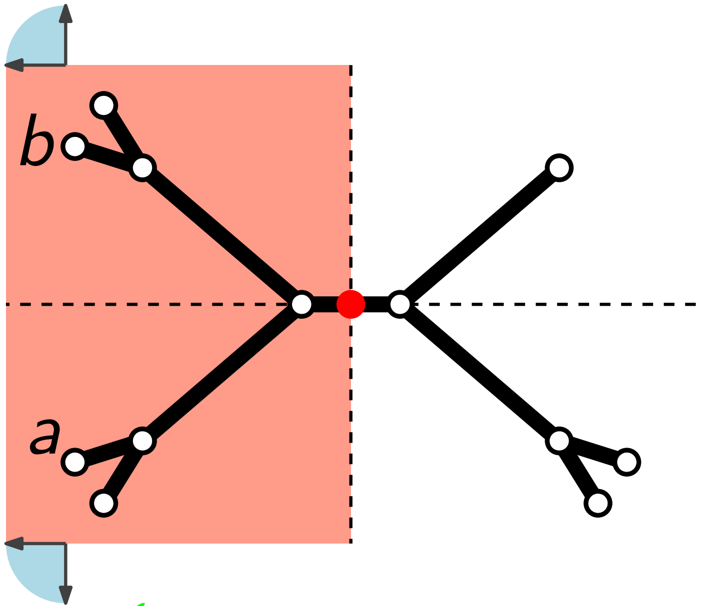
Case 1: a and b on common root-leaf path ✓

Case 2: a and b in opposite sectors ✓

Case 3: else



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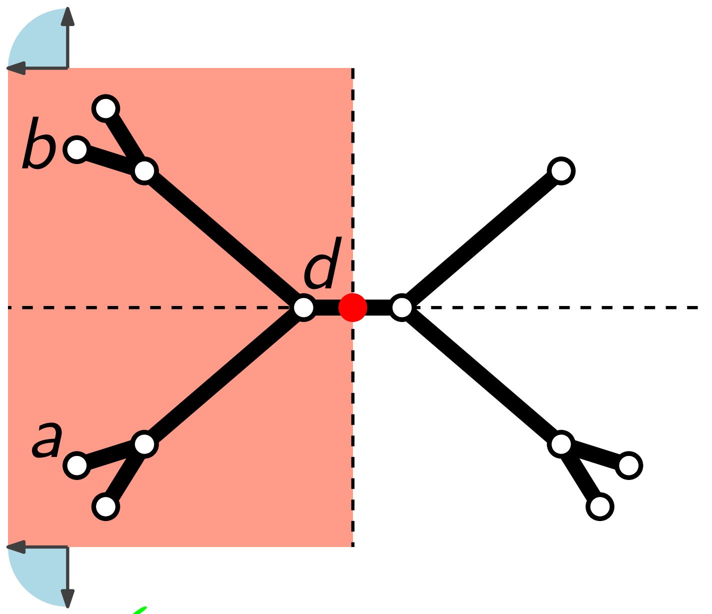

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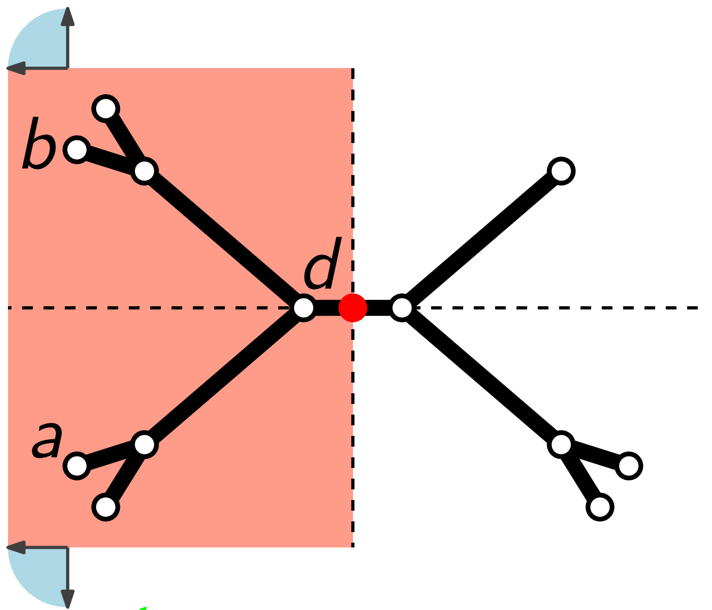

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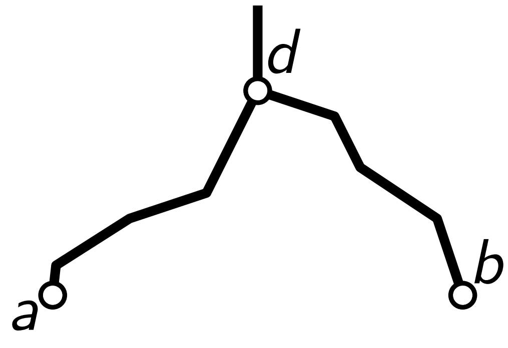

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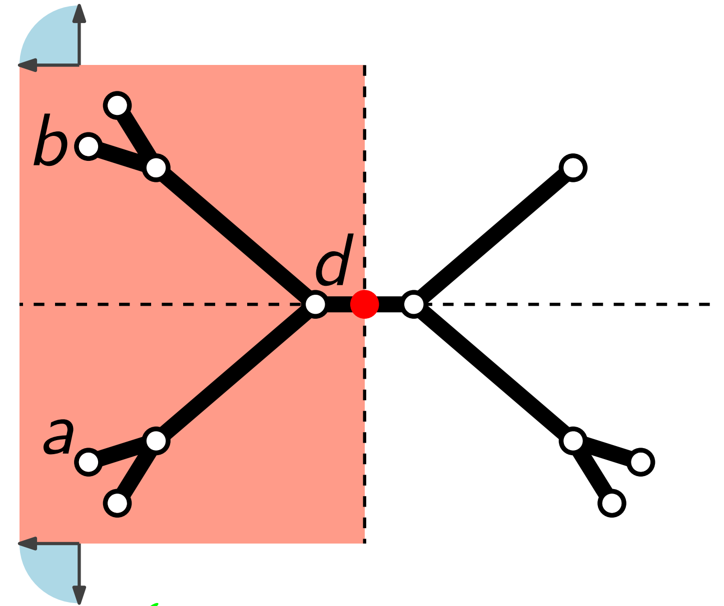

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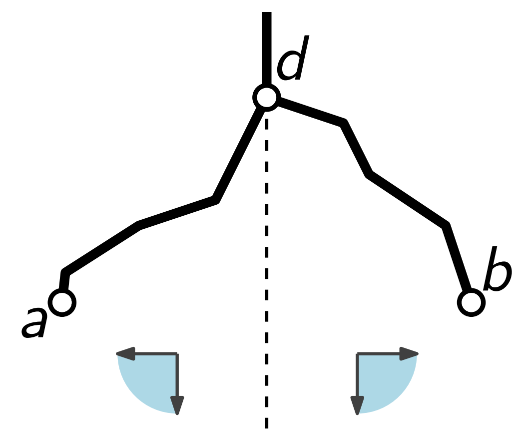

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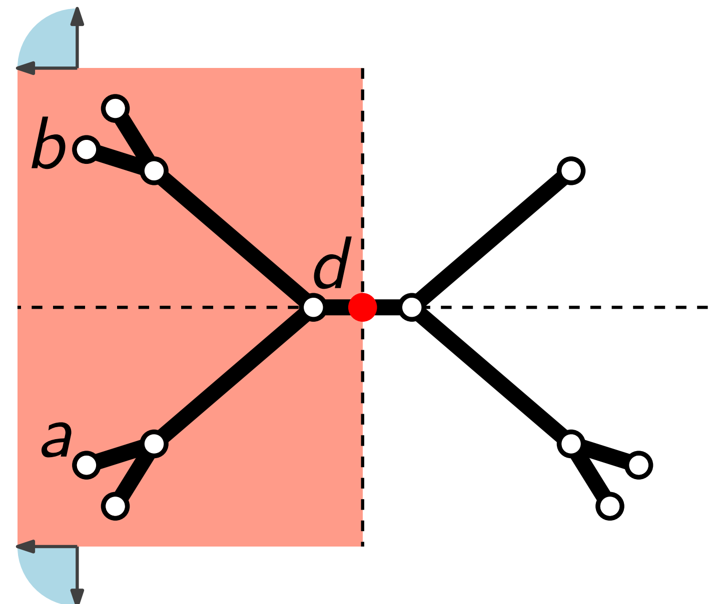

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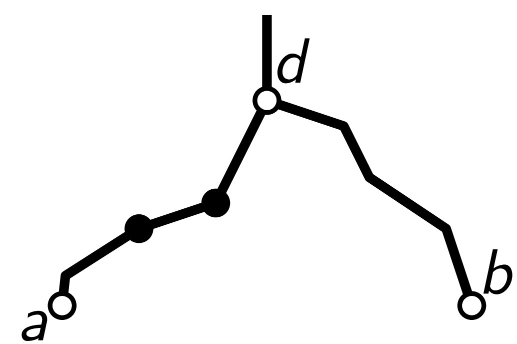
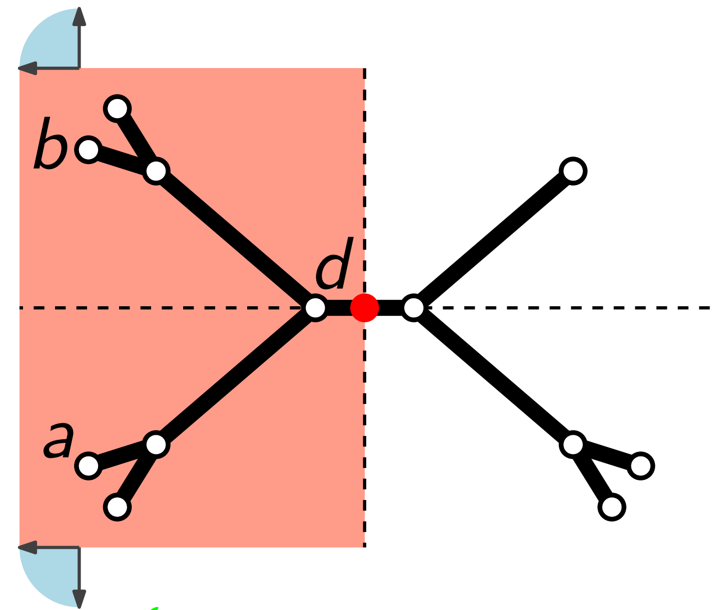


Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?



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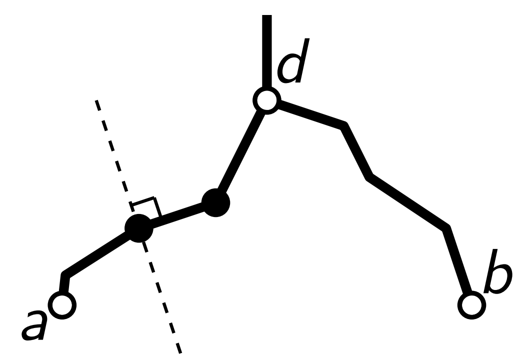

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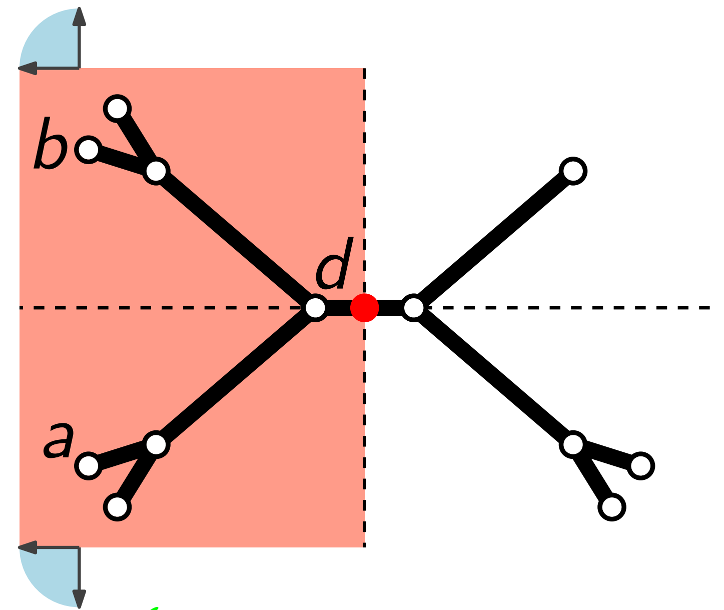

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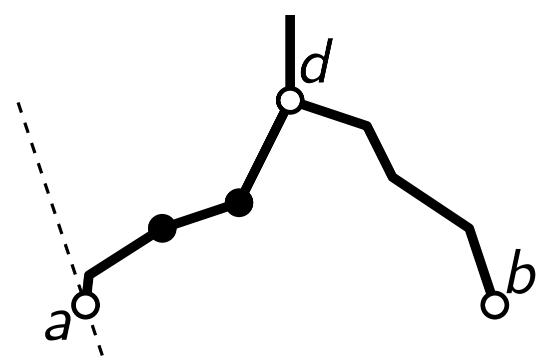

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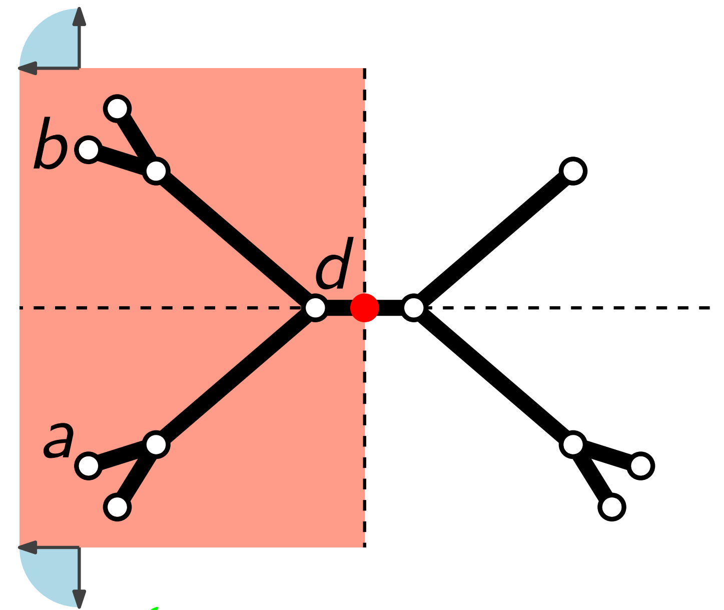

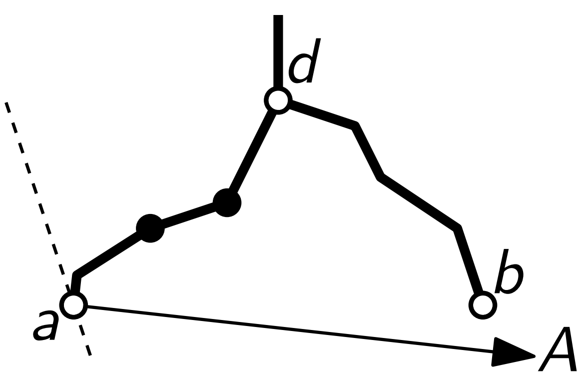

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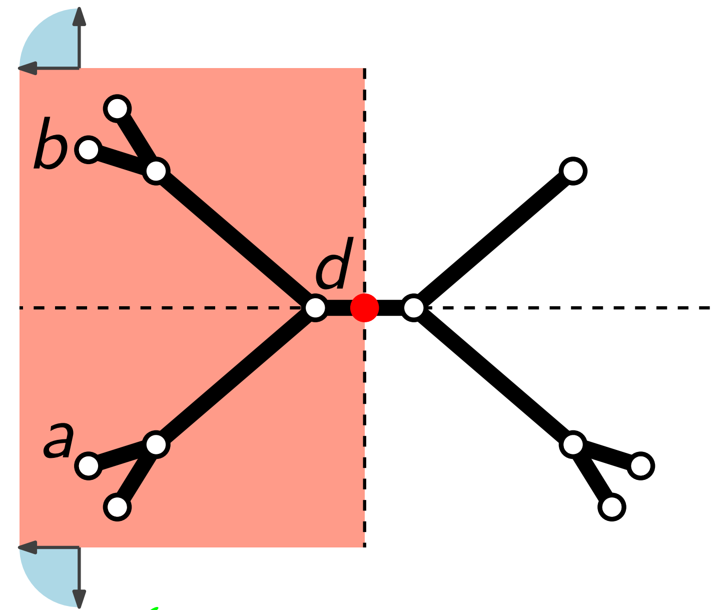

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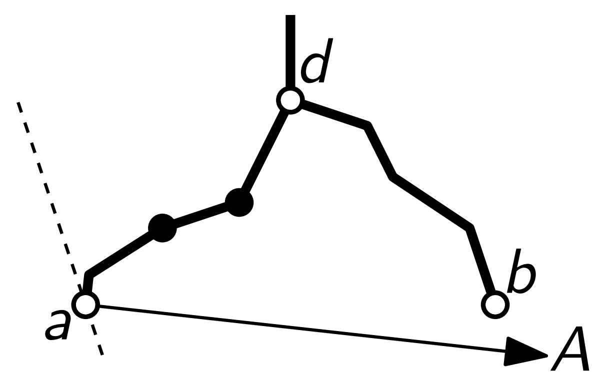
a-d-path monotone to A



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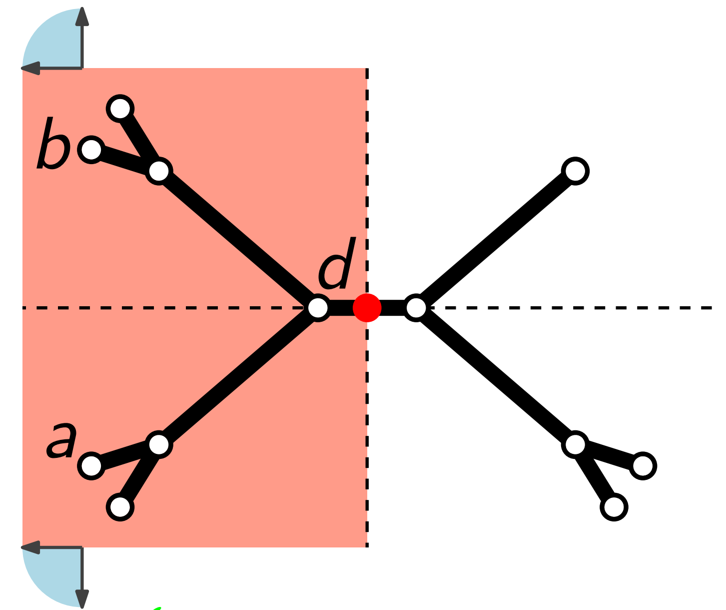

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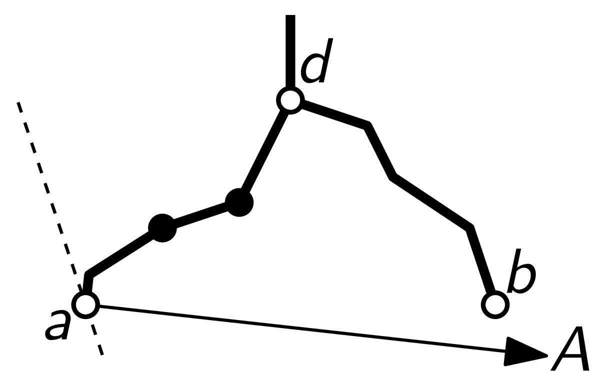

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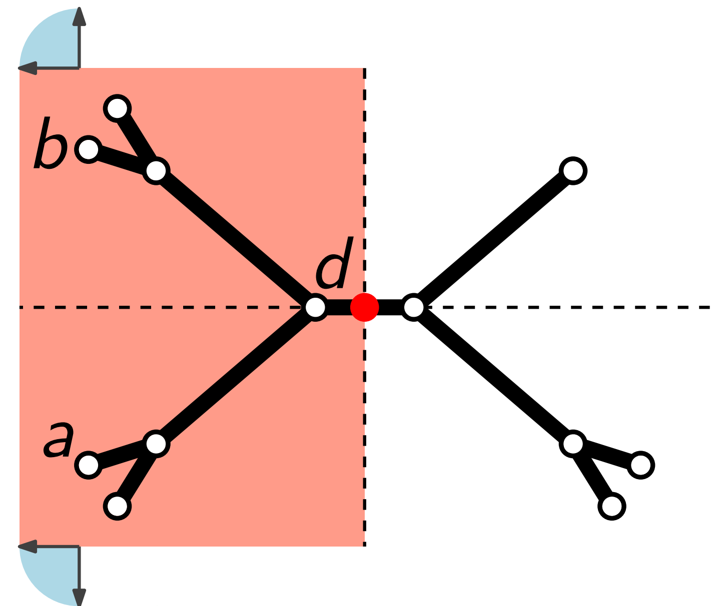

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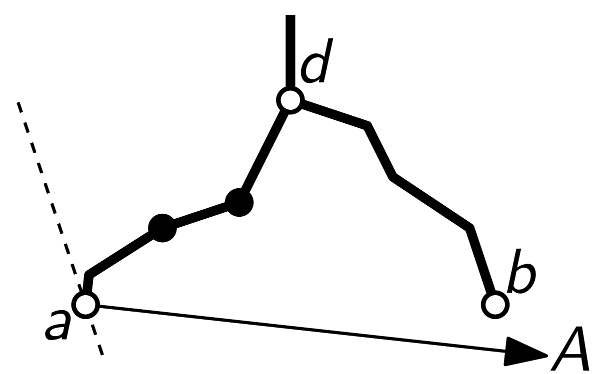
$\Rightarrow$ a-b-path monotone to A



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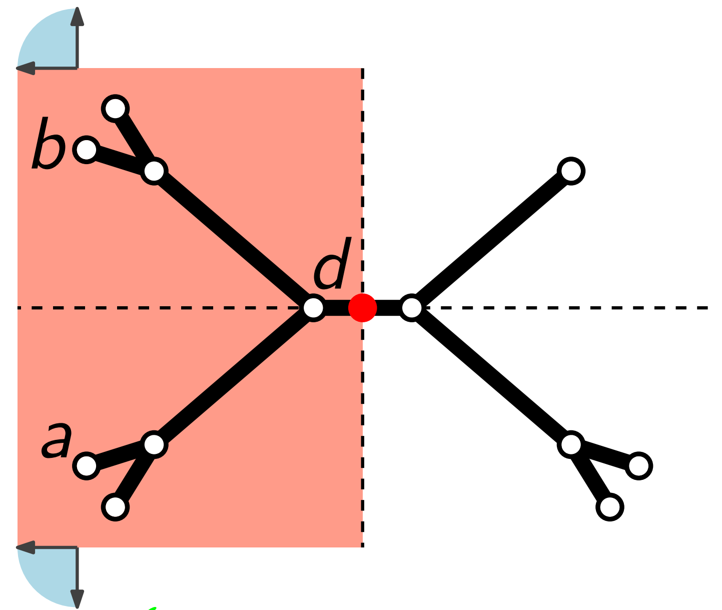

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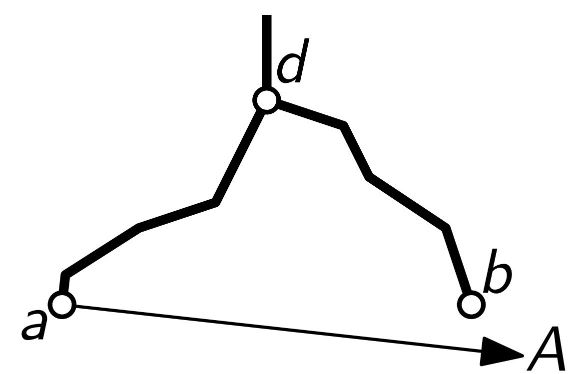

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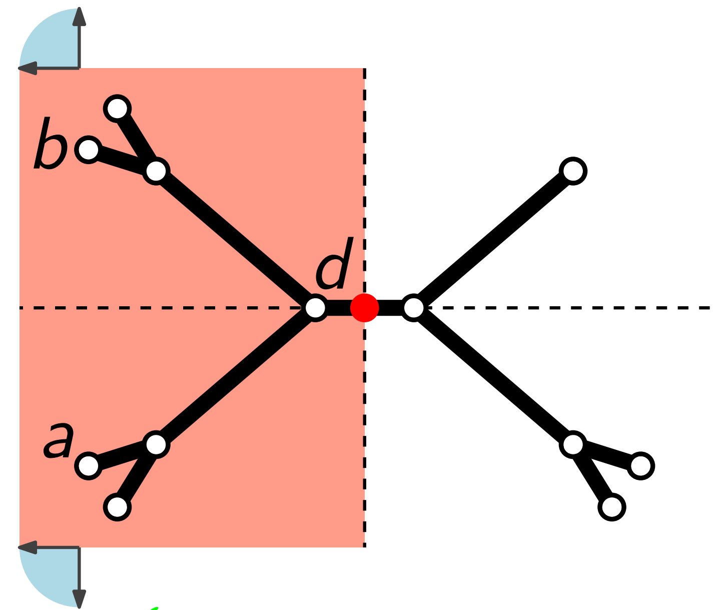

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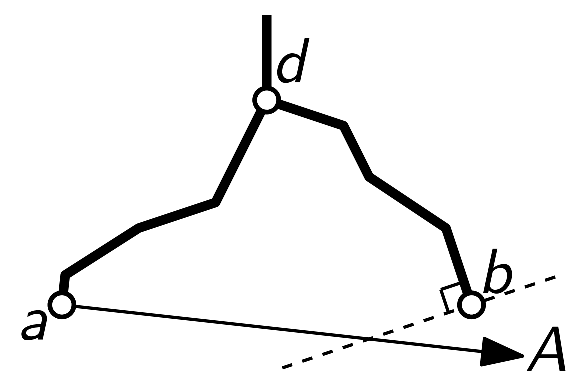

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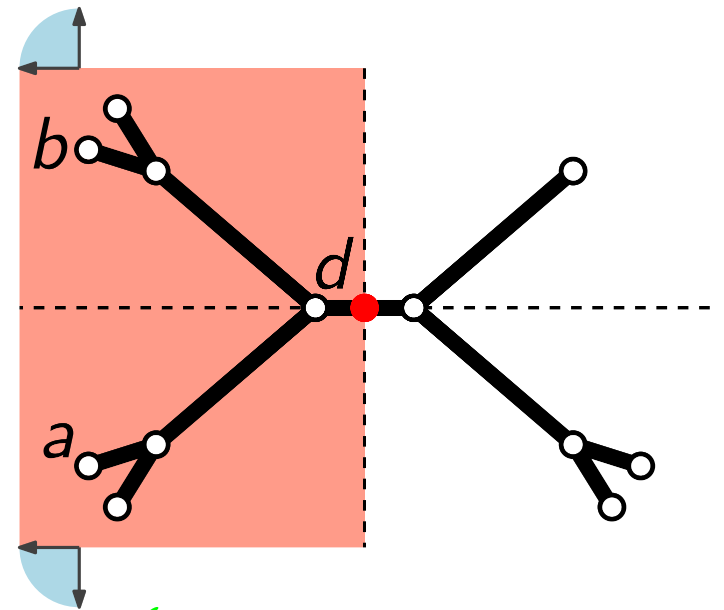

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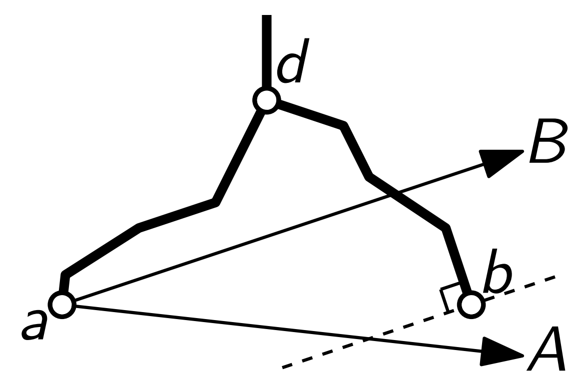

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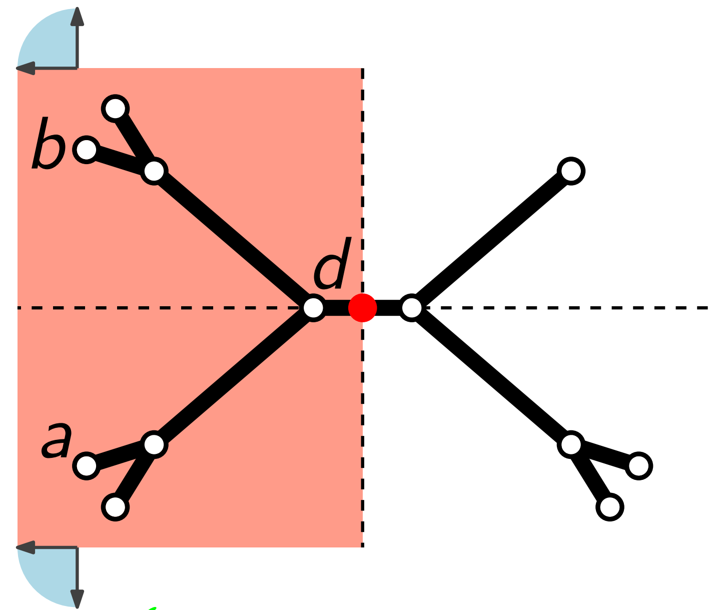

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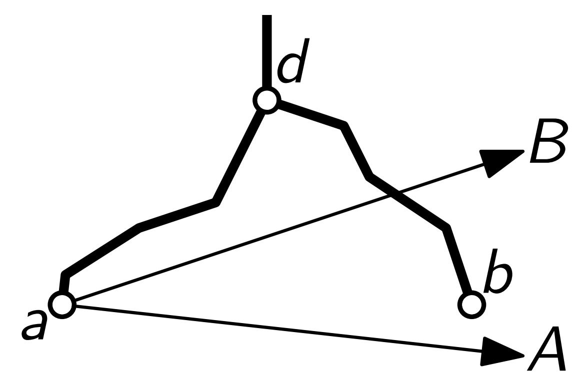

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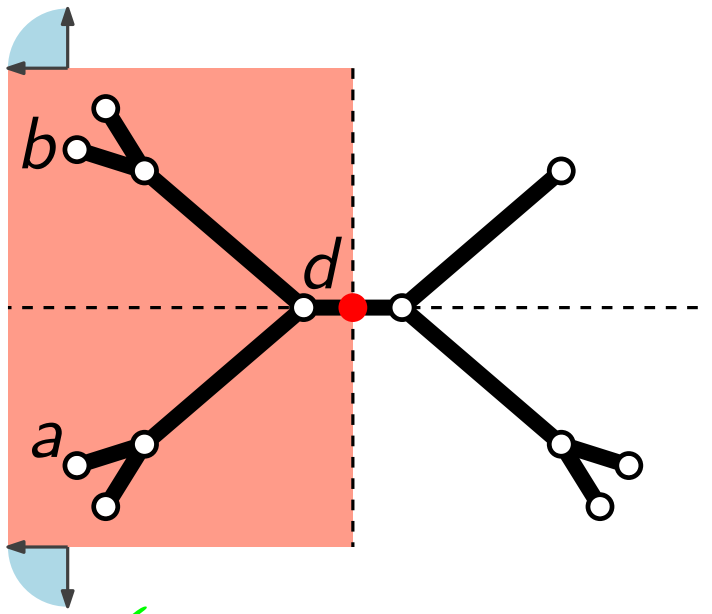

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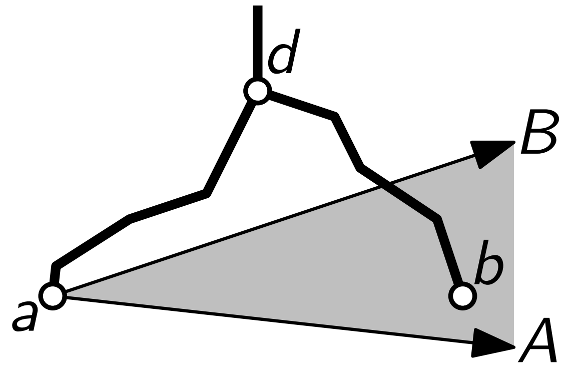

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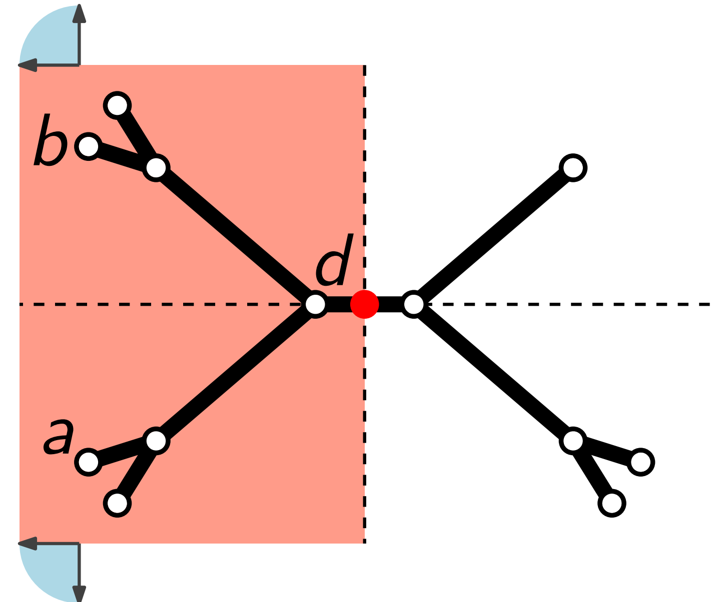

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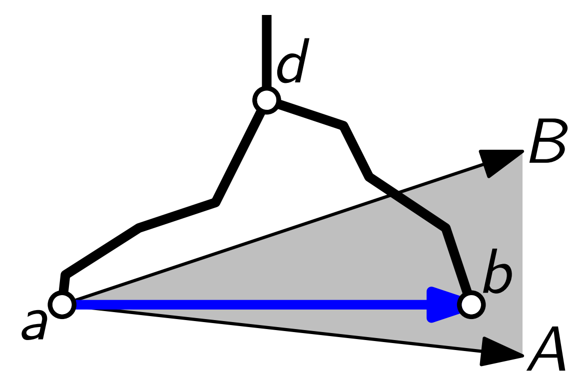

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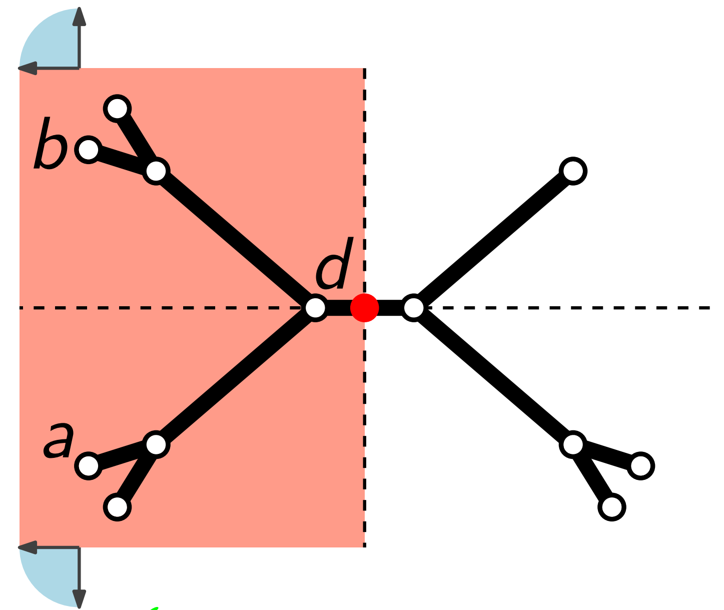

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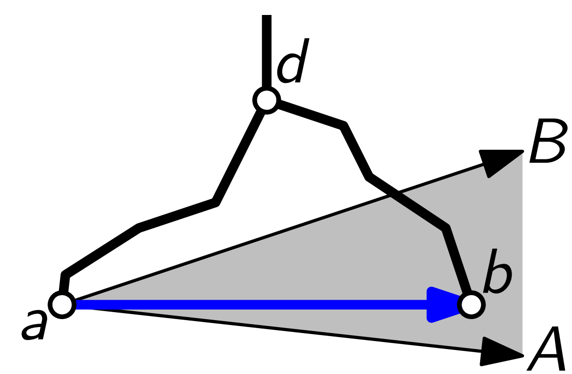
a-b-path *strongly* monotone



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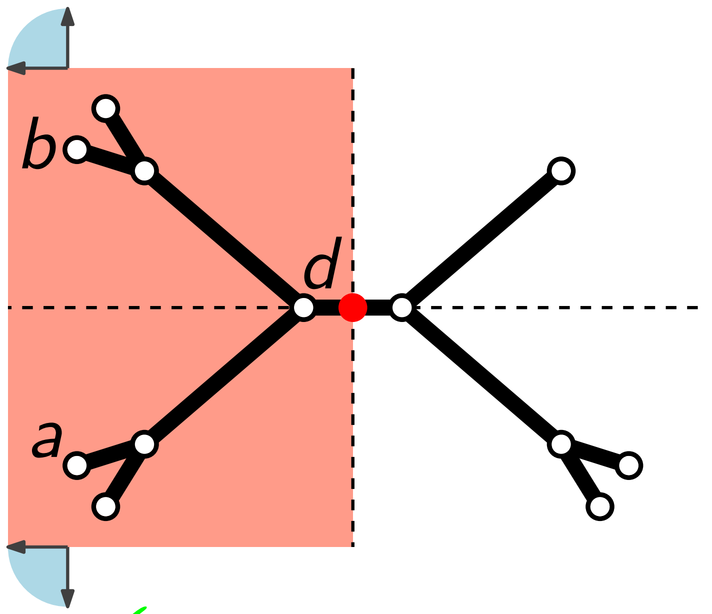

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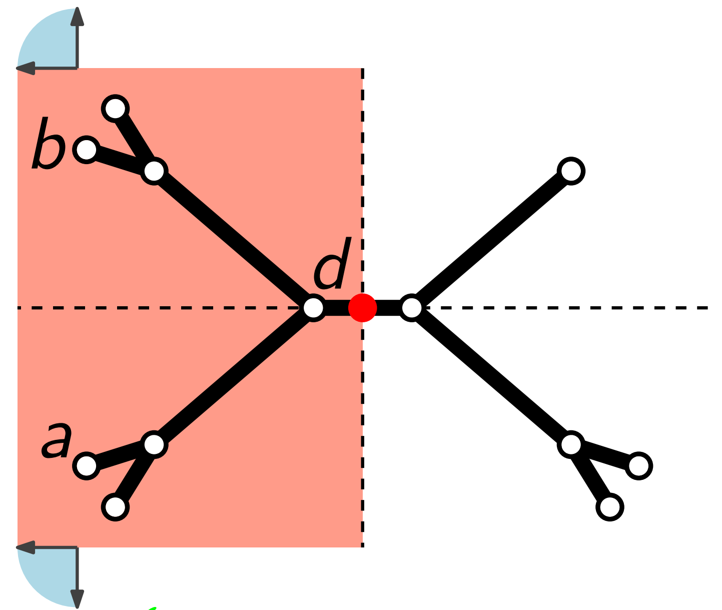

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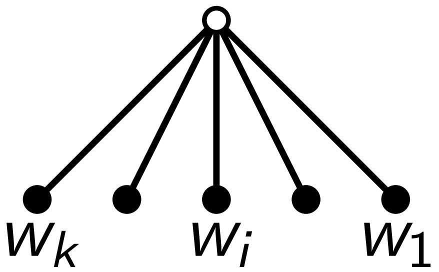
Theorem.
Any proper binary tree has a strongly monotone and strictly convex drawing.

General Trees



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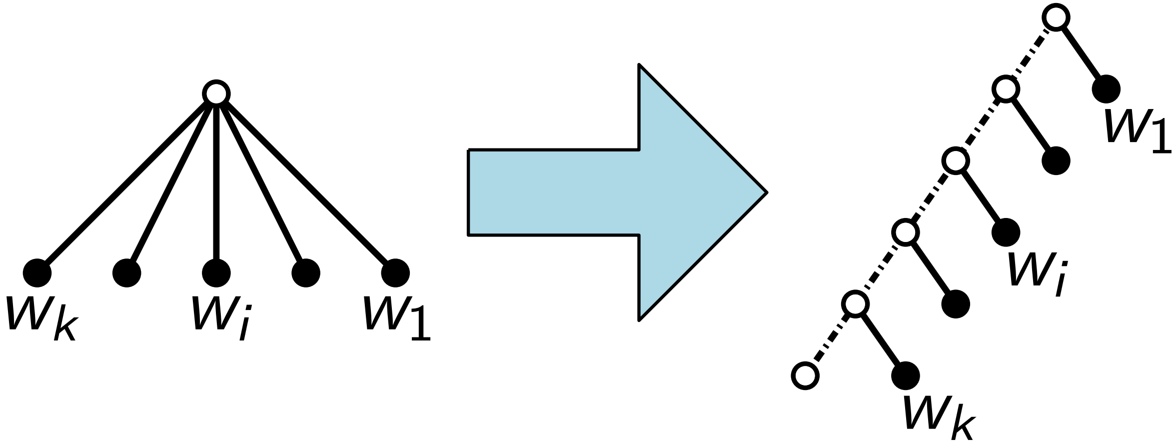
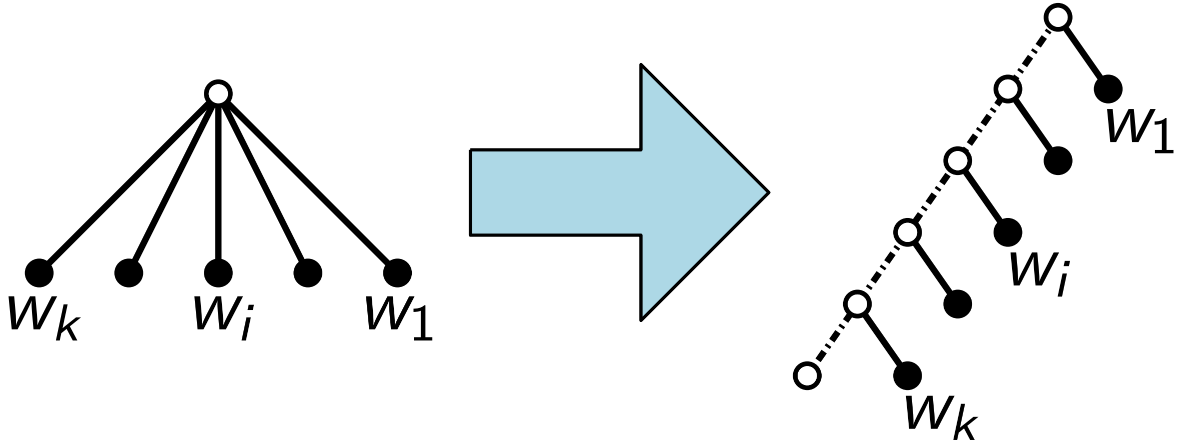
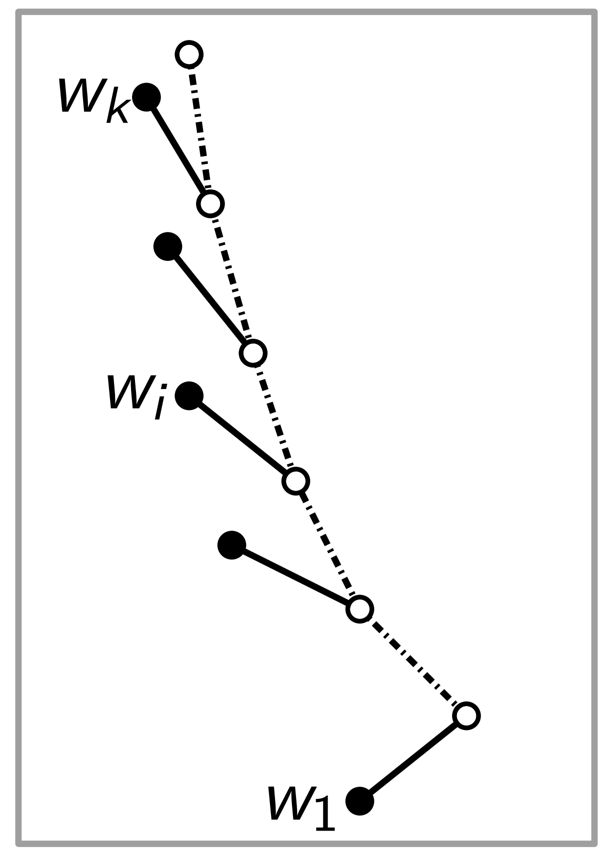


1. Substitute high-degree vertices by paths

General Trees



1. Substitute high-degree vertices by paths
2. Draw Proper Binary Tree



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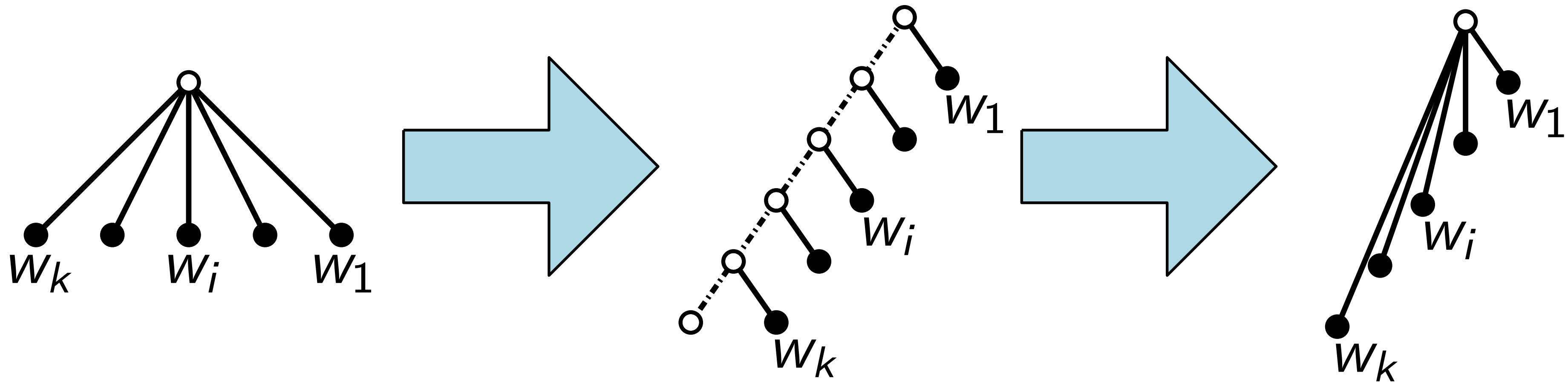
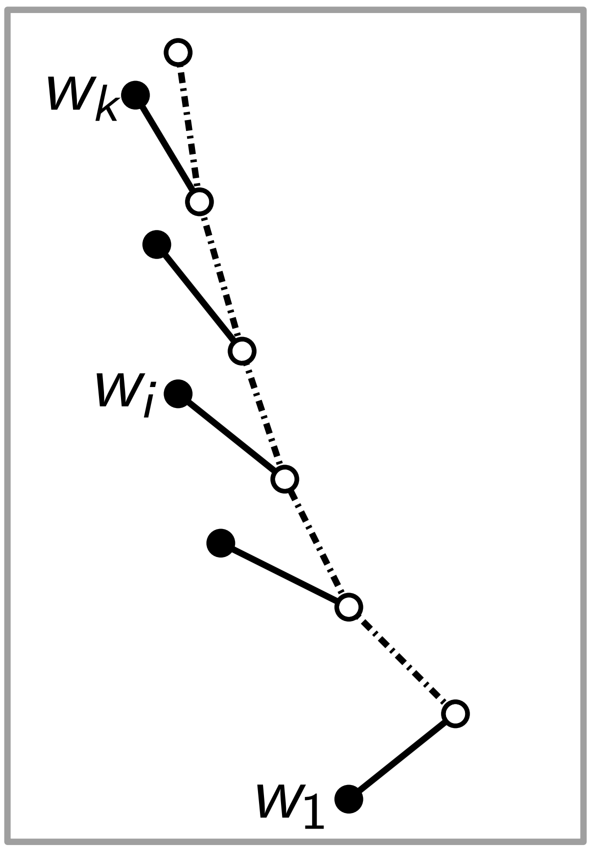


1. Substitute high-degree vertices by paths
2. Draw Proper Binary Tree
3. Shortcut edges



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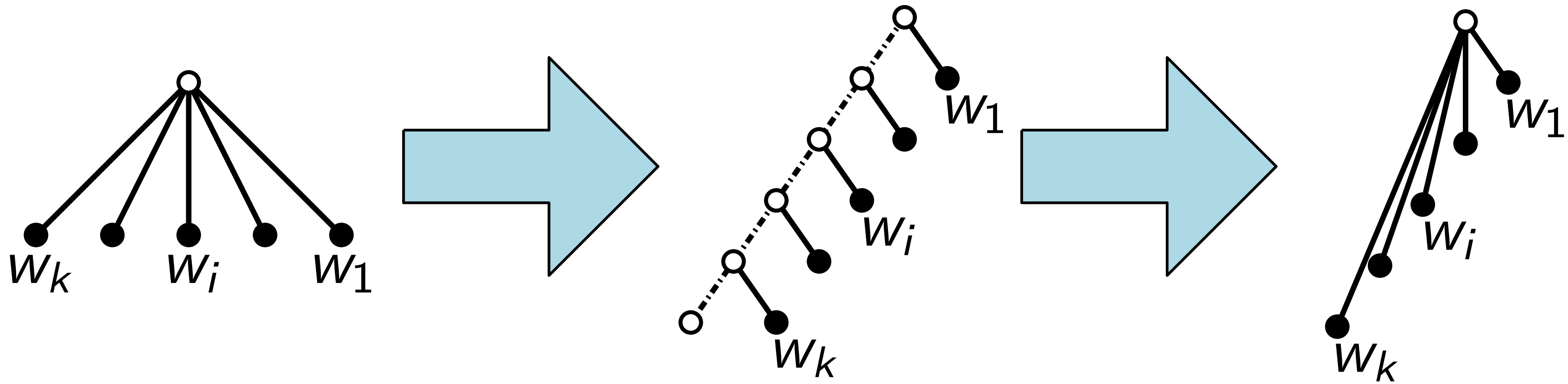

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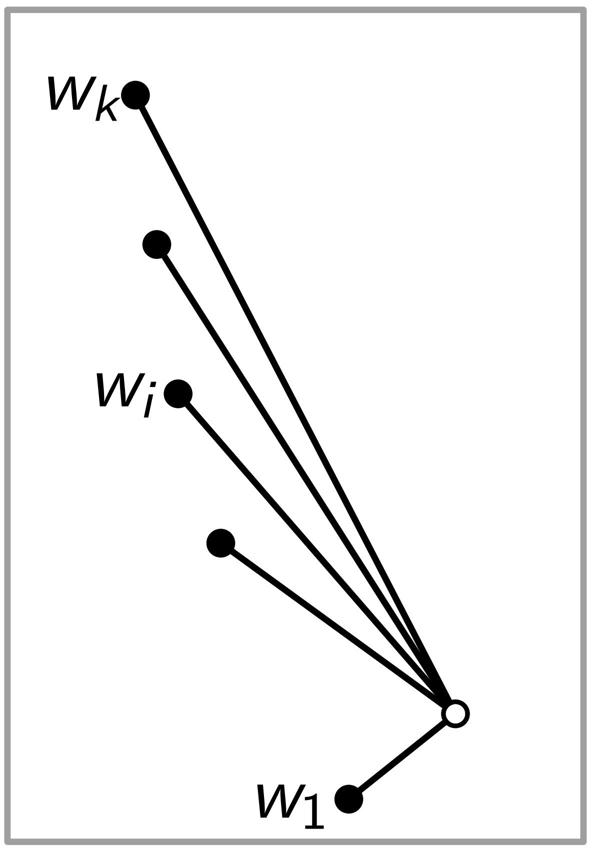

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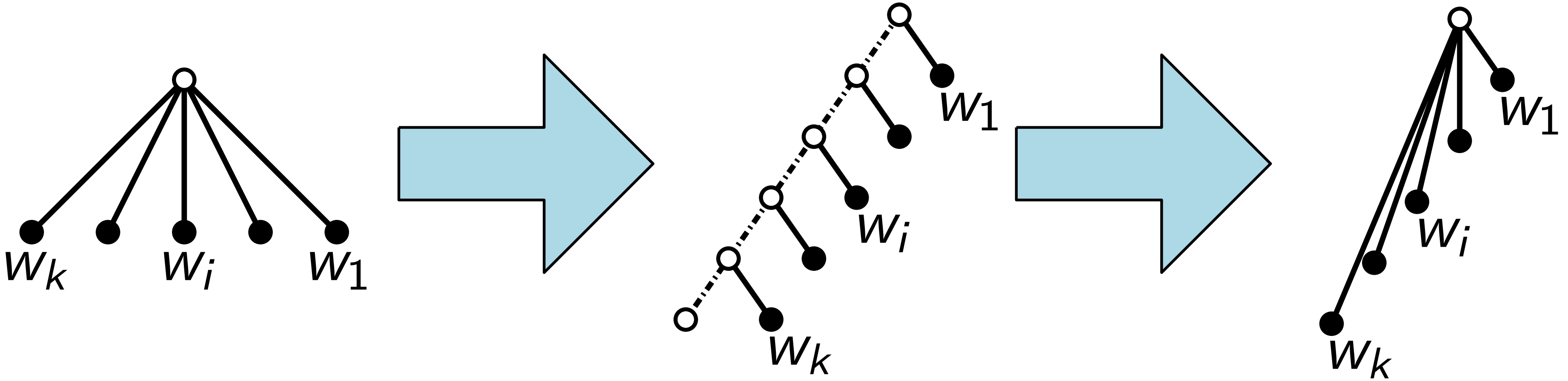

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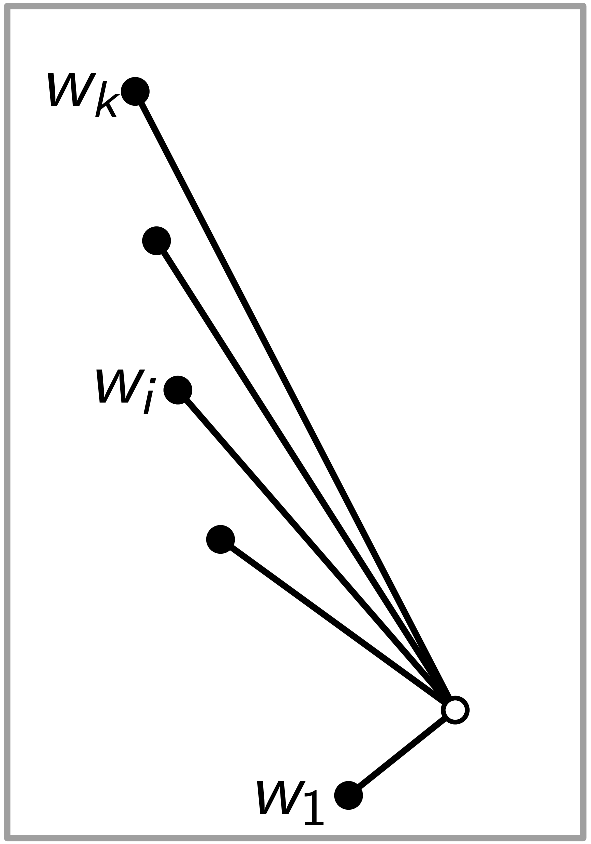
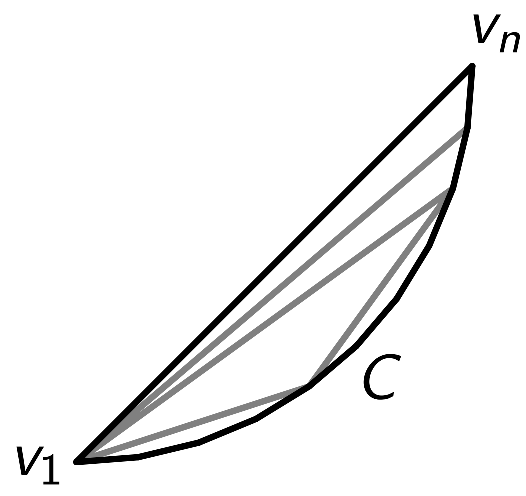


Theorem.
Any tree has a strongly monotone drawing.

Planar Graphs

Theorem.
Any biconnected outerplanar graph has a strongly monotone and strictly convex drawing.



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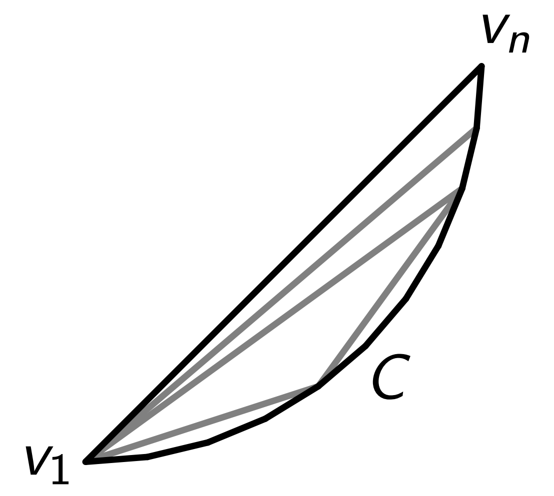
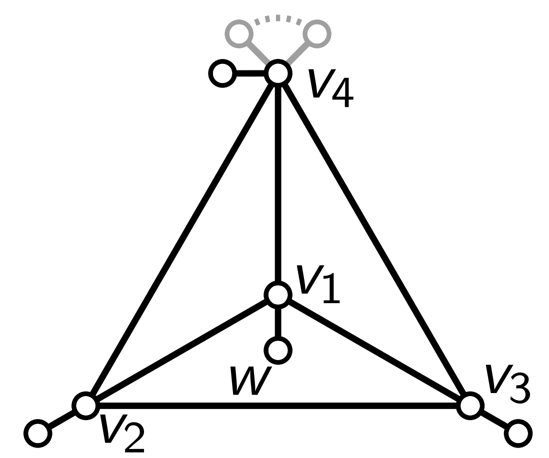


Theorem.
There is an infinite family of connected planar graphs that do not have a strongly monotone drawing in any combinatorial embedding.



Open Problems

- Does any tree have a strongly monotone drawing on a grid of polynomial size?

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- Is there a triconnected (or biconnected) planar graph that does not have any strongly monotone drawing?
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- Are our drawings for general trees also convex?
  If yes, then all Halin graphs would automatically have convex and strictly monotone drawings, too.